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SUPERVISOR'S USE ONLY

91267

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NEW ZEALAND QUALIFICATIONS AUTHORITY
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QUALIFY FOR THE FUTURE WORLD
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Level 2 Mathematics and Statistics, 2015

91267 Apply probability methods in solving problems

2.00 p.m. Tuesday 10 November 2015
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability methods in solving problems.	Apply probability methods, using relational thinking, in solving problems.	Apply probability methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Resource Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Excellence

TOTAL 23

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QUESTION ONE

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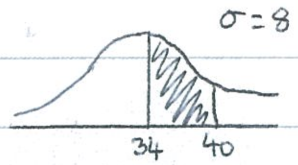
(a) The waiting time for a patient attending a medical centre before seeing a doctor is approximately normally distributed, with a mean of 34 minutes and a standard deviation of 8 minutes.

$\mu = 34 \quad \sigma = 8$

(i) Find the probability that a patient will wait between 34 and 40 minutes.

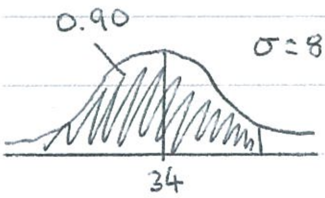


Correct answer. GC used.

$P = 0.2734$ (4 dp)

a

(ii) After how many minutes will 90% of patients have begun being seen by a doctor?



Correct answer. GC used.

44.25 minutes

r

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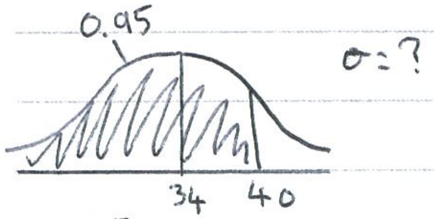
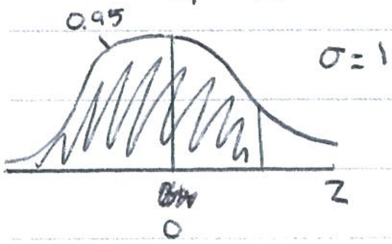
(iii) It is decided that waiting times must be changed so that at least 95% of patients will be seen by a doctor within 40 minutes.

Because of the administration required, the mean time cannot change, but it is known that for each doctor added to the duty teams, the standard deviation will reduce by 0.4 minutes.

How many doctors must be added to meet the new requirement?





Correct z-value identified.

Standard deviation found.

No. of doctors correct.

$z = 1.64485363$

$z = \frac{x - \mu}{\sigma}$

$1.64485363 = \frac{(40 - 34)}{\sigma}$

$\sigma = 3.65$ (2dp)

$8 - 3.65$

$= 4.35 \div 0.4$

$= 10.875$

∴ 10 or 11 doctors would be needed to get the waiting time changed so that 95% of patients will be seen within 40 minutes

t

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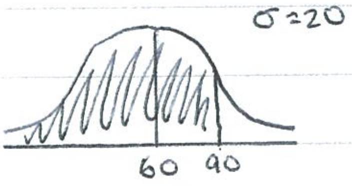
(b) At reception, patients are assessed on the urgency of their condition. This is done within two minutes of arrival.

It is thought that the waiting time before an assessment is done is approximately normally distributed with a mean of 60 seconds and standard deviation of 20 seconds.

(i) What proportion of patients would be assessed at reception within 90 seconds of arrival?

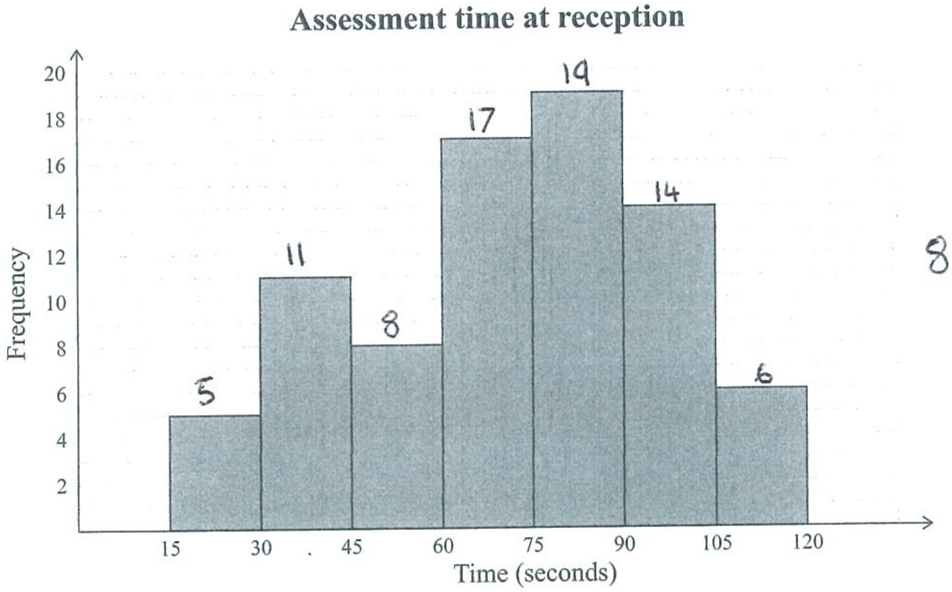
Correct answer. GC used.



