3

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91577

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NZQA

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QUALIFY FOR THE FUTURE WORLD
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Level 3 Calculus, 2017

91577 Apply the algebra of complex numbers in solving problems

9.30 a.m. Thursday 23 November 2017
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply the algebra of complex numbers in solving problems.	Apply the algebra of complex numbers, using relational thinking, in solving problems.	Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Achievement

TOTAL 09

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QUESTION ONE

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(a) If $u = 2 + 3i$ and $v = 1 - 4i$, find $\bar{u} - 3v$, giving your solution in the form $a + bi$.

$\bar{u} = 2 - 3i \qquad 3v = 3(1 - 4i) = 3 - 12i$

$\bar{u} - 3v = (2 - 3i) - (3 - 12i) = -1 + 9i$

(b) Write $\dfrac{36}{5 - \sqrt{7}}$ in the form $a + b\sqrt{7}$, where a and b are integers.

$\dfrac{36}{(5 - \sqrt{7})} \times \dfrac{(5 + \sqrt{7})}{(5 + \sqrt{7})} = \dfrac{180 + 36\sqrt{7}}{25 + 5\sqrt{7} - 5\sqrt{7} - 7} = \dfrac{180 + 36\sqrt{7}}{18}$

$= 10 + 2\sqrt{7}$

(c) Solve the following equation for x in terms of p:

$$p\sqrt{x - 2} - 5\sqrt{x} = 0$$

$p\sqrt{x - 2} = 5\sqrt{x}$

$p^2(x - 2) = 25x$

$p^2 = \dfrac{25x}{(x - 2)}$

(d) One solution of the equation $z^3 - 2z^2 + Bz - 30 = 0$ is $z = -2 - i$.

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If B is a real number, find the value of B and the other two solutions of the equation.

$z^2 = (-2-i)(-2-i) = 4 + 2i + 2i + i^2 = (3 + 4i)$

$z^3 = (3+4i)(-2-i) = -6 - 3i - 8i - 4i^2 = (-2 - 11i)$

$(-2 - 11i) - (2(3+4i)) + B(-2-i) - 30 = 0$

$(-2 - 11i) - (6 + 8i) + (-2B - Bi) - 30 = 0$

$-8 - 19i - 2B - Bi - 30 = 0$

(e) Find the Cartesian equation of the locus described by $|z + 2 - 7i| = 2|z - 10 + 2i|$.

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Write your answer in the form $(x + A)^2 + (y + B)^2 = K$.

$x + iy + 2 - 7i = 2|x + iy - 10 + 2i|$

$x + iy + 2 - 7i = 2x + 2iy - 20 + 4i$

$-iy - 11i = x - 22$

QUESTION TWO

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(a) Dividing $x^3 - 2x^2 + 5x + d$ by $(x - 3)$ gives a remainder of 13.

Find the value of d.

$3^3 - (2(3)^2) + (5 \times 3) + d = 13$

$27 - 18 + 15 + d = 13$

$24 + d = 13$

$d = -11$

(b) Simplify, as far as possible, the expression $\sqrt{2k}\left(\sqrt{18k} - \sqrt{8k}\right)$.

$6k - 4k$

$= 2k$

(c) z and w are complex numbers such that $z = -2 + 3i$ and $zw = 15 - 3i$.

Find an exact value of arg(w).

$w(-2 + 3i) = (15 - 3i)$

$w = \dfrac{(15 - 3i)}{(-2 + 3i)}$

$w = -7.5 - i$

~~$w = -6.5$~~

~~$w = +5$~~

$\tan^{-1}\left(\dfrac{-7.5}{-1}\right) = \theta$

$\arg(w) = 1.438$

(d) Solve the equation $z^4 = \frac{m}{\sqrt{2}} + \frac{m}{\sqrt{2}}\mathrm{i}$, where m is real and positive.

Write your solutions in polar form in terms of m.

