2

SUPERVISOR'S USE ONLY

91267

912670

NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 2 Mathematics and Statistics, 2015

91267 Apply probability methods in solving problems

2.00 p.m. Tuesday 10 November 2015
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability methods in solving problems.	Apply probability methods, using relational thinking, in solving problems.	Apply probability methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Resource Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Achievement

TOTAL 12

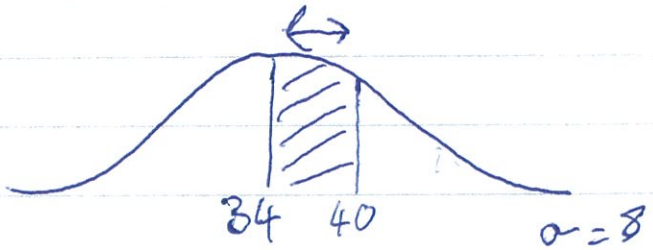
ASSESSOR'S USE ONLY

QUESTION ONE

ASSESSOR'S USE ONLY

(a) The waiting time for a patient attending a medical centre before seeing a doctor is approximately normally distributed, with a mean of 34 minutes and a standard deviation of 8 minutes.

(i) Find the probability that a patient will wait between 34 and 40 minutes.



$P = 0.273372$

Correct answer. GC used.

u

(ii) After how many minutes will 90% of patients have begun being seen by a doctor?

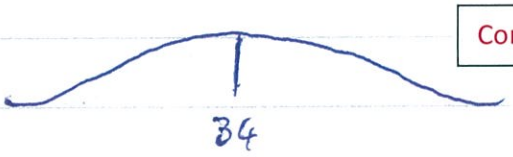
$x \text{ Inv} = $ ~~23.7475 minutes~~ 44.25 minutes

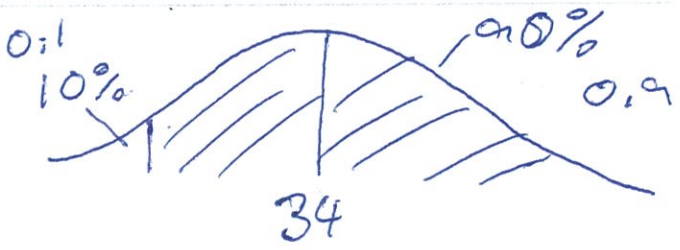
~~44.25 minutes~~

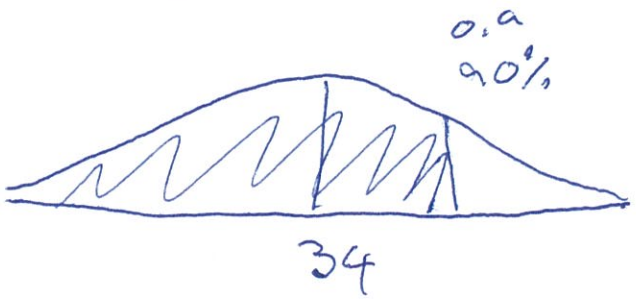
~~23.747~~

r

Correct answer. GC used.





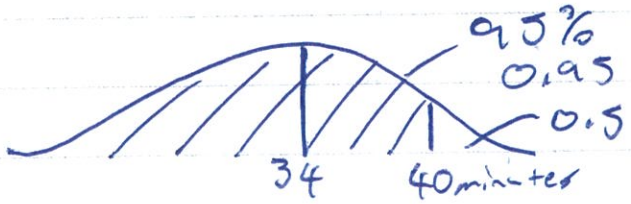


ASSESSOR'S USE ONLY

(iii) It is decided that waiting times must be changed so that at least 95% of patients will be seen by a doctor within 40 minutes.

Because of the administration required, the mean time cannot change, but it is known that for each doctor added to the duty teams, the standard deviation will reduce by 0.4 minutes.

How many doctors must be added to meet the new requirement?



$\sigma =$

$$z = \frac{x - \mu}{\sigma}$$

$z = 1.281$

$$\frac{x - 34}{8}$$

$1.281 = \frac{x - 34}{8}$

$(1.281 \times 8) + 34 = 44.248$

$x = 44.248$

44 more doctors would have to be added to meet the new requirement.

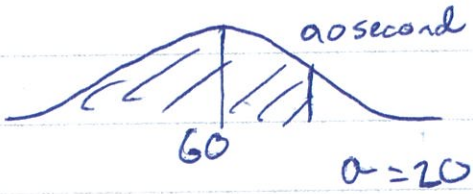
Incorrect z-value used.
Incorrect answer.

ASSESSOR'S USE ONLY

(b) At reception, patients are assessed on the urgency of their condition. This is done within two minutes of arrival.

It is thought that the waiting time before an assessment is done is approximately normally distributed with a mean of 60 seconds and standard deviation of 20 seconds.

(i) What proportion of patients would be assessed at reception within 90 seconds of arrival?



