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91577

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NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
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QUALIFY FOR THE FUTURE WORLD
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Level 3 Calculus, 2016

91577 Apply the algebra of complex numbers in solving problems

9.30 a.m. Wednesday 23 November 2016
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply the algebra of complex numbers in solving problems.	Apply the algebra of complex numbers, using relational thinking, in solving problems.	Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–8 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Achievement

TOTAL 09

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QUESTION ONE

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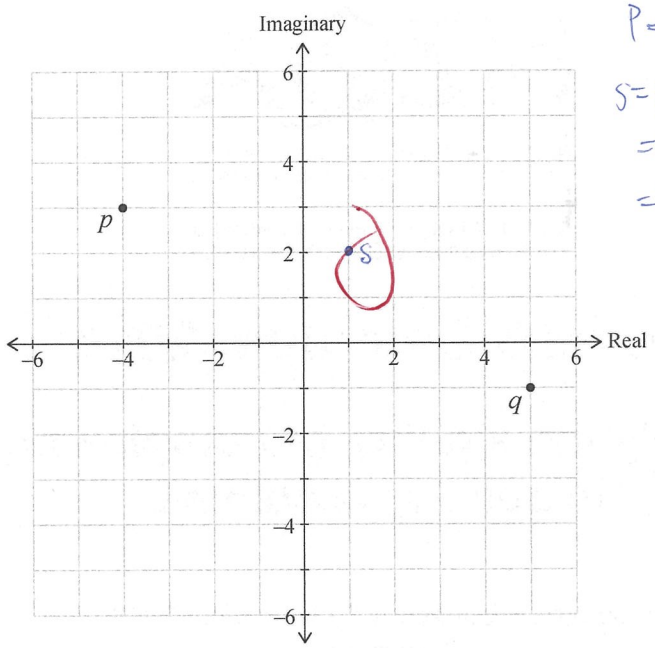
(a) Complex numbers p and q are represented on the Argand diagram.

If $s = p + q$, then show s on the Argand diagram below.



$P=(-4+3i)$, $q=(5,-i)$

$s=p+q$
$=(-4+3i)+(5-i)$
$=(1+2i)$

(b) Dividing $2x^3 + 5x^2 + Ax + 7$ by $x + 3$ gives a remainder of 16.

What is the value of A?

$f(-3) = 2x^3+5x^2+Ax+7+R$
$=2(-3)^3+5(-3)^2+A(-3)+7+16$
$=14-3A$
$3A=14$
$A=4\frac{2}{3}$

(c) Solve the equation $5-\sqrt{x}=\sqrt{x-p}$ for x in terms of p.

$\frac{5-(x)^{\frac{1}{2}}}{5+(x)^{\frac{1}{2}}} = \frac{(x-p)^{\frac{1}{2}}}{5+(x)^{\frac{1}{2}}}$

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(d) If $w = 1 + 2i$, find the value of $w^2 + \frac{w}{\overline{w}}$, giving your answer in the form $a + bi$, where a and b are real.

You must clearly show each step of your working.

$w^2 = (1+2i)(1+2i)$

$= 1 + 4i + 4i^2$

$[i^2 = -1]$

$= 1 + 4i + 4(-1)$

$= -3 + 4i$

$\overline{w} = 1 - 2i$

$\frac{w}{\overline{w}} = \frac{1+2i}{1-2i} \quad \frac{(1+2i)}{(1+2i)}$

$= \frac{(-3+4i)}{1^2 + 2i^2} \quad [i^2 = -1]$

$= \frac{(-3+4i)}{-1}$

$\frac{w}{\overline{w}} = (+3 - 4i)$

$w^2 + \frac{w}{\overline{w}} = (-3+4i) + (3-4i)$

$= 0 + 0i$

(e) The locus described by $|z - 2 + 3i| = |z - 1|$ is a straight line.

Find the gradient of that line.

$|z - 2 + 3i| = |z - 1|$

$|z + 3i| = |z + 1|$

$3i = 1$

$(1 - 3i) = 0$

QUESTION TWO

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(a) Solve the equation $x^2 - 6x + 12 = 0$.

Write your answer in the form $a \pm \sqrt{b}\,i$, where a and b are rational numbers.

$\Delta = b^2 - 4ac$

$= (-6)^2 - 4(1)(12)$

$= -36 - 48$

$= -84$

(b) $u = 2 + 3i$ and $v = 5 + mi$.

Find the value of m if $uv = 22 + 7i$.