P = 0.933 (3dp)

u

(ii) A survey is carried out on 80 patients who arrive at reception. Patients are selected at random on a particular day. The results are shown in the frequency histogram below.



80

What proportion of patients in the survey were assessed at reception within 90 seconds of arrival?

60 ÷ 80

= 0.75

P = 0.75

Correct answer.

u

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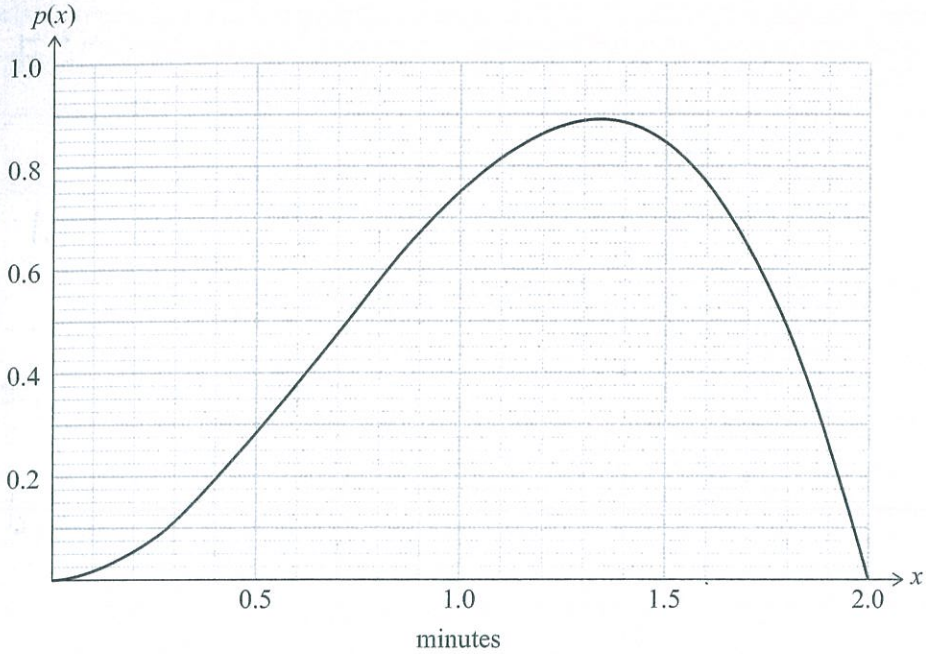
(iii) A statistician states that the assessment times are not normally distributed, but are more likely to approximate the distribution $p(x)$ below.



The associated probabilities (with minutes converted to seconds) are given in the following table:

Assessment Time (seconds)	0 –	15 –	30 –	45 –	60 –	75 –	90 –	105 – 120
Probability	0.01	0.05	0.10	0.16	0.21	0.22	0.17	0.08

Compare the frequency histogram for the survey of 80 patients with the distribution curve $p(x)$.

You should comment on the comparative shape, centre, and spread of the two distributions.

It is important to give numerical values to support your statements where possible.

- The frequency histogram is not normally distributed as it is skewed to the left, having a larger number of data (60) compared to 20 on the right hand side of the mode. Distribution curve also skewed left.
- The two graphs both share the same median and mode of 75 – 90.
- The frequency histogram has a range of 105. Where as the distribution curve has a range of 120.

There is more space for your answer on the following page.

