2

SUPERVISOR'S USE ONLY

91267

912670

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 2 Mathematics and Statistics, 2017

91267 Apply probability methods in solving problems

2.00 p.m. Friday 24 November 2017
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability methods in solving problems.	Apply probability methods, using relational thinking, in solving problems.	Apply probability methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–15 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Merit

TOTAL 14

ASSESSOR'S USE ONLY

QUESTION ONE

ASSESSOR'S USE ONLY

Regular surveys are taken of New Zealanders to find out about the state of their health and well-being.

A random sample of 2500 young adults from the age group 15–24 years gave the following results for obesity.

Table 1

	Obese	**Not Obese**	**Total**
Male	222	983	1205
Female	285	1010	1295
Total	507	1993	2500

(a) (i) What proportion of obese young adults in the sample were male?

$= \frac{222}{507}$

$= 0.44$ (3dp)

(ii) At the time of the survey, there were known to be about 585 000 young adults in the age group 15–24 years in New Zealand.

From the results of this survey, how many young adults in this age group would you estimate to be obese?

~~2500~~ $\frac{507}{2500} = 0.2028 = 20.28\%$

$585000 \times 20.28\% = 11863.8$

(iii) A newspaper uses the survey results in an article with the following introduction.

Kiwi Girls More Obese than Boys A recent survey of young adults aged 15 – 24 years shows that females are more than 20% more likely to be obese than their male counterparts.

Do you agree with the article's introduction?

Use the data from Table 1 to support your answer, showing full calculations.

ralative Risk.

$$\frac{\frac{285}{1295}}{\frac{222}{1205}} = \frac{0.22}{0.18} = 1.19$$

I agree with this introduction, the kiwi girl is auctually more obese than Boys, by looking at the Ralative Risk, shown above, the girls has 1.19 times more then Boys Obese. But the no. of Female got more ~~the~~ ~~the~~ percentage ~~than~~ ~~boys~~ no. in the newspaper than the boys got.

ASSESSOR'S USE ONLY

ASSESSOR'S USE ONLY

(b) Table 1 from Page 2 is repeated below.

Table 1

	Obese	Not Obese	Total
Male	222	983	1205
Female	285	1010	1295
Total	507	1993	2500

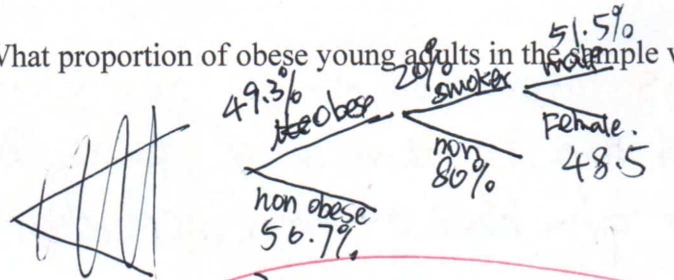
$\frac{53}{103}$

$\frac{103}{507}$

The survey also obtained information about the current smoking habits of participants.

It was found that of the young adults in the survey who were defined as obese, 103 were current smokers, and that 53 of the current smokers were male.

(i) What proportion of obese young adults in the sample were female non-smokers?



$\frac{103}{507} = 0.20$ (3dp) $\frac{53}{103} = 51.5\%$

$P = 0.507 \times 0.8 \times 0.485 = 0.197$ (3sf) $= 19.7\%$

(ii) Table 2 below gives further information on the participants in the survey who were in the age group 15–24 years.

Table 2

	Obese	Not Obese	Total
Current smoker	103	317	420
Non-smoker	404	1676	2080
Total	507	1993	2500

It is claimed that young adult smokers are more at risk of being obese than young adult non-smokers.

Do the results of the survey support this claim?

Support your answer with appropriate calculations.