$uv = 22 + 7i$

$(2+3i)(5+mi)$

$10 - 2mi - 15i + 3mi^2 \quad [i^2 = -1]$

$10 - 2mi - 15i + 3m(-1)$

$10 - 5mi - 15i = 22 + 7i$

$-12 - 5mi - 22i = 0$

$-12 - i(5m + 22) = 0$

$-12 - (5m + 22)i = 0$

$-12 = 5m + 22i$

(c) Solve the equation $z^3 = -8k^6$, where k is real.

Write your solutions in polar form in terms of k.

$z^3 = -8k^6$

$z = (-8k^6)^{\frac{1}{3}}$

(d) Prove that $\left|\frac{4+2i}{1+i}\right| = \sqrt{10}$.

You must clearly show each step of your working.

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$= \frac{(4+2i)(1-i)}{(1+i)(1-i)}$

$= \frac{4-4i+2i-2i^2}{1^2+i^2}$ $[i^2=-1]$

$= \frac{4-2i-2(-1)}{1-(-1)}$

$= \frac{6-2i}{2}$

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(e) Find the value of k if the equation $8-x+2\sqrt{2x+k}=0$ has equal roots.

N2

QUESTION THREE

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(a) Write $\frac{5}{2+\sqrt{3}}$ in the form $a+b\sqrt{c}$.

$= \frac{5}{2+\sqrt{3}} \frac{(2-\sqrt{3})}{(2-\sqrt{3})}$

$= \frac{10-5\sqrt{3}}{2^2-3}$

$= 10-5\sqrt{3}$

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(b) If $v = 4\,\text{cis}\,\frac{3\pi}{4}$ and $w = 6\,\text{cis}\,\frac{2\pi}{3}$, write the exact value $\frac{v}{w}$ in polar form.

$\frac{v}{w} = \frac{4\,\text{cis}\,\frac{3\pi}{4}}{6\,\text{cis}\,\frac{2\pi}{3}}$

$= \left(\frac{4}{6}\right)\text{cis}\left(\frac{3\pi}{4}-\frac{2\pi}{3}\right)$

$= \frac{2}{3}\,\text{cis}\left(\frac{\pi}{12}\right)$

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(c) $z = 3 - 4i$ is one solution of the equation

$z^3 - 8z^2 + Bz - 50 = 0$.

Find the value of B.

$z^3 = (3-4i)(3-4i)(3-4i)$

$= (9-12i-12i+16i^2)(3-4i)$

$= (9-24i+16i^2)(3-4i)$

$= 27-36i-72i+96i^2+48i^2-64i^3$

$= 27-108i+144i^2-64i^3$

Substitute z: $(3-4i)^3 - 8(3-4i)^2 + B(3-4i) - 50 = 0$

$(-117-44i) - (56-192i) + (3B-B4i) - 50 = 0$

$3B - B4i = 223+148i$

$B(3-4i) = 223+148i$

$B = \frac{223+148i}{(3-4i)}$

$= \frac{77}{25} + \frac{1336}{25}i$

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(d) If u and v are complex numbers, prove that $\overline{uv} = \overline{u} \cdot \overline{v}$.

let $u = (a+bi)$, $v = (c+di)$

$\overline{u} = (a-bi)$, $\overline{v} = (c-di)$

$\overline{uv} = (a-bi)(c-di)$

$= ac - adi - bci + bdi^2$

$\overline{uv} = (a+bi)(c+di)$

$= (a+bi) - (c+di)$

(e) u and v are two complex numbers, such that $|u+v|^2 = |u-v|^2$.

Prove that $u\overline{v}$ is purely imaginary.