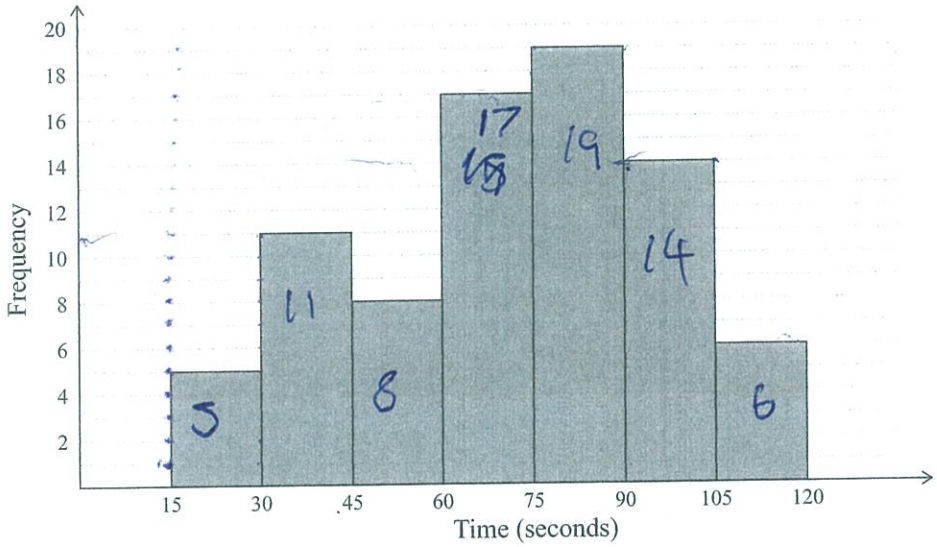
$P = 0.9331$

Correct answer. GC used.

u

(ii) A survey is carried out on 80 patients who arrive at reception. Patients are selected at random on a particular day. The results are shown in the frequency histogram below.

Assessment time at reception



What proportion of patients in the survey were assessed at reception within 90 seconds of arrival?

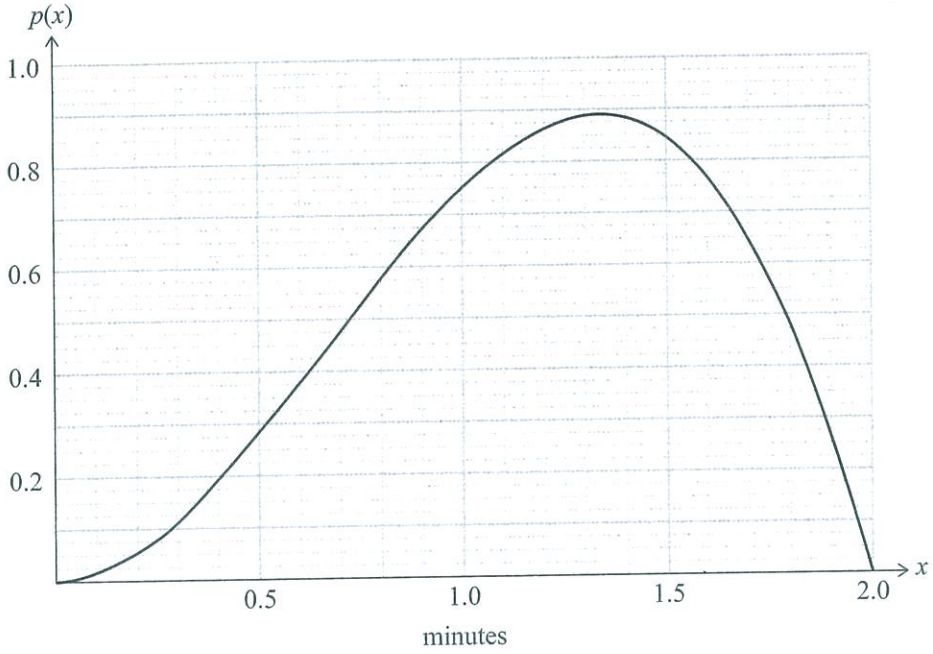
$\frac{60}{80} = \frac{3}{4}$ or 0.75

Correct answer.

u

ASSESSOR'S USE ONLY

(iii) A statistician states that the assessment times are not normally distributed, but are more likely to approximate the distribution $p(x)$ below.



The associated probabilities (with minutes converted to seconds) are given in the following table:

Assessment Time (seconds)	0 –	15 –	30 –	45 –	60 –	75 –	90 –	105 – 120
Probability	0.01	0.05	0.10	0.16	0.21	0.22	0.17	0.08

Compare the frequency histogram for the survey of 80 patients with the distribution curve $p(x)$.