No support this claim.

because $\frac{\frac{103}{507}}{\frac{2080}{\cancel{2500}}}$

$= 0.29$

the young adult non-smokers are 0.29 times

ASSESSOR'S USE ONLY

more than young adult smokers
~~are mo~~ of being obsese.
Hense I don't support this claim

M5

QUESTION TWO

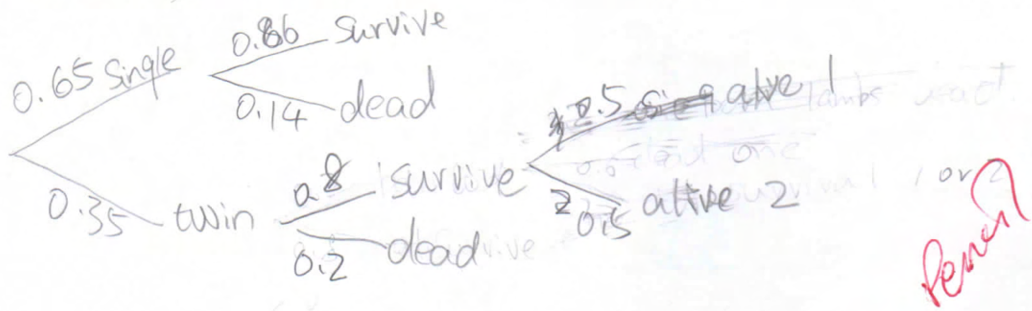
ASSESSOR'S USE ONLY

(a) Merino ewes that produce lambs usually have either single lambs or twins. Multiple births other than twins are extremely rare.

A long-term study has shown that 65% of Merino ewes that produce a lamb will have a single lamb, and of those lambs, 86% survive until they are weaned (separated from their mothers).

Of the ewes that produce twins, about one in five lose both lambs before they are weaned. Approximately equal numbers of one twin or both twins survive until they are weaned.

https://milligansganderhillfarm.wordpress.com/2013/06/06/merino-sheep/



(i) Find the probability that a ewe gives birth to a single lamb that survives until it is weaned.

$0.65 \times 0.86 = 0.559 = 55.9\%$

(ii) What proportion of ewes give birth to twins that both survive until they are weaned?

$0.35 \times 0.8 \times 0.5 = 0.14 = 14\%$

ASSESSOR'S USE ONLY

(iii) What is the probability that a randomly selected lamb that survives until it is weaned will be from a ewe that produced a single lamb?

Hint: Remember that there is an equal number of one twin or both twins surviving until they are weaned.

$0.65 \times 0.86 = 0.559$

$0.35 \times 0.8 \times 0.5 = 0.14$

$0.14 + 0.559 = 0.669 = 66.9\%$

(iv) 'Lambing percentage' is the number of **lambs** that survive until they are weaned compared to the number of **breeding ewes**, expressed as a percentage.

It is known that about 85% of breeding Merino ewes actually produce lambs.

What was the lambing percentage for this long-term study?

ASSESSOR'S USE ONLY

(b) On Highbrook Station there are two breeds of sheep, Merino and Romney.

Table 3 below gives information about the lambs born in the 2016 lambing season. It shows the proportion of ewes that did not produce a lamb, had a single birth, or had multiple births, for each breed of sheep.

Table 3

	No lamb	Single	Multiple
Merino	0.13	0.62	0.25
Romney	0.06	0.48	0.46

After the lambs were weaned, the ewes were sorted. Some were culled (not kept) and others were kept for breeding in the 2017 lambing season.

Table 4 shows the proportion of Romney ewes that were either culled or kept for the 2017 lambing season.

Table 4

	No lamb	Single	Multiple
Romney ewes culled	0.88	0.68	0.40
Romney ewes kept	0.12	0.32	0.60

The ratio of Romney to Merino breeding ewes on Highbrook Station at the beginning of the season was approximately 3:2.

According to the data in tables 3 and 4, at the end of the 2016 lambing season, what proportion of the total breeding ewes on Highbrook Station were Romneys that were 'empty' (did not produce a lamb) and were culled?