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(e) Find all possible values of k that make $u = \dfrac{k+4\mathrm{i}}{1+k\mathrm{i}}$ a purely real number.

A3

QUESTION THREE

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(a) If $u = p^3 \text{cis} \frac{\pi}{3}$ and $v = p \text{cis} \frac{\pi}{8}$, write $\frac{u}{v}$ in polar form.

$p^3 \text{cis} \frac{\pi}{3} \div p \text{cis} \frac{\pi}{8}$

$= p^2 \text{cis} \left(\frac{\pi}{3} - \frac{\pi}{8} \right)$

$= p^2 \text{cis} \frac{5\pi}{24}$

(b) Solve the equation $x^2 - 6x + 14 = 0$.

Give your solution in the form $a \pm \sqrt{b}\,\text{i}$, where a and b are rational numbers.

$(-6)^2 - 4 \times 1 \times 14 = -20$

$= \frac{6 \pm \sqrt{-20}}{2}$

$= \frac{6 \pm \sqrt{20}\,\text{i}}{2}$

$= 3 \pm \sqrt{20}\,\text{i}$

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(c) $\dfrac{3x^3+8x^2-2x+11}{x+2} = 3x^2+Ax+B+\dfrac{C}{x+2}$, where A, B, and C are integers.

Find the values of A, B, and C.

$(x+2)\left(3x^2+Ax+B+\dfrac{C}{(x+2)}\right)$

$= 3x^3+Ax^2+Bx+\dfrac{Cx}{(x+2)}+6x^2+2Ax+2B+\dfrac{2C}{(x+2)}$

(d) Solve the equation $\dfrac{8+x}{x} = \sqrt{3}$, writing your solution in the form $x = a+b\sqrt{3}$.

$\left(\dfrac{8+x}{x}\right)^2 = (\sqrt{3})^2$

$= \dfrac{64+16x+x^2}{x^2} = 3$

$= x^2+16x+64 = 3x^2$

$= +2x^2-16x-64 = 0$

$(-16)^2 - 4\times 2\times -64$

$= 256 - -512$

$= 768$

$\dfrac{16+\sqrt{768}}{4}$

Question Three continues on the following page.

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(e) z is a complex number such that $z = \dfrac{a+b\mathrm{i}}{a-b\mathrm{i}}$, where a and b are real numbers.

Prove that $\dfrac{z^2+1}{2z} = \dfrac{a^2-b^2}{a^2+b^2}$.

$z^2 = \dfrac{(a+bi)}{(a-bi)} \times \dfrac{(a+bi)}{(a-bi)} = \dfrac{a^2+abi+abi+bi^2}{a^2-abi-abi+bi^2} = \dfrac{a^2+2abi-b}{a^2-2abi-b}$