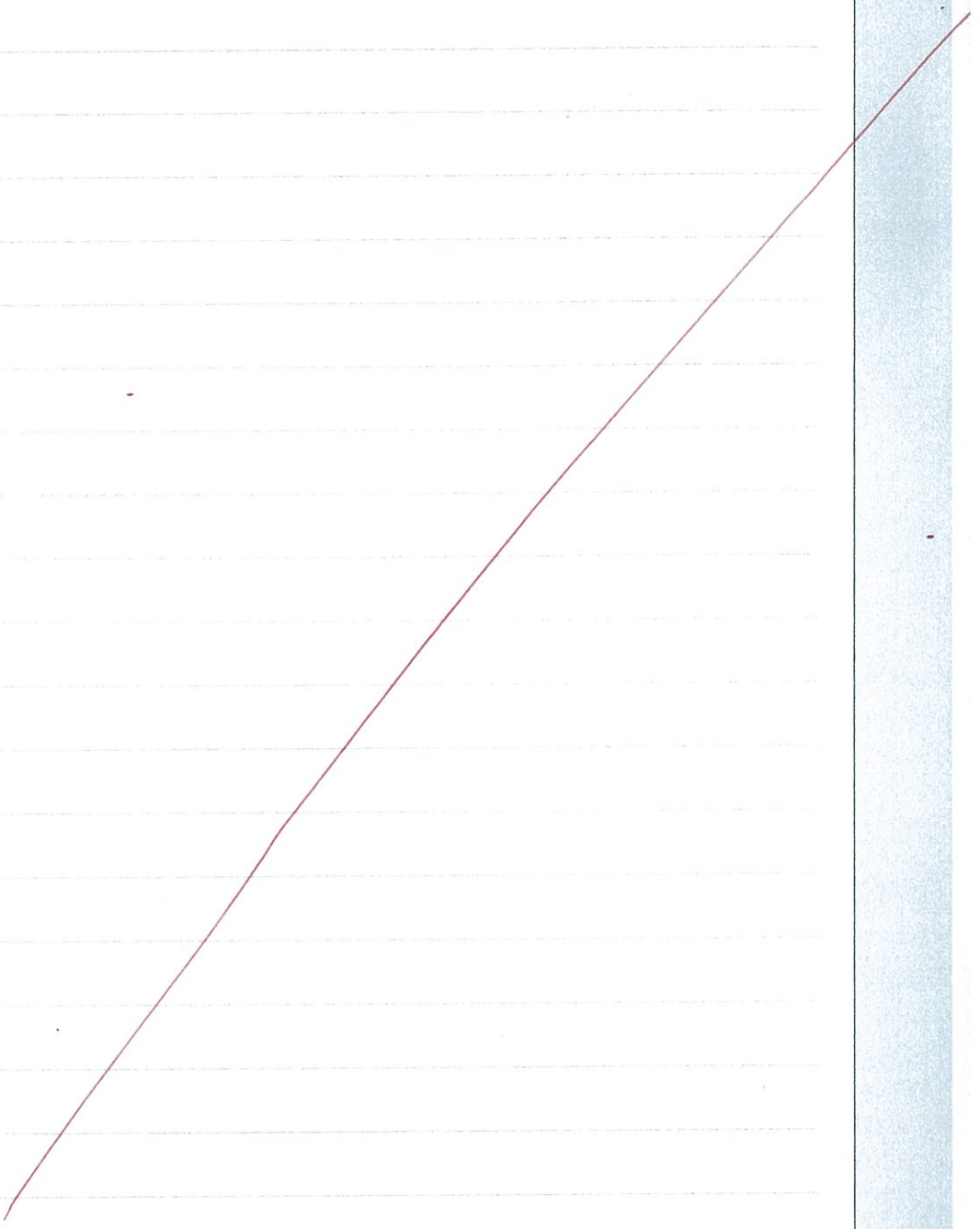
You should comment on the comparative shape, centre, and spread of the two distributions.

It is important to give numerical values to support your statements where possible.

There is a skewness to the right shifting the median to the right.

Comment on shape but little else.

There is more space for your answer on the following page.



ASSESSOR'S USE ONLY

M5

ASSESSOR'S USE ONLY

QUESTION TWO

A study is conducted of 1500 randomly selected candidates for an international examination to investigate whether Year 12 candidates were as successful as those from Year 13.

The results are summarised in the table below:

	Year 12	Year 13	Total
Passed	347	853	1200
Failed	33	267	300
Total	380	1120	1500

(a) (i) What proportion of candidates in the study passed the examination?

$\frac{1200}{1500}$ or $\frac{4}{5}$ or 0.8

Correct answer.

(ii) What proportion of candidates who failed the examination were from Year 12?

$\frac{33}{300}$ or $\frac{11}{100}$ or 0.11

Correct answer.

(iii) There were about 52 500 candidates from Year 12 and Year 13 who attempted the examination.

Using the results of this study, how many candidates would be expected to be from Year 13, and pass the examination?

$\frac{1120}{1500} \times \frac{853}{1120}$

Incorrect answer.

$= 0.7466 \div 7616 = 0.980 \times 52,500$

Year 13

51450 students would pass the exam

ASSESSOR'S USE ONLY

(iv) It is claimed that Year 13 candidates are four times more likely to fail the examination than Year 12 candidates.

State whether or not you agree with this claim, showing full calculations to support your view.

Incorrect as the Probability of them passing is 0.761 which is higher than 0.50 and 0.5. ~~Compared to~~ year 12s who are

Insufficient response.