A3

ASSESSOR'S USE ONLY

QUESTION THREE

(a) Salmon are grown in sea pens. Each pen contains several thousand salmon.

After one year in the pens, male salmon have weights that are approximately normally distributed, with mean 4125 grams and standard deviation 65 grams.

www.technologybloggers.org/wp-content/uploads/2013/06/big-glory-bay.jpg

(i) Find the probability that after one year in a pen, a randomly selected male will weigh between 4125 and 4200 grams.

$z = \frac{4125-4125}{65} = 0$

$z = \frac{4200-4125}{65} = 1.153$

$z = 0.37496 = 37.50\%$

(ii) What is the maximum weight of the lightest 10% of salmon?

$z = \overset{-0.4}{\cancel{0.1}}\ \cancel{4} = -1.281$

$-1.281 = \frac{x-4125}{65}$

$x = 4041.735$ grams

ASSESSOR'S USE ONLY

(iii) After one year in the pens, female salmon have weights that are approximately normally distributed with mean 3975 grams.

μ 3975

0.4

If 40% of female salmon exceed 4000 grams, then what would be the standard deviation?

$z = 0.4 = 1.281$

$1.281 = \dfrac{4000 - 3975}{\sigma}$

$1.281 \times \sigma = 25$

$\sigma = 19.52$ (4dp) grams // CON

(iv) The pens contain approximately equal numbers of male and female salmon.

When they are harvested, the weights of all the salmon are approximately normally distributed, with mean 4050 grams and standard deviation 84 grams.

When the salmon are harvested, each member of the harvest team is given two salmon to take home.

μ 4050 σ 849 50%

If these two salmon are selected at random, what is the probability that **both** of the salmon will each weigh more than 4025 grams?