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- The frequency histogram has a standard deviation of 62.5% which is very similar to a normal distribution but it canot be classified as a normal distribution as it is skewed to the left
- Both graphs are unimodel, each having a mode of 75-90
- If the ~~histo~~ frequency histogram did not ~~a~~ have a as large value of 30-45 (11 people) and it was around 6 or 7 people instead then it would come close to being considered a normal distribution as it would relatively resemble a bell ~~shaph~~ shape, although being skewed left.
- The distribution curve doesn't ~~show~~ ~~the~~ ~~amount~~ reflect the amount of people (11) between 30-45 seconds fairly as on the frequency histogram this is a major feature but on the distribution curve it goes unnoticed

t

Comparative statements made about shape, centre and spread with supporting evidence.

Comment on 'dip' in the frequency histogram.

E8

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QUESTION TWO

A study is conducted of 1500 randomly selected candidates for an international examination to investigate whether Year 12 candidates were as successful as those from Year 13.

The results are summarised in the table below:

	Year 12	Year 13	Total
Passed	347	853	1200
Failed	33	267	300
Total	380	1120	1500

(a) (i) What proportion of candidates in the study passed the examination?

1200 ÷ 1500

= 0.8

Correct answer.

u

(ii) What proportion of candidates who failed the examination were from Year 12?

33 ÷ 300

= 0.11

Correct answer.

u

(iii) There were about 52 500 candidates from Year 12 and Year 13 who attempted the examination.

Using the results of this study, how many candidates would be expected to be from Year 13, and pass the examination?

853 ÷ 1120

= 0.7616 (4dp)

0.7616 x 52500 = 39984 students

n

Incorrect answer.

1120 used instead of 1500 – a common error.

(iv) It is claimed that Year 13 candidates are four times more likely to fail the examination than Year 12 candidates.

State whether or not you agree with this claim, showing full calculations to support your view.

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$P(\text{Yr 13 fail}) = 267 \div 1120$
$= 0.2384 \ (4\text{dp})$
$P(\text{Yr 12 fail}) = 33 \div 380$
$= 0.0868 \ (4\text{dp})$

$\cancel{\frac{0.2384}{0.0868}}$ $\cancel{\frac{0.0868}{0.2384}}$ $\frac{0.2384}{0.0868}$
~~2.7465 (4dp)~~ $= 2.7465 \ (4\text{DP})$

∴ No I do not agree with this claim as Year 13 candidates are not four times more likely to fail the examination than Year 12 candidates. This is because as shown above Year 13 ~~sth~~ candidates are approx. ~~2.7~~ 2.7465 times more likely ~~(…)~~ to fail the examination than Year 12 candidates. This claim is misleading and should be changed to approx. 2 and a half times more likely instead of 4.

t

Correct absolute risks and relative risk found.

Correct supporting statement.

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(b) The same study also considered the number of subjects the candidates were taking in their normal academic courses. It found that of the same sample of 1500 candidates, 682 were taking six subjects, while the rest were taking five subjects. Of the candidates who were taking five subjects, 192 failed the examination.

The table from page 7 is repeated here to help you answer the questions that follow.

	Year 12	Year 13	Total
Passed	347	853	1200
Failed	33	267	300
Total	380	1120	1500

	5 subjects	6 subjects	Total
Passed	626	574	1200
Failed	192	108	300
Total	818	682	1500

(i) What proportion of candidates in the study took six subjects and passed?

574 ÷ 682

= 0.8416 (4 dp)

Incorrect answer.

682 used instead of 1500 – a common error.

(ii) On the evidence of this study, would you recommend that candidates take six subjects?

Support your answer with numerical calculations that consider the absolute and relative risks. You may also wish to comment on the sensibility of drawing any conclusions on this evidence.

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P (Took 6 subjects and passed) = 0.8416

626 ÷ 818

P (Took 5 subjects and passed) = 0.7653 (4dp)

$\frac{0.8416}{0.7653}$

= 1.0997 (4dp)

Correct absolute risks and relative risk found.

Correct supporting statement.

