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91261



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NEW ZEALAND QUALIFICATIONS AUTHORITY
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SUPERVISOR'S USE ONLY

Level 2 Mathematics and Statistics, 2018

91261 Apply algebraic methods in solving problems

9.30 a.m. Wednesday 14 November 2018
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply algebraic methods in solving problems.	Apply algebraic methods, using relational thinking, in solving problems.	Apply algebraic methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You are required to show algebraic working in this paper. Guess-and-check methods, and correct answer(s) only, will generally limit grades to Achievement.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Merit
18
TOTAL

ASSESSOR'S USE ONLY

QUESTION ONE

ASSESSOR'S
USE ONLY

- (a) Simplify fully
- $(25m^{16})^{\frac{1}{2}}$

$$\underline{\underline{5m^8}}$$

- (b) Simplify fully
- $\left(\frac{4}{3a}\right)^{-2}$
- , leaving your answer with a positive index.

$$\frac{4^{-2}}{3^{-2}a^{-2}}$$

$$\underline{\underline{\frac{9a^2}{16}}}$$

- (c) Write
- $4 - \frac{b+8c}{3c}$
- as a single fraction in its simplest form.

$$\frac{4}{1} - \frac{b+8c}{3c}$$

$$\frac{12c - b - 8c}{3c}$$

$$\underline{\underline{\frac{4c - b}{3c}}}$$

$$\frac{4-b}{3}$$

- (d) Factorise fully
- $4bx + 2xy - 6ab - 3ay$

$$2x(2b+y) - 3a(2b+y)$$

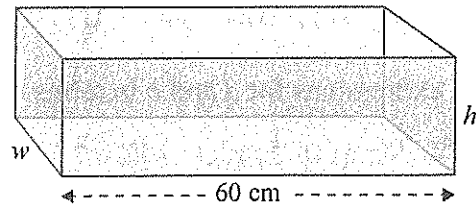
$$\underline{\underline{(2x-3a)(2b+y)}}$$

- (e) A rectangular box has no lid.

The length of the base is 60 cm.

Its height is one quarter of the sum of its width and length.

The total area of the base and the four sides of the box is 7400 cm^2 .



ASSESSOR'S
USE ONLY

Find the height of the box.

$$4h = w + 60$$

$$w = 4h - 60$$

$$A = 60(4h - 60) + 60h + 60h + (4h - 60)h + (4h - 60)h$$

$$74000 = 240h - 1200 + 60h + 60h + 4h^2 - 60h + 4h^2 - 60h$$

$$74000 = 8h^2 - 240h - 1200$$

$$0 = 8h^2 - 240h - 74200 \quad \text{G.C}$$

$$a = 8 \quad b = -240 \quad c = -74200$$

$$h = 112.468 (3dp) \quad \text{or} \quad h = -82.468 (3dp)$$

$$h = 112.468 \text{ cm} \quad \text{because } h \text{ cannot be negative}$$

- (f) $(3x + y)(x - 12y) - (2x + y)(x - 16y)$ can be written in the form $(a + b)^2$.

Find expressions for a and b in terms of x or y .

$$3x^2 - 36xy + xy - 12y^2 - 2x^2 + 16y^2 - xy + 32xy$$

$$\underline{x^2 - 4xy + 4y^2}$$

$$\begin{aligned} (x + 2y)(x + 2y) \\ (x + 2y)^2 \end{aligned}$$

$$a = x \quad b = 2y //$$

QUESTION TWO

ASSESSOR'S
USE ONLY

- (a) Find
- x
- if
- $\log_x 243 = 5$

$$243 = x^5$$

$$3 = x$$

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- (b) Find
- m
- if
- $\log_3(4m - 1) = 2$

$$4m - 1 = 3^2$$

$$4m - 1 = 9$$

$$4m = 10$$

$$m = 2.5$$

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- (c) Find an expression for
- x
- in terms of
- w
- if
- $\frac{3^{4x+1}}{9^x} = 27^{\frac{w}{3}}$

$$\frac{3^{4x+1}}{9^x} = \sqrt[3]{27^w}$$

$$\frac{3^{4x+1}}{9^x} = 9^w$$

$$3^{4x+1} = 81^{w+x}$$

Take log on both sides

$$4x+1 \log 3 = w+x \log 81$$

$$\frac{4x+1}{w+x} = \frac{\log 81}{\log 3}$$

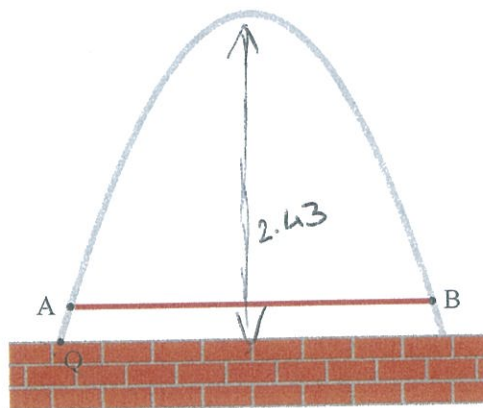
$$\frac{4x+1}{w+x} = 4$$

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- (d) An equestrian jump has a parabolic arch mounted on a wall. Horses and riders jump through the arch.



Source: <http://luxequestrian.com/slideshow/incredible-jumps-brody-robertson>



The arch rises 2.43 metres **above the wall**.

The arch can be modelled by a function of the form $h(x) = kx(3.6 - x)$, where k is a constant, h metres is the height above the wall, and x metres is the horizontal distance from Q.