$z = \dfrac{4051 - 4050}{84}$

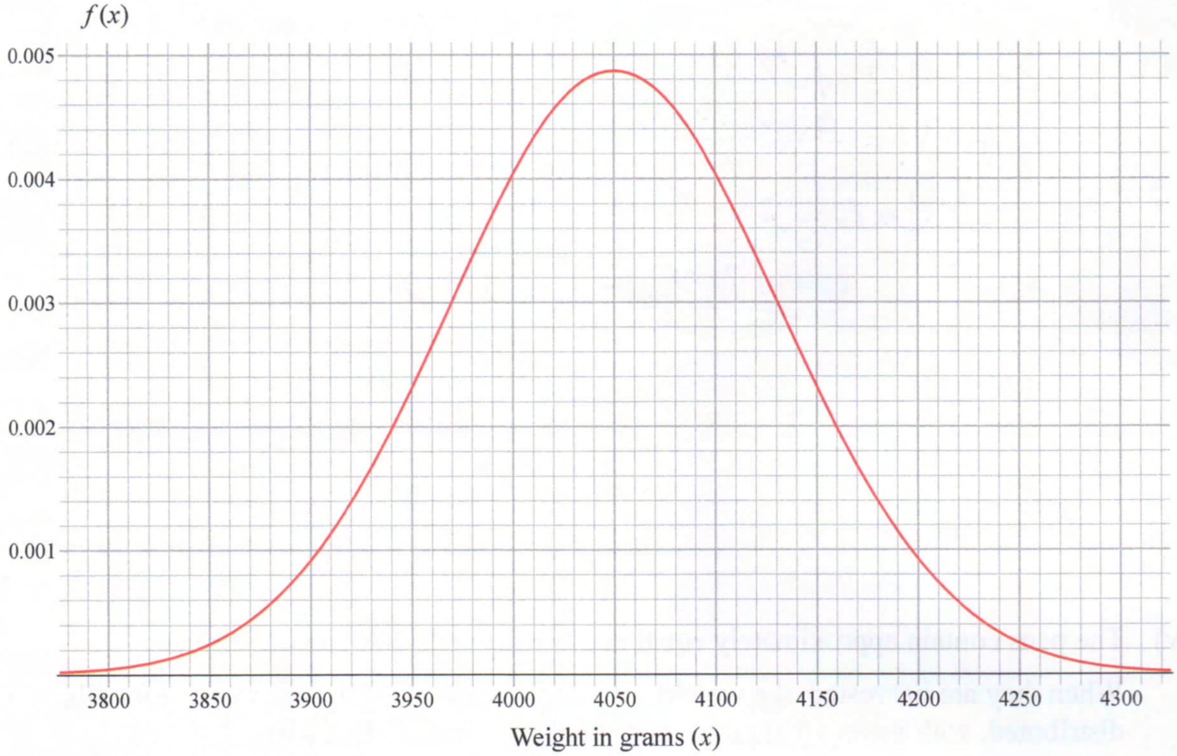
$z = 0.012$ (2sf) $= \cancel{}$

$= 0.00404 = 0.4\%$ // A

ASSESSOR'S USE ONLY

(b) When a pen of salmon is harvested, the weights of the salmon are expected to have the probability distribution shown in Figure 1 below.

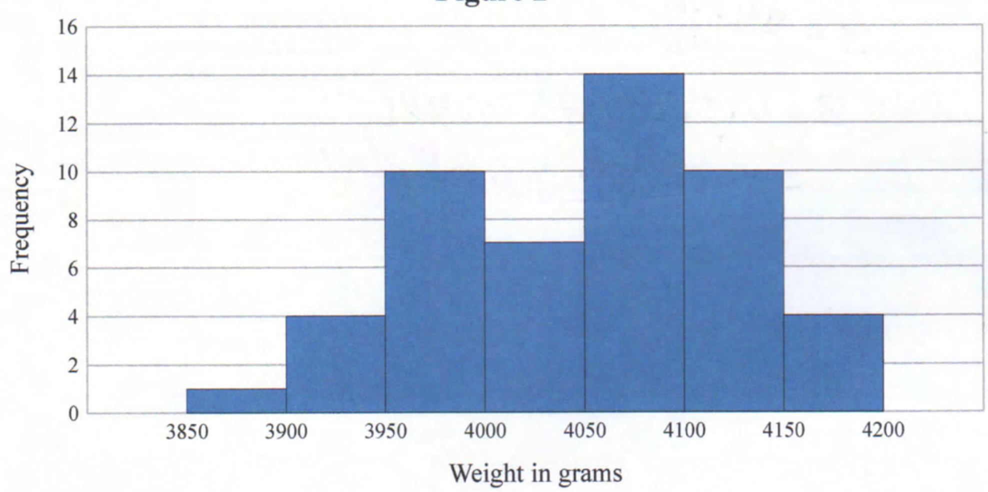
Figure 1



Once harvested, a random sample of 50 salmon was taken and weighed.

A histogram of the weights of the sampled salmon is shown in Figure 2 below.

Figure 2



(i) What proportion of salmon in the sample had weights which exceeded 4000 grams?

In Figure 1 is $\frac{3}{4} = 0.75 = 75\%$

Figure 2 is $\frac{2}{3} = 0.67 = 67\%$

(ii) Compare the probability distribution and the histogram that resulted from the sample results.

In your answer you should consider the shape, centre, and spread of both distributions, and should provide numerical evidence where appropriate.

ASSESSOR'S USE ONLY

Compareing ~~the~~ Figure1 and Figure2.
Figure1 ~~have~~ has more simmetry shape than Figure2 has, and Figure2 is more skew to the ~~left~~ left side than Figure 1, Figure2 is also more spread to right out of weight where Figure1 is no any spread to one side or skew. The centre of mean of Figure1 is lie in 4050 ~~mg~~ ~~g~~ where Figure2 is lie in between 4050 - 2400 g.

