91261

2

912610

NZQA
NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

SUPERVISOR'S USE ONLY

# Level 2 Mathematics and Statistics, 2016

## 91261 Apply algebraic methods in solving problems

9.30 a.m. Thursday 24 November 2016
Credits: Four

| Achievement | Achievement with Merit | Achievement with Excellence |
|---|---|---|
| Apply algebraic methods in solving problems. | Apply algebraic methods, using relational thinking, in solving problems. | Apply algebraic methods, using extended abstract thinking, in solving problems. |

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

**You should attempt ALL the questions in this booklet.**

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

**You are required to show algebraic working in this paper. Guess-and-check methods and correct answer(s) only will generally limit grades to Achievement.**

Check that this booklet has pages 2–11 in the correct order and that none of these pages is blank.

**YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.**

Merit

**TOTAL** 14

ASSESSOR'S USE ONLY

## QUESTION ONE

ASSESSOR'S USE ONLY

(a) Simplify $\left(\frac{3b}{c^2}\right)^{-4}$ leaving your answer with positive indices.

$\frac{1}{\left(\frac{3b}{c^2}\right)^4} = \frac{1}{\left(\frac{81b^4}{c^8}\right)}$ A

(b) Write $x^2 - 8x + 10$ in the form $(x-p)^2 + q$.

$(x-4)^2 - 6$

(c) (i) Show that the solutions of the equation $x^2 + x - 56 = 0$ are four times the solutions of the equation $4x^2 + x - 14 = 0$.

$(x+8)(x-7)$ root $= 7, -8$ A

$(4x-7)(x-2)$ root $= \frac{7}{4}, -2$ A

(ii) Find the relationship between the solutions of the equation $dx^2 + ex + f = 0$ and the solutions of the equation $x^2 + ex + df = 0$, where $d$, $e$, and $f$ are real numbers.

A $\frac{e}{d}$ = sum of the roots for $dx^2 + ex + f = 0$

A $e$ = sum of the roots for $x^2 + ex + df = 0$

$\frac{f}{d}$ = product of the root for $dx^2 + ex + f = 0$

$df$ = sum of the roots for $x^2 + ex + df = 0$

The sum of the roots is times by $d$ and the product of the roots is times by $d^2$

ASSESSOR'S USE ONLY

(d) A quadratic equation of the form $ax^2 + bx + c = 0$ has solutions $-\frac{1}{2}$ and $\frac{2}{3}$.

Find a possible set of values for $a$, $b$, and $c$.

$(2x - 1)(3x + 2)$

$6x^2 + x - 2$ CON

$a = 6$

$b = 1$

$c = -2$ CON

(e) Find positive integer value(s) for $k$ so that the quadratic equation $2x^2 + 4kx + (2k^2 + 3k - 11) = 0$ has **real rational** solutions.

Justify your answer.

$b^2 - 4ac \geq 0$

$16k^2 - 8(2k^2 + 3k - 11) \geq 0$

$16k^2 - 16k^2 - 24k + 88 \geq 0$

$-24k + 88 \geq 0$

$88 \geq 24k$

$k \leq 3\frac{2}{3}$

because if the discriminant is $< 0$ the roots are non real so k must be $\leq 3\frac{2}{3}$ A

M5

## QUESTION TWO

ASSESSOR'S USE ONLY

(a) Find the discriminant of the quadratic equation $x^2 = 10x + 3$.

$x^2 - 10x - 3 = 0$

$10^2 - 12 = 88$

(b) Simplify $\frac{4\log(u^3)}{\log u}$.

$u^{11}$

(c) Marie buys a new car for \$24 990.

The car's value decreases continuously by 12% each year.

The value of the car, \$$P$, $t$ years after she first bought it, can be modelled by a function of the form $P = A(r)^t$.

How long will it take for the value of the car to halve? Set P to ~~substitute in~~ 12495

$\$P = 24990 \times 0.88^t$

$12495 = 24990 \times 0.88^t$

$t = 5.42$

5.43

it will take [illegible] years for the value to halve

ASSESSOR'S USE ONLY

(d) (i) Solve the equation $\log_8 x = \frac{2}{3}$.