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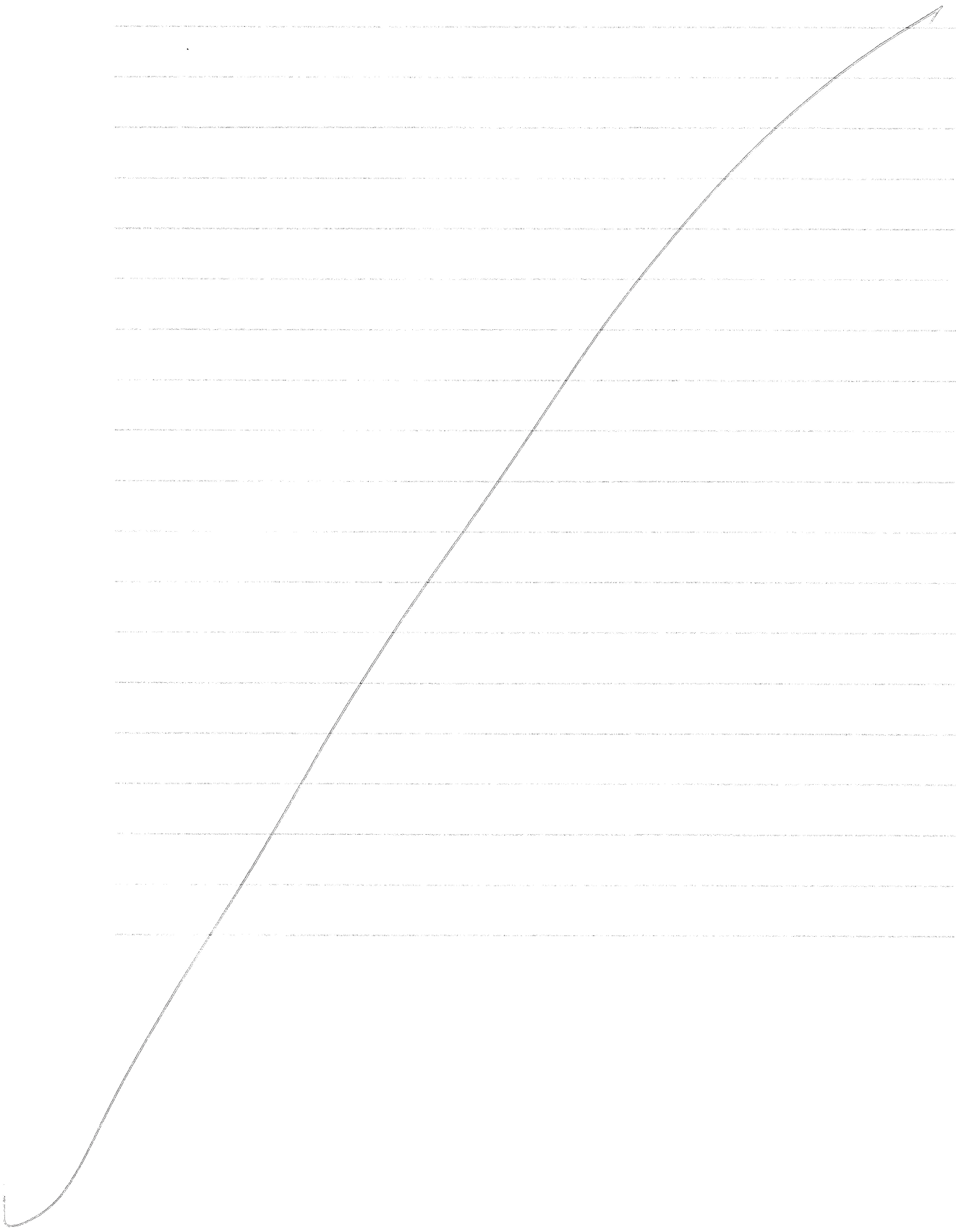