1

^

M6

Exemplar 002: Merit

Question One

Part	Annotation (if any)
A i	Incorrect description of rounding not penalised
A ii	A whole number would be more sensible since we are talking about the number of people
A iii	Correct calculation of relative risk needs to be connected with the article to improve the grade
B i	
B ii	
Overall	

Question Two

Part	Annotation (if any)
A i	
A ii	
A iii	
A iv	
B	
Overall	

Question Three

Part	Annotation (if any)
A i	It is not true that z=0.37496 but this is not penalised
A ii	
A iii	This student has misinterpreted the situation, as their diagram shows, but is awarded at least some credit as their working is clear.
A iv	
B i	The question referred to the sample.
B ii	While the answer does try to cover the areas suggested for comment, the meaning is not clear so does not gain U
Overall	

91267

912670

2

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Merit

TOTAL 19

ASSESSOR'S USE ONLY

QUESTION ONE

ASSESSOR'S USE ONLY

Regular surveys are taken of New Zealanders to find out about the state of their health and well-being.

A random sample of 2500 young adults from the age group 15–24 years gave the following results for obesity.

Table 1

	Obese	Not Obese	Total
Male	222	983	1205
Female	285	1010	1295
Total	507	1993	2500

(a) (i) What proportion of obese young adults in the sample were male?

~~P(Young~~ P(Obese male) = 222/507

= 0.4378698225

= ~~0.44 (2dp)~~ 0.438 (3sf)

= 43.8%

(ii) At the time of the survey, there were known to be about 585 000 young adults in the age group 15–24 years in New Zealand.

From the results of this survey, how many young adults in this age group would you estimate to be obese?

P(Obese) = 507/2500

= 0.203 (3sf)

Estimate = 585,000 × 0.203

= 118755 estimated obese young adults

(iii) A newspaper uses the survey results in an article with the following introduction.

ASSESSOR'S USE ONLY

> **Kiwi Girls More Obese than Boys**
> A recent survey of young adults aged 15 – 24 years shows that females are more than 20% more likely to be obese than their male counterparts.

Do you agree with the article's introduction?

Use the data from Table 1 to support your answer, showing full calculations.

P(Female and Obese) = $285/1295$

= 0.220 (3sf)

P(Male and Obese) = $222/1205$

= 0.184 (3sf)

Relative Risk = $0.220/0.184$

= 1.20

The newspaper introduction is correct as females are 1.2 (20%) times more likely to be obese when compared to men.

ASSESSOR'S USE ONLY

(b) Table 1 from Page 2 is repeated below.

Table 1

	Obese	Not Obese	Total
Male	222	983	1205
Female	285	1010	1295
Total	507	1993	2500

The survey also obtained information about the current smoking habits of participants.

It was found that of the young adults in the survey who were defined as obese, 103 were current smokers, and that 53 of the current smokers were male.

(i) What proportion of obese young adults in the sample were female non-smokers?

		Smoke	Don't smoke	Total
~~Male~~ Obese	Male	53	169	222
Obese	Female	50	235	285
	Total	103	404	507

P(Female Non-Smoker) = 235/507

= 0.464 (3sf)

= 46.4%

(ii) Table 2 below gives further information on the participants in the survey who were in the age group 15–24 years.

Table 2

	Obese	Not Obese	Total
Current smoker	103	317	420
Non-smoker	404	1676	2080
Total	507	1993	2500

It is claimed that young adult smokers are more at risk of being obese than young adult non-smokers.

Do the results of the survey support this claim?

Support your answer with appropriate calculations.

~~P(Smoke & Obese) = 103/507~~

P(Obese) =

ASSESSOR'S USE ONLY

P(Smoker & Obese) = 103/420

= 0.250 (3sf)

P(Non Smoker & Obese) = 404/2080

= 0.194 (3sf)

Relative Risk = 0.250/0.194

= 1.30

You are 1.3 times more likely to be obese if you smoke rather than not smoke. Therefore I do support this claim is there is a 30% higher chana you will be obese if you smoke.