$x = 4$ //

(ii) Solve the equation $6(\log_8 x)^2 + 2\log_8 x - 4 = 0$.

make $\log_8 \cancel{4}x = u$

$6u^2 + 2u - 4 = 0$

$\left(\frac{2}{3}, -1\right)$

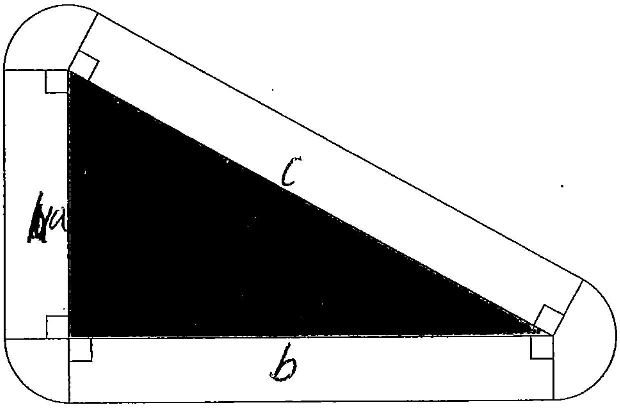
A

$x = 4$ or $x = 0.25$ //

A

(e) The diagram below shows a triangular garden with a path around it.

ASSESSOR'S USE ONLY




The triangular garden has sides with lengths in the ratio 3:4:5.

The path is 1 m wide.

At each corner of the garden, the path is a sector (part) of a circle with a radius of 1 m.

The difference between **twice** the **total** area of the path and the area of the garden is $2\pi$ m$^2$.

Find the length of the longest side of the garden.

(Area of circle = $\pi r^2$)

~~Area of path = $\pi + a + b + c$~~

~~Area of garden = $\frac{1}{2} \times b \times a$~~

~~$2(\pi + a + b + c)\ 2\pi = \frac{1}{2} \times a \times b$~~

~~$4\pi^2 + 4\pi a + 4\pi b + 4\pi c = \frac{1}{2} a \times b$~~

~~$4\pi c = \frac{1}{2} ab + 4\pi a + 4\pi b + 4\pi^2$~~

~~$C = \frac{ab}{2\pi} + a + b + \pi$~~

Area of garden = $3x \times 4x \times \frac{1}{2} = 6x$

Area of path = $\pi + 3x + 4x + 5x$

$\pi + 3x + 4x + 5x =$ $3x \pm 2\pi$

$x = 0.34$

Longest side is $5x$

Longest side = $1.7$ m

M6

ASSESSOR'S USE ONLY

**QUESTION THREE**

(a) Where would the graph of $y = 12x^2 - x - 6$ cut the $x$-axis?

At $x = \frac{3}{4}$ or $-\frac{2}{3}$ //

(b) For what value(s) of $x$ does $\log_x(216) = 3$?

$x = 6$ //

(c) Rearrange the following formula to make $x$ the subject: $\frac{4x}{5} = \frac{y(x+3)}{2}$.

$8x = 5y(x+3)$ A $\quad \frac{8x}{x+3} = 5y$ //

~~$8x = 5yx + 15y$~~

~~$8 = 5y + \frac{15y}{x}$~~

~~$-\frac{15y}{x}$~~ ~~$= 8 - 5y$~~

Question Three continues on the following page.

ASSESSOR'S USE ONLY

(d) Solve the equation $9^{8n+6} = 27^{n^2-1} \times 3^{1-3n}$.

$(8n+6)\log 9 = (n^2-1)\log 27 \times (1-3n)\log 3$

$8n+6 = (n^2-1)\log 27 \times (1-3n)\log 3 \div \log 9$

$\dfrac{8n+6}{(n^2-1)(1-3n)} = \dfrac{\log 27 \times \log 3}{\log 9}$ //

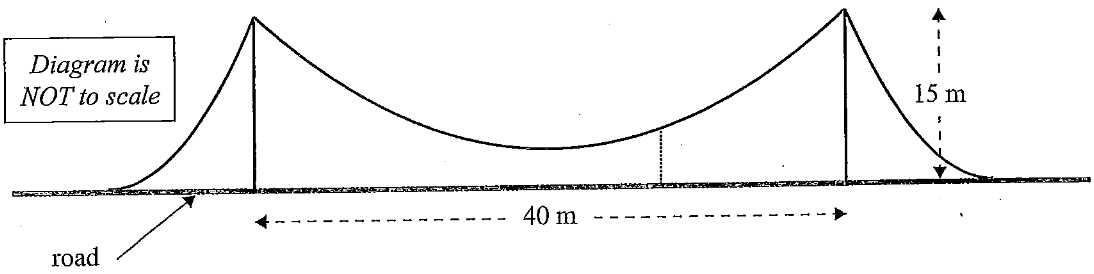
$-3n^3 + n^2 + 3n - 1$ //

(e) A symmetrical bridge has its central cable in the shape of a parabola, as shown in the diagram below.

The towers supporting the cable are each 15 m high and 40 m apart.

At the point midway between the towers, the height of the cable above the road is 3 m.

A vertical post (shown dotted in the diagram) is placed 10 m from the centre of the bridge and just touches the cable.