ASSESSOR'S USE ONLY

(b) The same study also considered the number of subjects the candidates were taking in their normal academic courses. It found that of the same sample of 1500 candidates, 682 were taking six subjects, while the rest were taking five subjects. Of the candidates who were taking five subjects, 192 failed the examination.

The table from page 7 is repeated here to help you answer the questions that follow.

	Year 12	Year 13	Total
Passed	347	853	1200
Failed	33	267	300
Total	380	1120	1500

682/1500 → 6 ÷ 818/1500 300/818

passed 5 subject

374/682 ← 682/1500 → six subjects, 818/1500 → 192/818 → 626/818

Total passed 1200

382 students

(i) What proportion of candidates in the study took six subjects and passed?

n.

682

374/682 passed on six subjects

Incorrect answer.

(ii) On the evidence of this study, would you recommend that candidates take six subjects?

Support your answer with numerical calculations that consider the absolute and relative risks. You may also wish to comment on the sensibility of drawing any conclusions on this evidence.

ASSESSOR'S USE ONLY

n

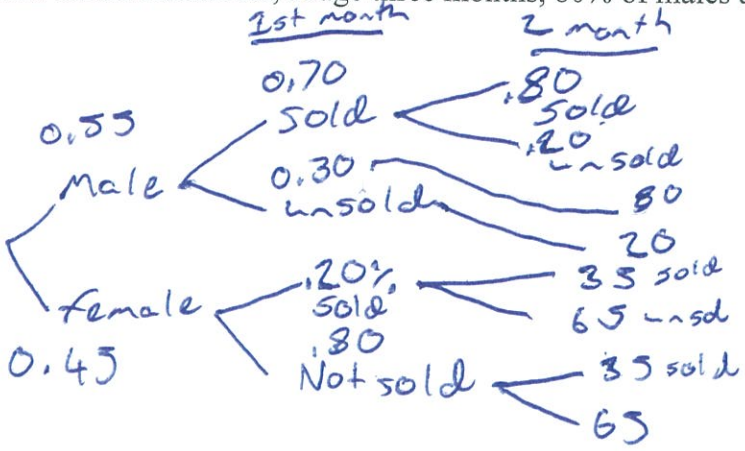
A3

QUESTION THREE

ASSESSOR'S USE ONLY

(a) When calves are born into a pedigree beef herd, decisions are made after they are one month old, and again when they are three months old, as to whether they will be kept in the herd or sold.

55% of calves born are male. At age one month, 70% of male and 20% of female calves are sold. Of the remainder, at age three months, 80% of males and 35% of females are also sold.



(i) Find the probability that a randomly chosen calf born into the herd will be male and sold at age one month.

0.55×0.70

0.385

Correct answer.

u

(ii) Find the probability that a randomly chosen calf born into the herd will be female and sold at age three months.

$0.45 \times 0.80 \times 0.35$

0.126

Correct answer.

u

(iii) What percentage of calves will eventually be kept in the pedigree herd?

$0.1410 \times 100 = 14.1\%$

$0.033 + 0.234$

0.033 found but incorrect answer.

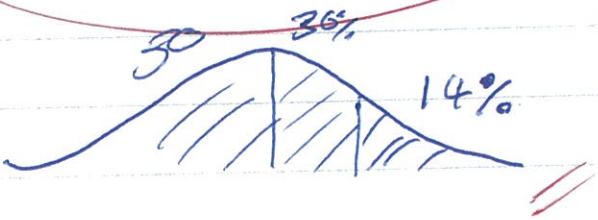
u

ASSESSOR'S USE ONLY

(iv) In a particular year 550 calves were born.

How many male calves can be expected to be kept in the pedigree herd?

14% of 550 calves.



Incorrect answer.

(v) The ratio of male to female calves being kept in the herd after three months is about one male to every seven females. This is to be changed to one male to every ten females.

If the number of male calves remains the same, what proportion of females would have been sold?

$\frac{1}{10} \times 550 = 55$

$\frac{55}{550}$

44 495 female calves

$\frac{1}{7} \times \frac{1}{10}$

Incorrect answer.

(b) New Zealand fantails are birds which are either pied or black.

ASSESSOR'S USE ONLY

Pied fantail Black fantail

Cherryl Mariner, www.nzbirdsonline.org.nz/species/new-zealand-fantail

They interbreed, and pairs with successful nests are found in the following proportions:

Pair	Two pied fantails	One pied and one black fantail	Two black fantails
Proportion	0.75	0.2	0.05

Successful nests have between one and four eggs. The proportions of eggs are given in the table below.

Pair	Two pied fantails	One pied and one black fantail	Two black fantails
One egg	0.15	0.2	0.3
Two eggs	0.3	0.35	0.5
Three eggs	0.4	0.35	0.15
Four eggs	0.15	0.1	0.05

(i) What proportion of pairs with two pied fantails will have a successful nest with more than one egg?

0.85
1.00

Correct answer.

U

(ii) A researcher claims that only one out of every 50 nests found with three eggs is likely to be from a pair of two black fantails.