E7

QUESTION TWO

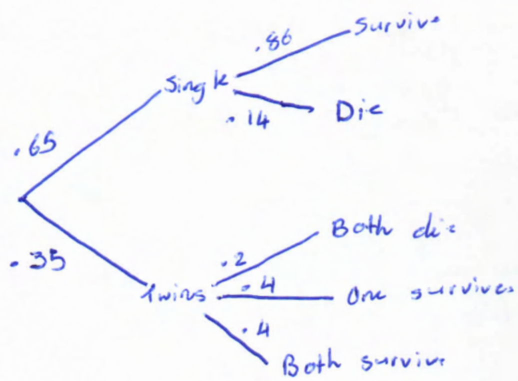
ASSESSOR'S USE ONLY

(a) Merino ewes that produce lambs usually have either single lambs or twins. Multiple births other than twins are extremely rare.

A long-term study has shown that 65% of Merino ewes that produce a lamb will have a single lamb, and of those lambs, 86% survive until they are weaned (separated from their mothers).

Of the ewes that produce twins, about one in five lose both lambs before they are weaned. Approximately equal numbers of one twin or both twins survive until they are weaned.

https://milligansganderhillfarm.wordpress.com/2013/06/06/merino-sheep/



(i) Find the probability that a ewe gives birth to a single lamb that survives until it is weaned.

p(Single Survive) = .65 × .86

= 0.559

(ii) What proportion of ewes give birth to twins that both survive until they are weaned?

P(Twins Both Survive) = .35 × .2

= 0.07

= 7%

ASSESSOR'S USE ONLY

(iii) What is the probability that a randomly selected lamb that survives until it is weaned will be from a ewe that produced a single lamb?

Hint: Remember that there is an equal number of one twin or both twins surviving until they are weaned.

P(Weaned lamb from single) = $\frac{.86}{.4 + .4}$

= 1.075

0.47515

(iv) 'Lambing percentage' is the number of **lambs** that survive until they are weaned compared to the number of **breeding ewes**, expressed as a percentage.

It is known that about 85% of breeding Merino ewes actually produce lambs.

What was the lambing percentage for this long-term study?

Lambing % = $(.85 \times .65) \times (.86) + (.85 \times .35) \times (.8)$

= 0.71315

= 71.3% (3sf)

Of the ~~lambs~~ ewes that gave birth 71.3% of their lambs survived till weaning.

(b) On Highbrook Station there are two breeds of sheep, Merino and Romney.

Table 3 below gives information about the lambs born in the 2016 lambing season. It shows the proportion of ewes that did not produce a lamb, had a single birth, or had multiple births, for each breed of sheep.

Table 3

	No lamb	Single	Multiple
Merino	0.13	0.62	0.25
Romney	0.06	0.48	0.46

After the lambs were weaned, the ewes were sorted. Some were culled (not kept) and others were kept for breeding in the 2017 lambing season.

Table 4 shows the proportion of Romney ewes that were either culled or kept for the 2017 lambing season.

Table 4

	No lamb	Single	Multiple
Romney ewes culled	0.88	0.68	0.40
Romney ewes kept	0.12	0.32	0.60

The ratio of Romney to Merino breeding ewes on Highbrook Station at the beginning of the season was approximately 3:2.

According to the data in tables 3 and 4, at the end of the 2016 lambing season, what proportion of the total breeding ewes on Highbrook Station were Romneys that were 'empty' (did not produce a lamb) and were culled?

P(Romney no lamb culled) = 0.0528

= 5.28%

A

M5

ASSESSOR'S USE ONLY

QUESTION THREE

(a) Salmon are grown in sea pens. Each pen contains several thousand salmon.

After one year in the pens, male salmon have weights that are approximately normally distributed, with mean 4125 grams and standard deviation 65 grams.

www.technologybloggers.org/wp-content/uploads/2013/06/big-glory-bay.jpg

(i) Find the probability that after one year in a pen, a randomly selected male will weigh between 4125 and 4200 grams.

$P(4125 < x < 4200)$

$= 0.375$

$= 37.5\%$

(ii) What is the maximum weight of the lightest 10% of salmon?

