*See back cover for an English translation of this cover*

3

91577M

915775

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

**QUALIFY FOR THE FUTURE WORLD**
**KIA NOHO TAKATŪ KI TŌ ĀMUA AO!**

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Tohua tēnei pouaka mēnā KĀORE koe i tuhi kōrero ki tēnei pukapuka

# Tuanaki, Kaupae 3, 2022

## 91577M Te whakahāngai i te taurangi o ngā tau tuatini i te wā e whakaoti rapanga ana

Ngā whiwhinga: E rima

| Paetae | Kaiaka | Kairangi |
|---|---|---|
| Te whakahāngai i te taurangi o ngā tau tuatini i te wā e whakaoti rapanga ana. | Te whakahāngai i te taurangi o ngā tau tuatini i te wā e whakaoti rapanga ana, mā roto i te whakaaro pānga. | Te whakahāngai i te taurangi o ngā tau tuatini i te wā e whakaoti rapanga ana, mā roto i te whakaaro waitara e whānui ana. |

Tirohia kia kitea ai e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

**Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.**

Tuhia ō whiriwhiringa KATOA.

Tirohia kia kitea ai kei a koe te pukapuka Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Ki te hiahia wāhi atu anō koe mō ō tuhinga, whakamahia ngā whārangi wātea kei muri o tēnei pukapuka.

Tirohia kia kitea ai e tika ana te raupapatanga o ngā whārangi 2–15 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi i ngā wāhanga e kitea ai te kauruku whakahāngai (). Ka tapahia pea taua wāhanga i te wā e mākahia ana te pukapuka.

**HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.**

## TE TŪMAHI TUATAHI

(a) Tuhia te $\dfrac{12k}{1+\sqrt{5}}$ i te takotoranga o te $ak + bk\sqrt{5}$, arā, he tau tōpū te $a$ me te $b$.

(b) Mehemea ko te $u = m^5 \text{cis} \dfrac{\pi}{3}$, ā, ko te $v = m^2 \text{cis} \dfrac{\pi}{5}$, tuhia te $\dfrac{u}{v}$ hei takotoranga ahuroa.

(c) Mehemea ko te $u = 3 + 2\text{i}$, ko te $v = 4 + 2\text{i}$, ā, ko te $w = 2 + k\text{i}$, whiriwhiria te uara o te $k$ mēnā ko te $\arg(uvw) = \dfrac{\pi}{4}$.

(d) Whiriwhiria te/ngā uara o te $p$ i runga i te mōhio kotahi anake te otinga tūturu mō te $x - 2\sqrt{x+p} = -5$.

(e) Mō ngā tau tuatini $w$ me te $z$, hāponotia te:

$$|w+z|^2 - |w-\bar{z}|^2 = 4\,\mathrm{Re}(w)\,\mathrm{Re}(z)$$

arā, ko te $\mathrm{Re}(w)$ te wāhanga tūturu o te $w$, ā, ko te $\mathrm{Re}(z)$ te wāhanga tūturu o te $z$.

**QUESTION ONE**

(a) Write $\dfrac{12k}{1+\sqrt{5}}$ in the form $ak + bk\sqrt{5}$, where $a$ and $b$ are integers.

(b) If $u = m^5\text{cis}\dfrac{\pi}{3}$ and $v = m^2\text{cis}\dfrac{\pi}{5}$, write $\dfrac{u}{v}$ in polar form.

(c) If $u = 3 + 2\text{i}$, $v = 4 + 2\text{i}$, and $w = 2 + k\text{i}$, find the value of $k$ if $\arg(uvw) = \dfrac{\pi}{4}$.

(d) Find the value(s) of $p$ for which the equation $x - 2\sqrt{x+p} = -5$ has only one real solution.

(e) For complex numbers $w$ and $z$, prove that:

$$|w+z|^2 - |w-\overline{z}|^2 = 4\,\mathrm{Re}(w)\,\mathrm{Re}(z)$$

where $\mathrm{Re}(w)$ is the real part of $w$, and $\mathrm{Re}(z)$ is the real part of $z$.

**TE TŪMAHI TUARUA**

(a) I te whakawehenga o te $x^3 - 3x^2 + bx + 9$ ki te $(x + 2)$, ka puta te 3 hei toenga.