N2

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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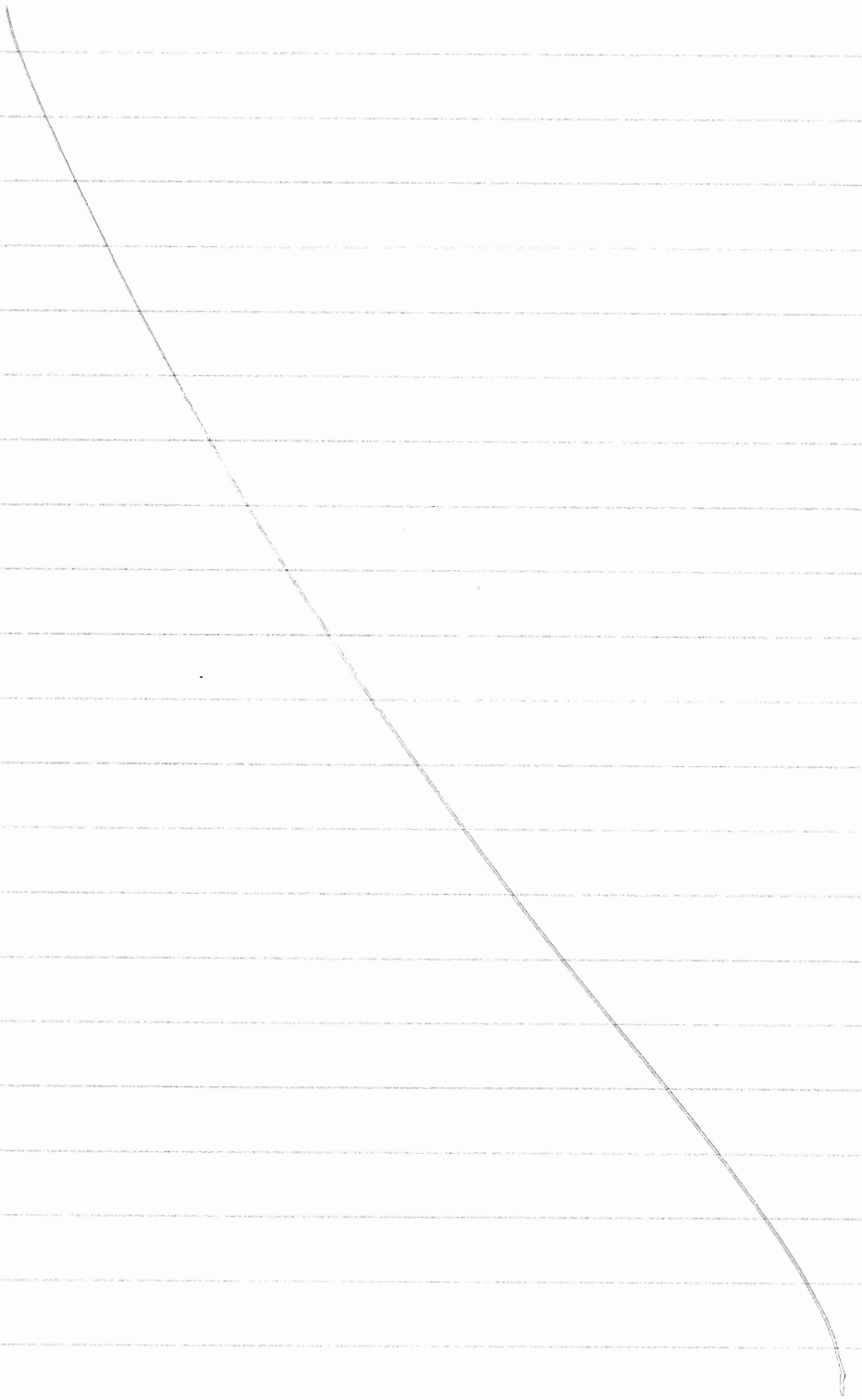

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Subject:	Calculus	Standard:	91577	Total score:	9

Q	Grade score	Annotation
1	A4	This question provides evidence for A4 because the candidate has gained 3 u grades for their efforts in parts a , b & c. a) The candidate has found $\bar{u} - 3v$ b) The candidate has correctly found an expression in the form, $a + b\sqrt{7}$ by rationalising the denominator. c) The candidate has correctly squared both sides of the equation but not given x in terms of p. d) The candidate has not made significant progress towards a solution. e) The candidate has not made significant progress towards a solution.
2	A3	This question provides evidence for A3 because the candidate has gained 2 u grades for their efforts in parts a & b. a) The candidate has correctly used remainder theorem to find the value of d. b) The candidate has correctly found an equivalent expression in its most simple form. c) The candidate has not successfully divided zw by z to give w. d) e)
3	N2	This question provides evidence for N2 because the candidate has gained 1 u grade for their efforts in part a) a) The candidate has correctly found an expression for, $\frac{u}{v}$ in polar form. b) The candidate has not correctly solved the equation due to an error in simplification. c) The candidate has not made significant progress towards a solution. d) The candidate has not made enough progress towards a solution. The solution is not in the required form and lacks the ± symbol signifying the two possible solutions. e) The candidate has not made significant progress towards a solution.