Heaviest weight of the lightest 10% = 4041.69915

= 4041.7 g (1dp)

ASSESSOR'S USE ONLY

(iii) After one year in the pens, female salmon have weights that are approximately normally distributed with mean 3975 grams.

If 40% of female salmon exceed 4000 grams, then what would be the standard deviation?

$z = 0.253$ (3sf)

$z = \frac{x - \mu}{\sigma}$

z

$0.253 = \frac{4{,}000 - 3{,}975}{\sigma}$

$0.253\sigma = 25$

$\sigma = 98.8$

(iv) The pens contain approximately equal numbers of male and female salmon.

When they are harvested, the weights of all the salmon are approximately normally distributed, with mean 4050 grams and standard deviation 84 grams.

When the salmon are harvested, each member of the harvest team is given two salmon to take home.

If these two salmon are selected at random, what is the probability that **both** of the salmon will each weigh more than 4025 grams?

P(Salmon weight > 4025g) = 0.617

P(2 salmon weigh > 4025g) = $(0.617)^2$

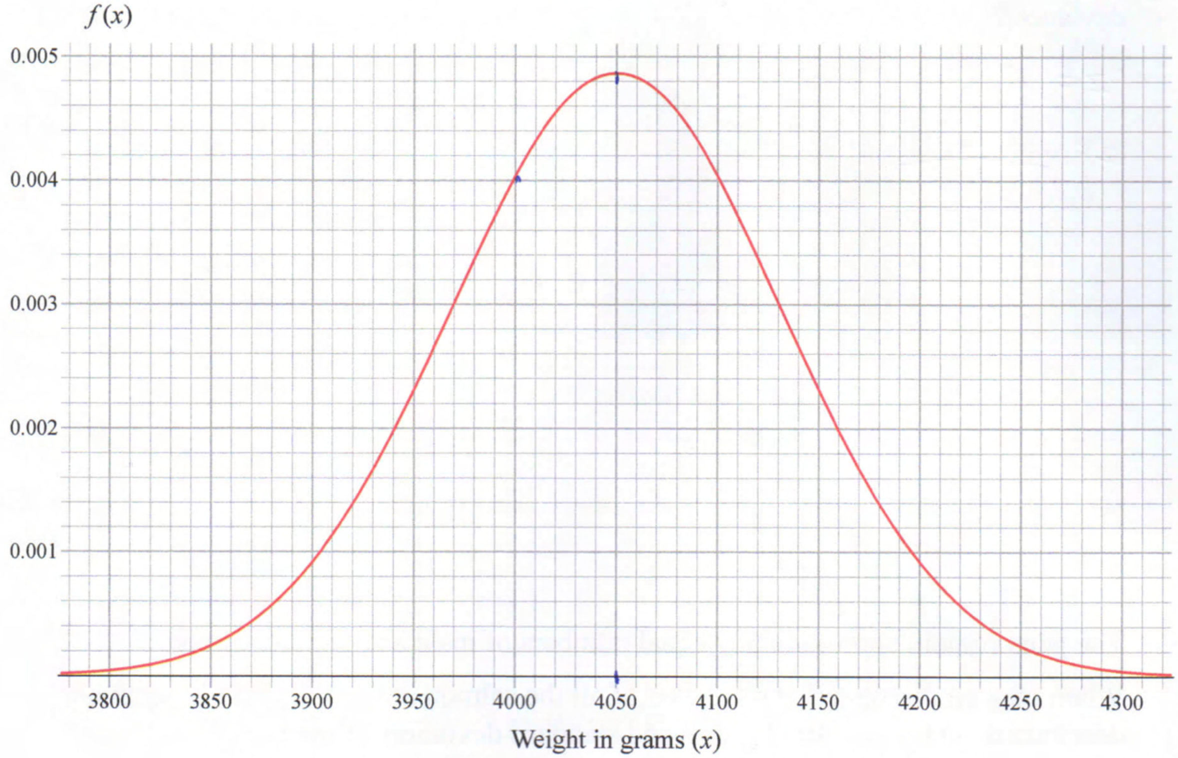
= 0.381

= 38.1%

(b) When a pen of salmon is harvested, the weights of the salmon are expected to have the probability distribution shown in Figure 1 below.