Use calculations to show that the researcher's claim is justified.

1/50

0.15

Incorrect answer

ASSESSOR'S USE ONLY

n

A4

91267

912670

2

SUPERVISOR'S USE ONLY

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 2 Mathematics and Statistics, 2015

91267 Apply probability methods in solving problems

2.00 p.m. Tuesday 10 November 2015
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability methods in solving problems.	Apply probability methods, using relational thinking, in solving problems.	Apply probability methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Resource Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Achievement

TOTAL 13

ASSESSOR'S USE ONLY

QUESTION ONE

ASSESSOR'S USE ONLY

(a) The waiting time for a patient attending a medical centre before seeing a doctor is approximately normally distributed, with a mean of 34 minutes and a standard deviation of 8 minutes.

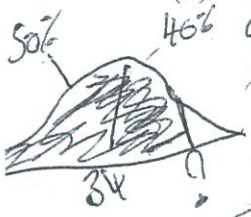
(i) Find the probability that a patient will wait between 34 and 40 minutes.

$40 - 34 = 6$ $6/8 = 0.75$ $0.75 = 0.2734$

u

Correct answer. GC used.

(ii) After how many minutes will 90% of patients have begun being seen by a doctor?



90% ÷ 2 = 45% 45% = 0.45 0.45 =

1.645 STD $1.645 \times 8 = 13.16$

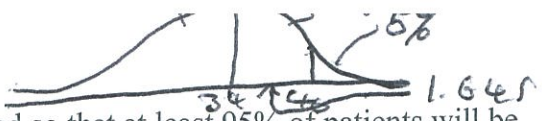
$34 - 13.16 = 20.84$ $34 + 13.16 = 47.16$

$0.4 = 1.281$ STD $1.281 \times 8 = 10.248$

$34 + 10.248 =$ 44.248 minutes

r

Correct answer. GC used.



ASSESSOR'S USE ONLY

(iii) It is decided that waiting times must be changed so that at least 95% of patients will be seen by a doctor within 40 minutes.

Because of the administration required, the mean time cannot change, but it is known that for each doctor added to the duty teams, the standard deviation will reduce by 0.4 minutes.

How many doctors must be added to meet the new requirement?

$0.45 = 1.645$ stD $1.645 \times 8 = 13.16$

$34 + 13.16 = 47.16$

$47.16 \div 9.6 = 4.9125$

$47.16 \div 9 = 5.1261$ (4dp)

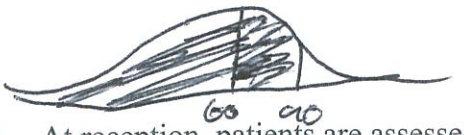
StD must be 9.6

$9.6 - 8 = 1.6$ $1.6 \div 0.4 = 4$

4 Doctors must be added

Correct z-value identified.

Answer incorrect.



ASSESSOR'S USE ONLY

(b) At reception, patients are assessed on the urgency of their condition. This is done within two minutes of arrival.

It is thought that the waiting time before an assessment is done is approximately normally distributed with a mean of 60 seconds and standard deviation of 20 seconds.

(i) What proportion of patients would be assessed at reception within 90 seconds of arrival?

90-60 = 30 seconds 30/20 = 1.5 s+D

1.5 = 0.4332 0.4332 + 0.5 = 0.9332

0.9332

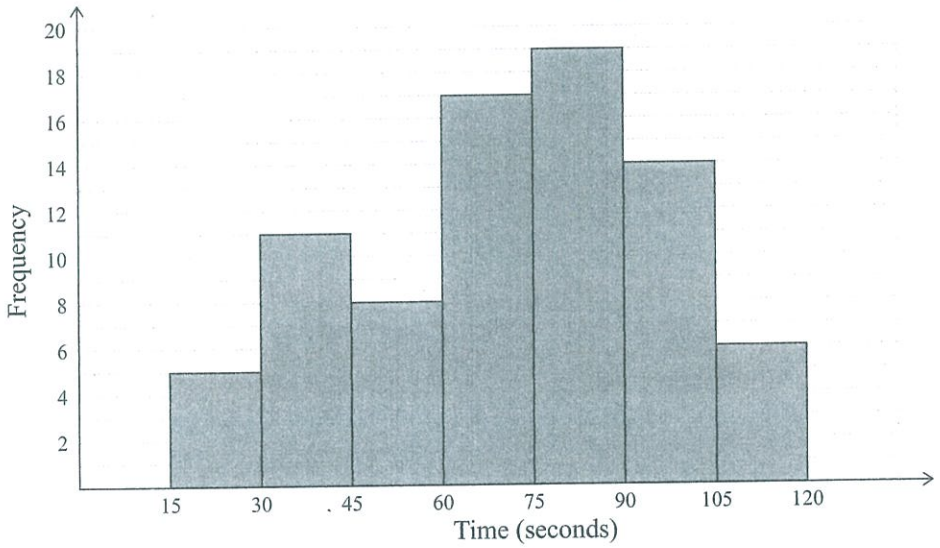
93.32%

Correct answer. GC used.

u

(ii) A survey is carried out on 80 patients who arrive at reception. Patients are selected at random on a particular day. The results are shown in the frequency histogram below.

