

91577M



915775

Tuhia he (X) ki te pouaka mēnā kāore koe i tuhi kōrero ki tēnei puka

☐

+



Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Te Tuanaki, Kaupae 3, 2023

91577M Te whakahāngai i te taurangi o ngā tau pohewa i te whakaoti rapanga

Ngā whiwhinga: E rima

| Paetae | Kaiaka | Kairangi |
|--|---|--|
| Te whakahāngai i te taurangi o ngā tau pohewa i te whakaoti rapanga. | Te whakahāngai i te taurangi o ngā tau pohewa i te whakaoti rapanga, mā roto i te whakaaro pānga. | Te whakahāngai i te taurangi o ngā tau pohewa i te whakaoti rapanga, mā roto i te whakaaro waitara e whānui ana. |

Tirohia kia kitea ai e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATO A kei roto i tēnei pukapuka.

Tirohia kia kitea ai kei a koe te pukapuka Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Whakaaturia ō whiriwhiringa KATO A.

Ki te hiahia wāhi atu anō koe mō ō tuhinga, whakamahia ngā whārangi wātea kei muri o tēnei pukapuka.

Tirohia kia kitea ai e tika ana te raupapatanga o ngā whārangi 2–23 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi i ngā wāhi e kitea ai te kauruku whakahāngai (). Ka tapahia taua wāhanga ka mākahia ana te pukapuka.

HOATU TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

(a) Tuhia te $(5 - 2\sqrt{p})^2$ kia pēnei te takoto $a + bp + c\sqrt{p}$, arā, he tau tōpū te a , te b , me te c .

(c) Mehemea ko te $z = p + qi$, ko te $w = a + bi$, \bar{a} , ko te $\operatorname{Re}\left(\frac{z}{w}\right) = 0$, whakaaturia ko te $ap = -bq$.

QUESTION ONE

- (a) Write $(5 - 2\sqrt{p})^2$ in the form $a + bp + c\sqrt{p}$ where a , b , and c are integers.

- (b) Find the value(s) of r so that the quadratic equation $4x^2 - 4x + 3r - 2 = 0$ has no real roots.

- (c) If $z = p + qi$ and $w = a + bi$ and $\operatorname{Re}\left(\frac{z}{w}\right) = 0$, then show that $ap = -bq$.

(d) Ko tētahi o ngā otinga o te whārite $z^3 - 8z^2 + 6z + d = 0$, ko te $z = 5 - i$.

Mehemea he tūturu te d , whiriwhiria te uara o te d me ērā atu otinga e rua o te whārite.

- If d is real, find the value of d and the other two solutions of the equation.

(e) Ko ngā tau pohewa u me te v , ko te $u = 3 + i$ me te $v = 1 + 2i$.

Whakatauria te/ngā uara e taea ana o te tau pūmau tūturu k mehemea ko te $\left| \frac{u}{v} + k \right| = \sqrt{k+2}$.

- (e) The complex numbers u and v are $u = 3 + i$ and $v = 1 + 2i$.

Determine the possible value(s) of the real constant k if $\left| \frac{u}{v} + k \right| = \sqrt{k+2}$.

(a) Mehemea ko te $u = q^6 \operatorname{cis} \frac{5\pi}{8}$, ā, ko te $v = q^2 \operatorname{cis} \frac{2\pi}{5}$, tuhia te $\frac{u}{v}$ kia pēnei te takoto $r \operatorname{cis} \theta$.

QUESTION TWO

- (a) If $u = q^6 \operatorname{cis} \frac{5\pi}{8}$ and $v = q^2 \operatorname{cis} \frac{2\pi}{5}$, write $\frac{u}{v}$ in the form $r \operatorname{cis} \theta$.

- (b) If $z = 1 + ki$ and $w = 7 - ki$, then find $|z - w|$, giving your answer in terms of k .

- (c) Find $\operatorname{Arg}(z)$ if $\frac{13z}{z+1} = 11 - 3i$.

Tuhia $t\bar{o}/\bar{o}$ otinga i te takotoranga ahuroa, e ai ki te m .

- (d) Solve the equation $z^3 + 64m^{12} = 0$, where m is a real constant.

Write your solution(s) in polar form, in terms of m .

- Whiriwhiria te whārite whātinga takirua (*Cartesian*) o te huanui ME te uara o te m .

- (e) The straight line with equation $y = mx - 1$, where m is a real constant and $m > 0$, is a tangent to the locus described by $|z - 2 + i| = \sqrt{3}$.

Find the Cartesian equation of the locus AND the value of m .

Te Tuanaki 91577M , 2023

(a) When the polynomial $2x^3 + px^2 + 7x - 3$ is divided by $x + 3$, the remainder is 30.

- Given that $\frac{u}{v} = 3 + 4i$, find the value of n .

- $$4\sqrt{(4x-w)} = 5 - 8\sqrt{x}$$

- (d)

**He whārangi anō ki te hiahiatia.
Tuhia te tau tūmahi mēnā e hāngai ana.**

Extra space if required.
Write the question number(s) if applicable.

QUESTION
NUMBER

**He whārangi anō ki te hiahiatia.
Tuhia te tau tūmahi mēnā e hāngai ana.**

[illegible]

Extra space if required.
Write the question number(s) if applicable.

QUESTION
NUMBER

English translation of the wording on the front cover

Level 3 Calculus 2023

91577M Apply the algebra of complex numbers
in solving problems

Credits: Five

91577M

| Achievement | Achievement with Merit | Achievement with Excellence |
|---|---|--|
| Apply the algebra of complex numbers in solving problems. | Apply the algebra of complex numbers, using relational thinking, in solving problems. | Apply the algebra of complex numbers, using extended abstract thinking, in solving problems. |

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–23 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (DO NOT WRITE). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.