∴ It is approx. 1.1 times more likely to pass while taking 6 subjects. But in saying this it is not sensible to draw a conclusion as there is to many variables in the data and also 1.1 times is not enough to confidently say that taking 6 subjects gives you a ~~..~~ better chance of passing. By too many variables I mean that the type of subjects where not listed and if one person was to take another subject to ~~then~~ increase the likelyhood of them passing, this subject may be relatively 'easy' and does not effect there learning greatly anyway.

t

E8

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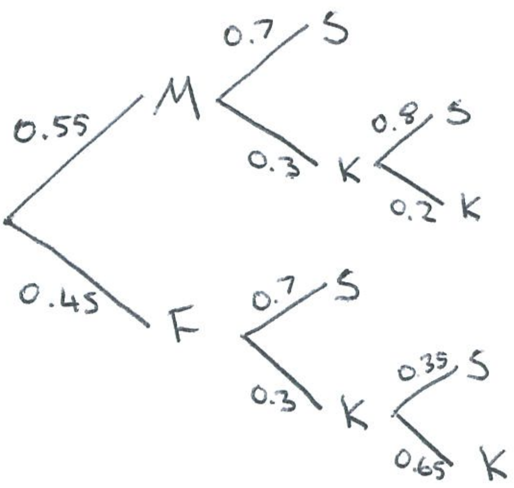
QUESTION THREE

(a) When calves are born into a pedigree beef herd, decisions are made after they are one month old, and again when they are three months old, as to whether they will be kept in the herd or sold.

55% of calves born are male. At age one month, 70% of male and 20% of female calves are sold. Of the remainder, at age three months, 80% of males and 35% of females are also sold.



(i) Find the probability that a randomly chosen calf born into the herd will be male and sold at age one month.

$P = 0.55 \times 0.7$

$P = 0.385$

Correct answer.

u

(ii) Find the probability that a randomly chosen calf born into the herd will be female and sold at age three months.

$P = 0.45 \times 0.3 \times 0.35$

$P = 0.04725$

Incorrect answer as a result of tree diagram.

n

(iii) What percentage of calves will eventually be kept in the pedigree herd?

$P = (0.55 \times 0.3 \times 0.2) + (0.45 \times 0.3 \times 0.65)$

$P = 0.12$ (2dp)

$= 12\%$

Incorrect answer as a result of tree diagram.

u

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(iv) In a particular year 550 calves were born.

How many male calves can be expected to be kept in the pedigree herd?

$0.55 \times 0.3 \times 0.2$

$= 0.033$

0.033×550

$= 18.15$

∴ 18 or 19 calves would be expected to be kept

Correct answer.

a

(v) The ratio of male to female calves being kept in the herd after three months is about one male to every seven females. This is to be changed to one male to every ten females.

1:7

1 : 10

If the number of male calves remains the same, what proportion of females would have been sold?

0. 033 = 1 male

Incomplete response.

n

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(b) New Zealand fantails are birds which are either pied or black.

Pied fantail Black fantail

Cherryl Mariner, www.nzbirdsonline.org.nz/species/new-zealand-fantail

They interbreed, and pairs with successful nests are found in the following proportions:

Pair	**Two pied fantails**	**One pied and one black fantail**	**Two black fantails**
Proportion	0.75	0.2	0.05

Successful nests have between one and four eggs. The proportions of eggs are given in the table below.

Pair	**Two pied fantails**	**One pied and one black fantail**	**Two black fantails**
One egg	0.15	0.2	0.3
Two eggs	0.3	0.35	0.5
Three eggs	0.4	0.35	0.15
Four eggs	0.15	0.1	0.05

(i) What proportion of pairs with two pied fantails will have a successful nest with more than one egg?

~~0.3 + 0.4 + 0.15~~ ~~P = 0.85~~

$0.3 + 0.4 + 0.15$

$= 0.85 \times 0.75$

$P = 0.6375$

n

Incorrect answer due to multiplication by 0.75 – a common error.

(ii) A researcher claims that only one out of every 50 nests found with three eggs is likely to be from a pair of two black fantails.

Use calculations to show that the researcher's claim is justified.

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$\frac{1}{50}$

~~0.18 = 1/56~~

P(black Fantails | 3 eggs)

$$P = \frac{(0.15 \times 0.05)}{(0.15 \times 0.05) + (0.35 \times 0.2) + (0.4 \times 0.75)}$$

P = 0.01987 (5dp)

P = 0.02 (rounded 0dp)

$\frac{1}{50} = 0.02$

Correct answer.

t

∴ researcher's claim is justified as it ~~is~~ probable that for every 50 nests 1 is found with 3 eggs from a pair of two black fantails $(0.02 = \frac{1}{50})$

E7