Assessment time at reception



What proportion of patients in the survey were assessed at reception within 90 seconds of arrival?

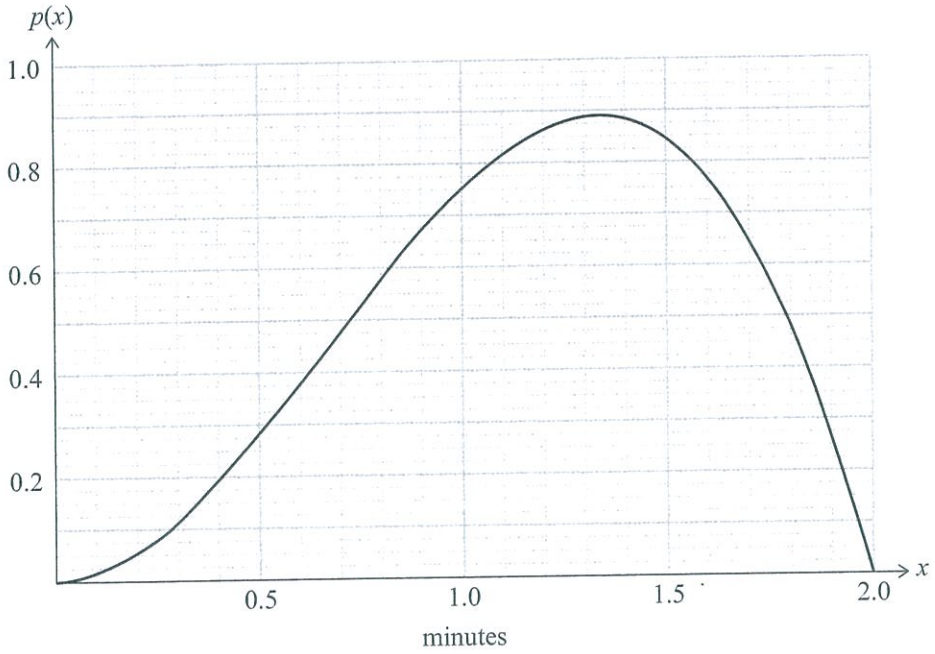
19+17+8+11+5=60 60/80 = 3/4 or 0.75

Correct answer.

u

ASSESSOR'S USE ONLY

(iii) A statistician states that the assessment times are not normally distributed, but are more likely to approximate the distribution $p(x)$ below.



The associated probabilities (with minutes converted to seconds) are given in the following table:

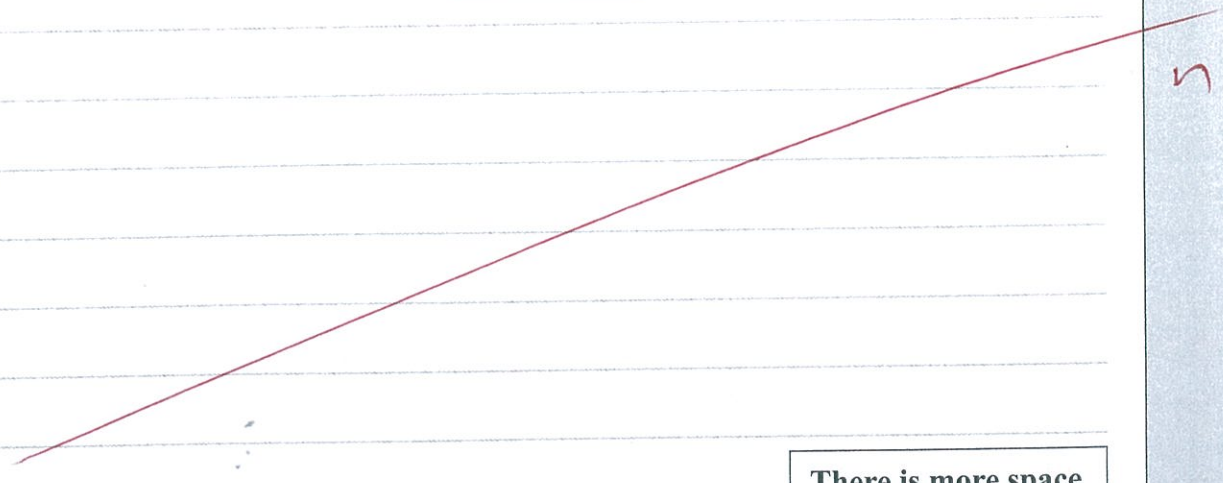
Assessment Time (seconds)	0 –	15 –	30 –	45 –	60 –	75 –	90 –	105 – 120
Probability	0.01	0.05	0.10	0.16	0.21	0.22	0.17	0.08

Compare the frequency histogram for the survey of 80 patients with the distribution curve $p(x)$.

You should comment on the comparative shape, centre, and spread of the two distributions.