AU

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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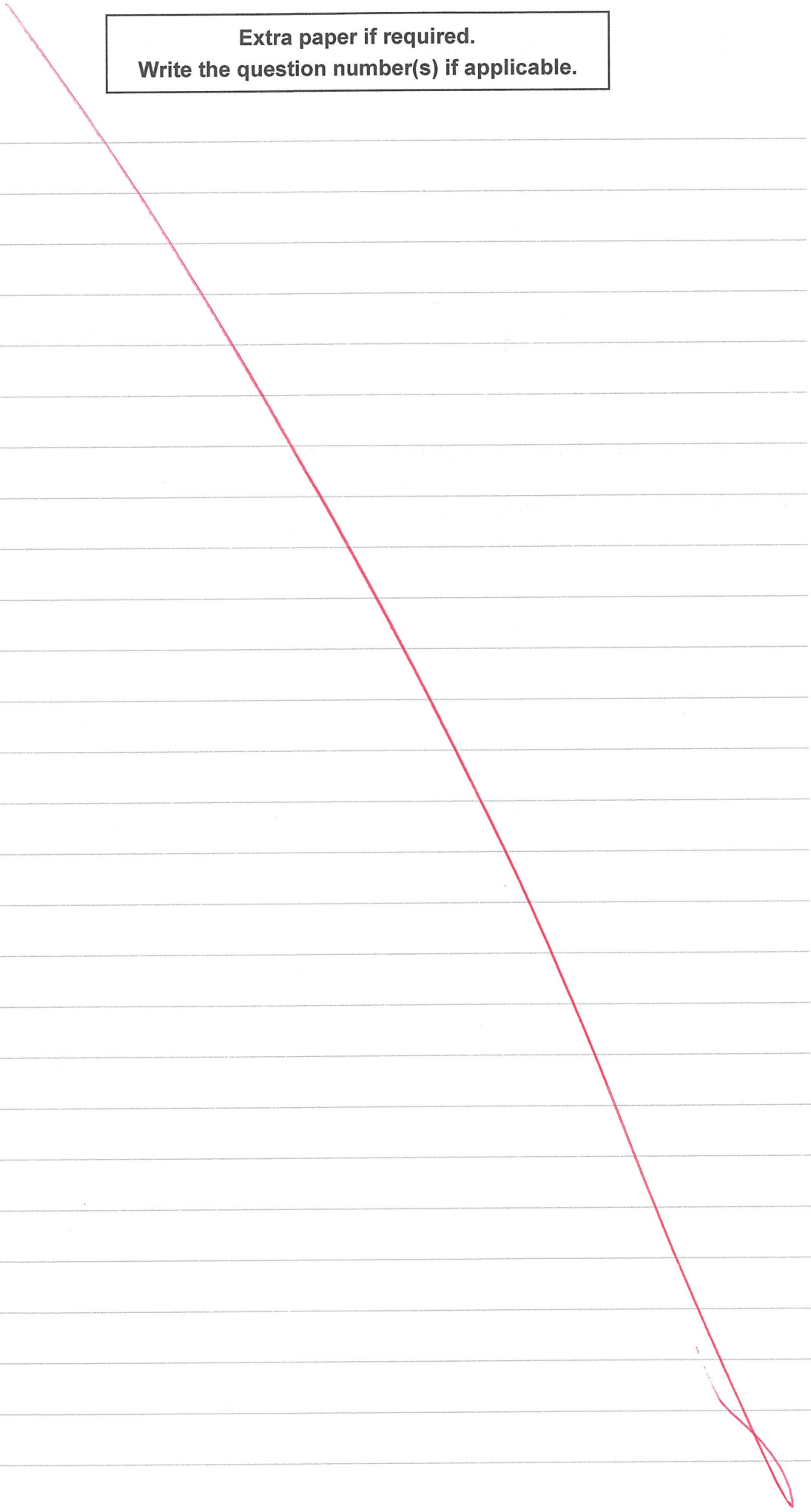

Annotated Exemplar Template

Achieved exemplar 2016

Subject:	Calculus	Standard:	91577	Total score:	09

Q	Grade score	Annotation
1	A3	This question provides evidence for A3 because the candidate has gained 2 u grades for their efforts in parts a) and d) a) The position of s on the Argand diagram is clearly identified. b) The candidate has not equated f(-3) to 16. c) The candidate has made no progress towards a solution d) The candidate has correctly calculated w^2, but has not successfully completed the problem. The candidate has not been able to correctly multiply (1+2i)(1-2i). e) The candidate has made no meaningful progress towards a solution.
2	N2	This question provides evidence for N2 because the candidate has gained 1 u grade for their efforts in part d) a) The candidate has supplied the discriminant, not the solution to the quadratic. b) All terms in line 2 should be positive. c) The candidate has made no progress towards a solution. d) The complex expression has been simplified but the modulus has not been calculated. e) Not attempted
3	A4	This question provides evidence for A4 because the candidate has gained 3 u grades for their efforts in parts a), b) and d) a) The denominator has been successfully rationalised b) The complex numbers have been successfully divided in polar form. c) The candidate has calculated z^3 but has an error in the next term, which should be +56. d) The candidate has not calculated the product of the conjugates, or simplified the conjugate of the product. e) Not attempted.