ASSESSOR'S USE ONLY

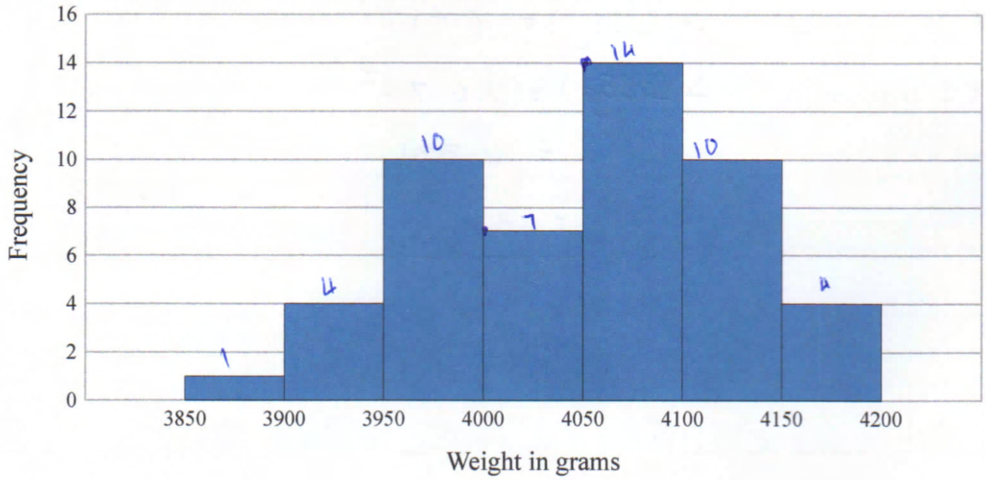
Figure 1



Once harvested, a random sample of 50 salmon was taken and weighed.

A histogram of the weights of the sampled salmon is shown in Figure 2 below.

Figure 2



(i) What proportion of salmon in the sample had weights which exceeded 4000 grams?

P (Weight > 4000g) = 35/50

= 0.7

= 70%

(ii) Compare the probability distribution and the histogram that resulted from the sample results.

In your answer you should consider the shape, centre, and spread of both distributions, and should provide numerical evidence where appropriate.

ASSESSOR'S USE ONLY

When comparing the histogram to the probability distribution curve there is a clear difference in shapes. The histogram roughly follows the shape of the distribution curve until 4000g where there is a sharp drop in the frequency of salmon at that particular weight when compared to the curve. The centre of both graphs is similar whereby they both peak in the 4,050g range of which is the mode and median for both sets of data. The histogram however does seem to be slightly skewed to the left and isn't symmetrical as there are 22 salmon found to the left mean/mode and only 14 to the right. Lastly the histogram only shows salmon weighing between 3850g - 4200g while the distribution curve ranges from 3,800g - 4,300g.

1

5+

E7

Exemplar 003: high Merit

Question One

Part	Annotation (if any)
A i	
A ii	
A iii	Correct use of the relative risk, connected to the article. However, to gain a T2 grade and E8, this response needs to realise that the article says “more than 20% more likely” whereas it is only because of rounding that they have calculated that it is 20%
B i	
B ii	Well-explained
Overall	

Question Two

Part	Annotation (if any)
A i	
A ii	
A iii	This is a very challenging question. This response has the beginnings of a correct approach but does not use the whole probability tree to gain the probabilities.
A iv	
B	
Overall	

Question Three

Part	Annotation (if any)
A i	
A ii	
A iii	
A iv	
B i	
B ii	Using a word like “centre” is too vague: use mean or median. The word centre is used in the question to remind students to think about either the mean or the median. A better response would also include the use of a probability to give another comparison between the 2 distributions.
Overall	