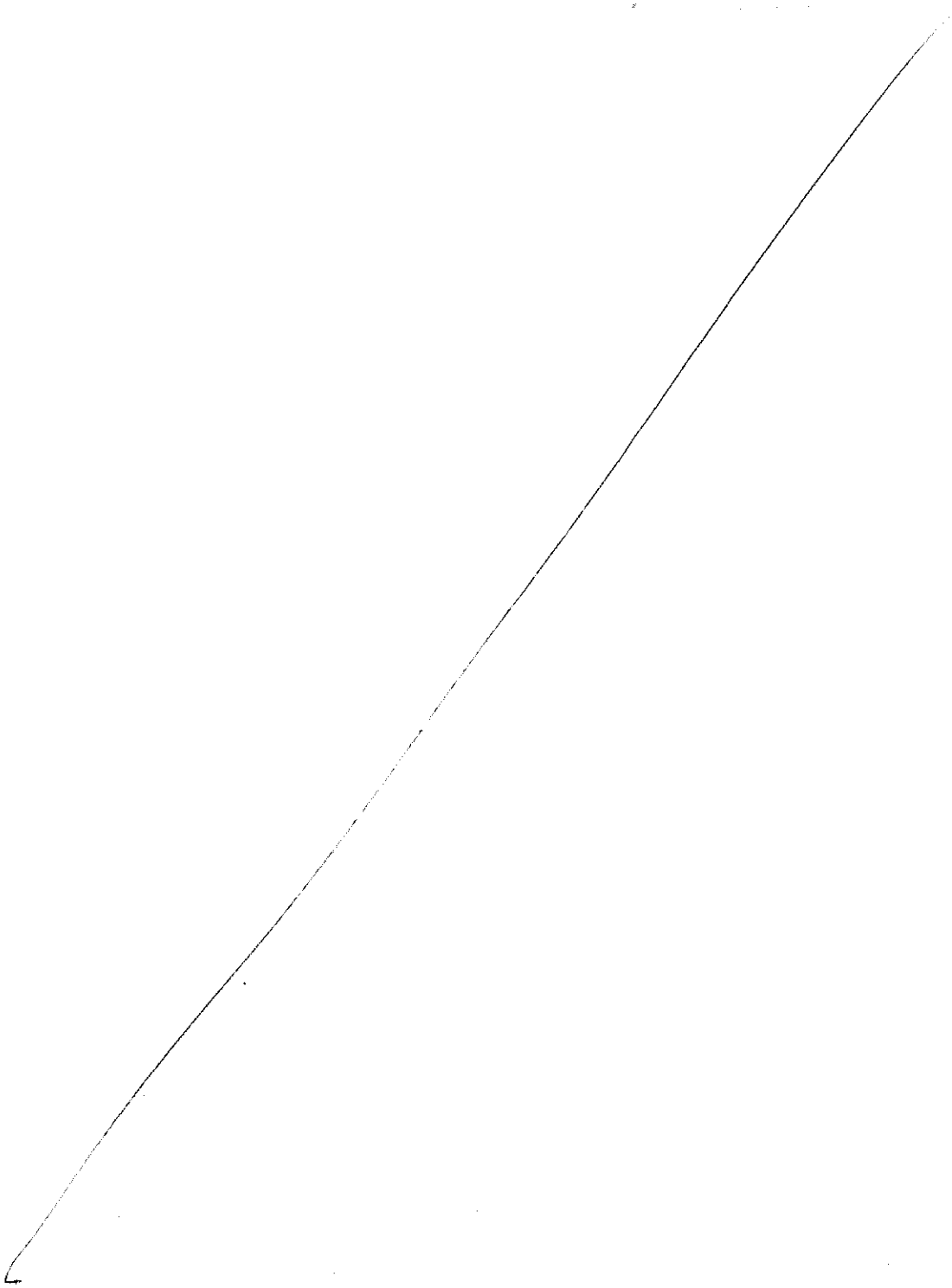
It is important to give numerical values to support your statements where possible.

No response.



There is more space for your answer on the following page.

ASSESSOR'S USE ONLY



MS

ASSESSOR'S USE ONLY

QUESTION TWO

A study is conducted of 1500 randomly selected candidates for an international examination to investigate whether Year 12 candidates were as successful as those from Year 13.

The results are summarised in the table below:

	Year 12	Year 13	Total
Passed	347	853	1200
Failed	33	267	300
Total	380	1120	1500

(a) (i) What proportion of candidates in the study passed the examination?

1200/1500 = 4/5 or 0.8

Correct answer.

(ii) What proportion of candidates who failed the examination were from Year 12?

33/300 = 11/100 or 0.11

Correct answer.

(iii) There were about 52 500 candidates from Year 12 and Year 13 who attempted the examination.

Using the results of this study, how many candidates would be expected to be from Year 13, and pass the examination?

52,500 × 0.8 = 42000 passed 853/1200

= Yr 13 pass rate 42000 × (853/1200)

= 29,855

Correct answer.

(iv) It is claimed that Year 13 candidates are four times more likely to fail the examination than Year 12 candidates.