Whiriwhiria te uara o te $b$.

(b) Whiriwhiria te tau tuatini $z$ mehemea ko te $z + 4\overline{z} = 15 + 12\mathrm{i}$.

(c) Ko tētahi o ngā whakautu o te $z^3 - 2z^2 + hz + 180 = 0$, ko te $z = -4$. (he tau tūturu te $h$).

Whiriwhiria ērā atu whakautu, i te takotoranga o te $a \pm b\mathrm{i}$, me te uara o te $h$.

**QUESTION TWO**

(a) Dividing $x^3 - 3x^2 + bx + 9$ by $(x+2)$ gives a remainder of 3.

Find the value of $b$.

(b) Find the complex number $z$ for which $z + 4\overline{z} = 15 + 12\text{i}$.

(c) One of the solutions of $z^3 - 2z^2 + hz + 180 = 0$ is $z = -4$. ($h$ is a real number).

Find the other solutions, in the form $a \pm b\text{i}$, and the value of $h$.

(d) Mehemea ko te $z = 1-\sqrt{3}\mathrm{i}$, ā, ko te $w = \frac{4}{z} - 2$, whiriwhiria te $\arg(w)$.

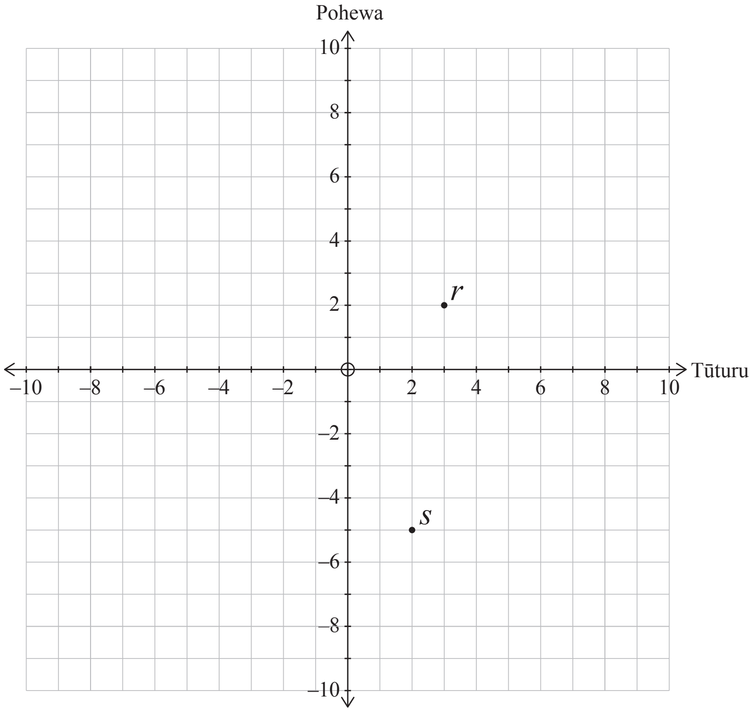
(e) Whiriwhiria te whārite whātuinga takirua (Cartesian) o te huanui e whakaahuatia nei kia $|z+\mathrm{i}| = 2|z-5\mathrm{i}|$ i te takotoranga o te $(x-a)^2 + (y-b)^2 = k^2$.

(d) If $z = 1 - \sqrt{3}\mathrm{i}$ and $w = \frac{4}{z} - 2$, find $\arg(w)$.

(e) Find the Cartesian equation of the locus described by $|z + \mathrm{i}| = 2|z - 5\mathrm{i}|$
in the form $(x-a)^2 + (y-b)^2 = k^2$.

**TE TŪMAHI TUATORU**

(a) E tohua ana ngā tau tuatini ki te $r$ me te $s$ i te papatau hiato (Argand) kei raro nei.




Mehemea ko te $v = 2r - s$, whiriwhiria te $v$, ka tohua ai ki te papatau hiato (Argand) kei runga nei.

(b) Whakaotia te whārite o te $z^2 + 6kz + 15k^2 = 0$ i runga i te mōhio he tau tūturu te $k$.