A rail AB can be placed above the wall and attached at each end to the arch. For one competition, the rail is placed 0.5 metres above the wall.

How long is the rail AB?

$$0 = k \times 0 (3.6 - 0)$$

$$2.43 = kx$$

$$2.43 = kx(3.6 - x) \rightarrow 2.43 = kx(3.6 - x)$$

$$2.43 = 3.6kx - kx^2$$

$$kx^2 - 3.6kx + 2.43 = 0$$

$$a = k \quad b = -3.6k \quad c = 2.43$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = x$$

$$\frac{3.6k \pm \sqrt{(-3.6k)^2 - 4k \times 2.43}}{2k} = x$$

$$\frac{3.6k \pm \sqrt{12.96k^2 - 9.72k}}{2k} = x$$

$$\frac{3.6 \pm \sqrt{12.96k - 9.72}}{2} = x$$

- (e) Interest is compounded on a principal investment, \$P\$, at the end of each year.

If the total amount of the investment after n years is \$A\$ then $A = P\left(1 + \frac{r}{100}\right)^n$

where $r\%$ is the compound interest rate per year.

- (i) Anushka invests \$20 000 at an interest rate of 3.85% (so $A = P(1.0385)^n$).

How many years will it take for her investment to be worth \$25 000?

$$25000 = 20000(1.0385)^n$$

$$1.25 = 1.0385^n \quad \text{take log on both sides}$$

$$\log 1.25 = n \log 1.0385$$

$$\frac{\log 1.25}{\log 1.0385} = n$$

$$5.9 \text{ (1dp)} = n$$

After 6 years.

- (ii) Semisi invests his money at a different interest rate than Anushka's investment.

His investment will double in value after twelve years.

What is the interest rate for Semisi's investment?

$$40000 = 20000\left(1 + \frac{r}{100}\right)^{12}$$

$$2 = \left(1 + \frac{r}{100}\right)^{12}$$

$$1.0595 \text{ (4dp)} = 1 + \frac{r}{100}$$

$$0.0595 = \frac{r}{100}$$

$$5.95 = r$$

interest rate of 5.95 %

QUESTION THREE

ASSESSOR'S
USE ONLY(a) Solve each of the following equations for x :

(i) $12x^2 - 5x = 2$

$$12x^2 - 5x - 2 = 0$$

$$a = 12 \quad b = -5 \quad c = -2 \quad \text{G.C}$$

$$x = \frac{2}{3} \quad \text{or} \quad x = -\frac{1}{4}$$

(ii) $x + 1 - \frac{3}{x} = 0$

$$x - \frac{3}{x} = -1$$

$$x - 3 = -x$$

$$2x - 3 = 0$$

$$2x = 3$$

$$x = \frac{3}{2}$$

(b) Show that the graph of the function $y = 2x^2 - 5x + 6$ does not cross the x -axis.

You must use algebra to support your explanation.

$$\Delta = b^2 - 4ac$$

$$\Delta = b^2 - 4ac$$

$$a = 2 \quad b = -5 \quad c = 6$$

$$b^2 - 4ac < 0$$

$$(-5)^2 - 4 \times 2 \times 6 < 0$$

$$25 - 48 < 0$$

$$-23 < 0$$

No real roots so $y = 2x^2 - 5x + 6$
does not cross the x axis.

Question Three continues
on the following page.

- (c) The equation $3x^2 + kx - 12 = 0$ has two real solutions.

If one of the solutions is $x = 3$, find the other solution.

~~$$b^2 - 4ac > 0 \quad a = 3 \quad b = k \quad c = -12$$~~

~~$$(3x + 4)(x - 3) = 0$$~~
~~$$3x$$~~

$$3x^2 + kx - 36 = 0$$

~~$$(3x + 4)(3x + 4)(3x - 9)$$~~

$$(3x + 4)(x - 9)$$

$$3x^2 - 5x - 12$$

$$k = -5$$



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- (d) Show that the roots of the equation $x^2 + 2(k+1)x - (k^2 + 2k + 5) = 0$, where k is a constant, can never be equal for any real number k .

$$b^2 - 4ac = \Delta \quad a=1 \quad b=2(k+1) \quad c=-(k^2+2k+5)$$

~~$$(2k+2)^2 - 4(k^2+2k+5) > 0$$~~

~~$$(2k+2)(2k+2) - 4(k^2+2k+5) > 0$$~~

~~$$4k^2+8k+4 - 4k^2-8k-20 > 0$$~~

~~$$8k^2+16k+24 > 0$$~~

~~$$a=8 \quad b=16 \quad c=24 \quad G.C$$~~

~~$$8(k^2+2k+3) > 0$$~~

~~$$8k^2+16k > 24$$~~

~~$$8k^2 > 24 - 16k$$~~

~~$$\frac{8k^2}{k} > 24 - 16$$~~

~~$$8k > 8$$~~

~~$$k > 1$$~~

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MS

Merit Exemplar 2018

Subject	Mathematics	Standard	91261	Total score	18
Q	Grade score	Annotation			
1	M6	1e Has substituted incorrect area into equation plus incorrect multiplication. 1f Has not correctly factorised the expression.			
2	E7	2c Has not expressed either side in terms of powers of 3. 2d Has not been able to find $k = 0.75$.			
3	M5	3aii Has been unable to manipulate to a quadratic equal to 0. 3c Has found $k = -5$ but not gone on to find the other root as required. 3d Candidate has correctly substituted into the discriminant but there is no recognition that equal roots would make the quadratic equal to 0.			