State whether or not you agree with this claim, showing full calculations to support your view.

ASSESSOR'S USE ONLY

Yr 12: $\frac{33}{380}$ = [illegible] 0.087

Yr 13 = $\frac{267}{1120}$ = 0.238

this claim is not supported by the ~~evi~~ evidence given in this sample this claim may be true for other samples. this sample shows yr 13 students are approximately 3 times more likely to fail. Given the relatively small sample of year 12 students compared ~~#~~ to the year 13 students ~~its~~ is entirely plausable then in another sample this statement could prove true.

Correct absolute risks found. No relative risk.

ASSESSOR'S USE ONLY

(b) The same study also considered the number of subjects the candidates were taking in their normal academic courses. It found that of the same sample of 1500 candidates, 682 were taking six subjects, while the rest were taking five subjects. Of the candidates who were taking five subjects, 192 failed the examination.

The table from page 7 is repeated here to help you answer the questions that follow.

	Year 12	Year 13	Total
Passed	347	853	1200
Failed	33	267	300
Total	380	1120	1500

(i) What proportion of candidates in the study took six subjects and passed?

682/1200 = 0.568 (3dp)

Incorrect answer.

(ii) On the evidence of this study, would you recommend that candidates take six subjects?

Support your answer with numerical calculations that consider the absolute and relative risks. You may also wish to comment on the sensibility of drawing any conclusions on this evidence.

ASSESSOR'S USE ONLY

I would not recomend taking 6 classes as 56.8% of students passed which means 47.2% failed. out of the students that took 5 subjects 76.53% passed and only 23.53% Failled.

1500 − 682 = 818 students took 5 classes 192 failed $\frac{192}{818} = 0.2347$

$1 - 0.2347 = 0.7653$ passed.

taking 5 subjects instead of 6 increases the chance of passing 56.8% to 76.53%

No absolute risk found for six subjects.

A4

QUESTION THREE

ASSESSOR'S USE ONLY

(a) When calves are born into a pedigree beef herd, decisions are made after they are one month old, and again when they are three months old, as to whether they will be kept in the herd or sold.

55% of calves born are male. At age one month, 70% of male and 20% of female calves are sold. Of the remainder, at age three months, 80% of males and 35% of females are also sold.

(i) Find the probability that a randomly chosen calf born into the herd will be male and sold at age one month.

$0.55 \times 0.7 = 0.385$

Correct answer.

u

(ii) Find the probability that a randomly chosen calf born into the herd will be female and sold at age three months.

$0.45 \times 0.8 \times 0.35 = 0126$

Correct answer.

u

(iii) What percentage of calves will eventually be kept in the pedigree herd?

$0.3 \times 0.8 \times 0.2 \times 0.65 = 0.0312$

$= 3.12\%$

Incorrect answer.

m

ASSESSOR'S USE ONLY

(iv) In a particular year 550 calves were born.

How many male calves can be expected to be kept in the pedigree herd?

550 x 55% = 302.5 302.5 x 30% = 90.75
90.75 x 20% = 18.15
18 male calves will be kept

u

Correct answer.

(v) The ratio of male to female calves being kept in the herd after three months is about one male to every seven females. This is to be changed to one male to every ten females.

If the number of male calves remains the same, what proportion of females would have been sold?

550 x 45% = 247.5 247.5 x 0.8 = 198
198 x 0.65 = 128.7 = 129 female calves
247.5 - 129 = 118.5 = 119
female calves sold.

n

Incorrect answer.

ASSESSOR'S USE ONLY

(b) New Zealand fantails are birds which are either pied or black.

Pied fantail Black fantail

Cherryl Mariner, www.nzbirdsonline.org.nz/species/new-zealand-fantail

They interbreed, and pairs with successful nests are found in the following proportions:

Pair	Two pied fantails	One pied and one black fantail	Two black fantails
Proportion	0.75	0.2	0.05

Successful nests have between one and four eggs. The proportions of eggs are given in the table below.

Pair	Two pied fantails	One pied and one black fantail	Two black fantails
One egg	0.15	0.2	0.3
Two eggs	0.3	0.35	0.5
Three eggs	0.4	0.35	0.15
Four eggs	0.15	0.1	0.05

(i) What proportion of pairs with two pied fantails will have a successful nest with more than one egg?

$0.75 \times (0.3 + 0.4 + 0.15) = 0.6375$

Incorrect answer due to multiplication by 0.75 – a common error.

(ii) A researcher claims that only one out of every 50 nests found with three eggs is likely to be from a pair of two black fantails.

ASSESSOR'S USE ONLY

Use calculations to show that the researcher's claim is justified.

pied = 50 × 0.75 × 0.4 = 15

one pied one black = 50 × 0.2 × 0.75 = 7.5

50 nests × 0.15 = 7.5 nests found with 3 eggs are from 2 black fantails.

7.5 × 0.05 = 0.375

0.05 ↑ Proportion of nests belong to 2 black fantails

there is a 37.5% chance that out of 50 nests with 3 eggs 1 nest will belong to 2 black fantails

Incorrect answer.

A4