Tuhia tō otinga i te takotoranga o te $ak \pm \sqrt{b}k\text{i}$, arā, he tau whakahau te $a$ me te $b$.

(c) Whakaotia te whārite o te $z^3 + k^6\text{i} = 0$, arā, he tau pūmau tūturu te $k$.

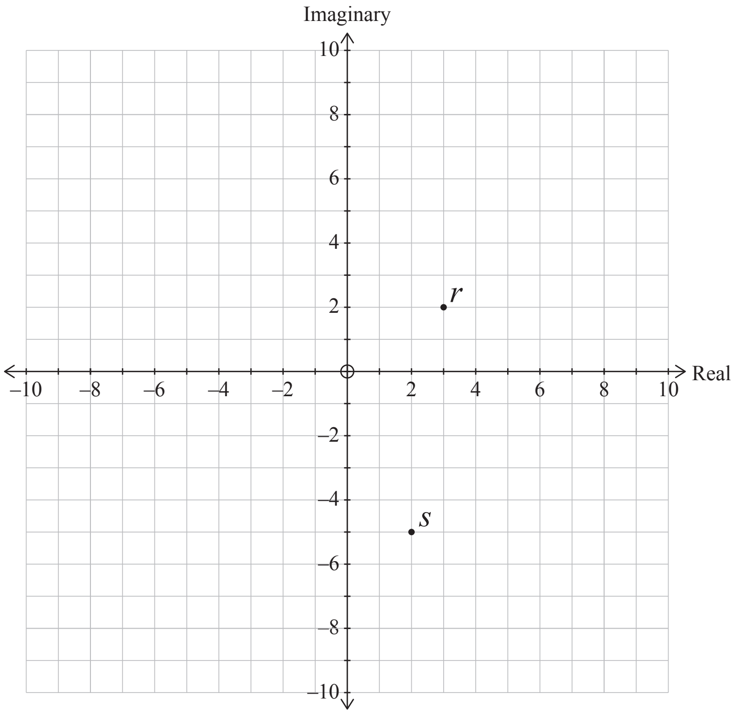
Tuhia tō/ō otinga i te takotoranga ahuroa mō te taurangi o te $k$.

(d) Hāponotia te whakaaro kāore he tau tuatini $z$ mā roto i te $|z| - z = \mathrm{i}$.

(e) Mehemea he tau tuatini te $z = a + b\mathrm{i}$, engari ehara i te kore, ā, ko te $\frac{\mathrm{i}}{z} + \frac{3}{\bar{z}} = 1$, whiriwhiria te uara o te $a$ me te $b$.

**QUESTION THREE**

(a) The complex numbers $r$ and $s$ are represented on the Argand diagram below.




If $v = 2r - s$, find $v$ and mark it on the Argand diagram above.

(b) Solve the equation $z^2 + 6kz + 15k^2 = 0$ in terms of real number $k$.

Give your solution in the form $ak \pm \sqrt{b}k\text{i}$, where $a$ and $b$ are rational numbers.

(c) Solve the equation $z^3 + k^6\text{i} = 0$, where $k$ is a real constant.

Give your solution(s) in polar form in terms of $k$.

(d) Prove that there is no complex number $z$ such that $|z| - z = \mathrm{i}$.

(e) If $z = a + b\mathrm{i}$ is a non-zero complex number, and $\frac{\mathrm{i}}{z} + \frac{3}{\overline{z}} = 1$, find the values of $a$ and $b$.

**He whārangi anō ki te hiahiatia.**
**Tuhia te tau tūmahi mēnā e hāngai ana.**

TE TAU TŪMAHI

**Extra space if required.**
**Write the question number(s) if applicable.**

QUESTION NUMBER

*English translation of the wording on the front cover*

# Level 3 Calculus 2022

## 91577M Apply the algebra of complex numbers in solving problems

Credits: Five

91577M

| Achievement | Achievement with Merit | Achievement with Excellence |
|---|---|---|
| Apply the algebra of complex numbers in solving problems. | Apply the algebra of complex numbers, using relational thinking, in solving problems. | Apply the algebra of complex numbers, using extended abstract thinking, in solving problems. |

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

**You should attempt ALL the questions in this booklet.**

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–15 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area. This area may be cut off when the booklet is marked.

**YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.**