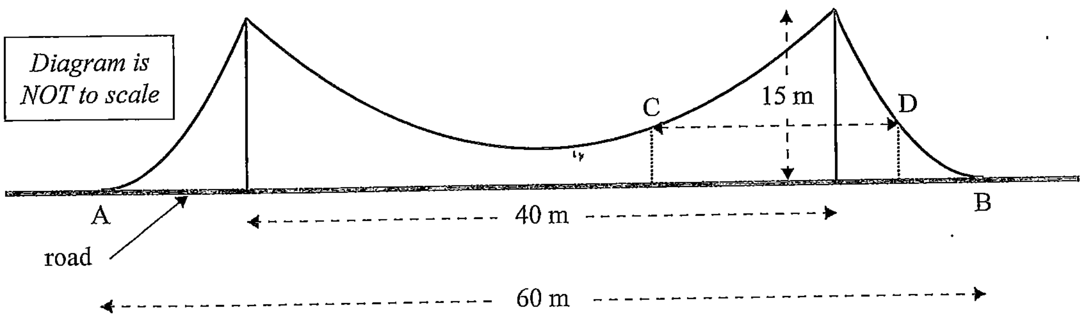

(i) Use ~~algebra to show~~ that the post is 6 m high.

$Ax^2 + bx + 3$ substitute in $(20, 40)$ //

ASSESSOR'S USE ONLY

(ii) The length of the bridge AB is 60 m.

The outside cables are also parabolic and symmetrical in shape, and touch the road at their vertices A and B.




Find the distance, CD, between the two parabolas at a height of 6 m above the road (the distance CD is shown in the diagram).

$ax^2 + bx + 0$ substitute in $(-10, 15)$

$ax^2 + 8.5x$ set $x$ to $6 = 87$

~~[illegible]~~

A3

# Annotated Exemplar Template

**Low Merit exemplar 2016**

| Subject: | Mathematics | Standard: | 91261 | Total score: | 14 |
|---|---|---|---|---|---|

| Q | Grade score | Annotation |
|---|---|---|
| 1 | M5 | 1a Power correctly applied but simplification incomplete<br>1ci Solutions to both equations found, relationship not stated or shown<br>1cii No equations formed correctly. Comparison of sum and product of roots flawed; incorrect sum of roots, no reference as to which equation is being referred to in relationship statement<br>1d Incorrect quadratic formed, values for a, b and c stated were consistent<br>1e Correct substitution into discriminant, inequality in solution for k. No integer values which may provide real rational solutions. |
| 2 | M6 | 2a Incorrectly calculated discriminant<br>2b No shown application of log rules, incorrect answer<br>2c Equation correctly formed and solved using logs with answer in context<br>2dii Correct quadratic with solutions, one correct solution to original equation found – no evidence of the other being a transfer error<br>2e Incorrect equation for area of garden, no quadratic formed for difference of areas, no evidence of how the x value was obtained. |
| 3 | A3 | 3c Attempted to make y the subject<br>3d Application of log rules incorrect with the multiplication of terms, no quadratic formed nor solved.<br>3ei Incomplete equation formed<br>3eii Equation for second parabola incomplete. |

91261

912610

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

2

SUPERVISOR'S USE ONLY

# Level 2 Mathematics and Statistics, 2016

## 91261 Apply algebraic methods in solving problems

9.30 a.m. Thursday 24 November 2016
Credits: Four

| Achievement | Achievement with Merit | Achievement with Excellence |
|---|---|---|
| Apply algebraic methods in solving problems. | Apply algebraic methods, using relational thinking, in solving problems. | Apply algebraic methods, using extended abstract thinking, in solving problems. |

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

**You should attempt ALL the questions in this booklet.**

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

**You are required to show algebraic working in this paper. Guess-and-check methods and correct answer(s) only will generally limit grades to Achievement.**

Check that this booklet has pages 2–11 in the correct order and that none of these pages is blank.

**YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.**

Merit

**TOTAL** 19

ASSESSOR'S USE ONLY

## QUESTION ONE

ASSESSOR'S USE ONLY

(a) Simplify $\left(\frac{3b}{c^2}\right)^{-4}$ leaving your answer with positive indices.

$\frac{(3b)^{-4}}{\cancel{(c^2)^{-4}}} = \frac{(c^2)^4}{(3b)^4}$

$= \frac{c^8}{81b^4}$ //

(b) Write $x^2 - 8x + 10$ in the form $(x - p)^2 + q$.

~~$x^2 - 8x + 10 = (x \quad)(x \quad)$~~ $(x - 4)^2 - 6$ //

$(x - p)^2 + q = x^2 - px - px + p^2 + q$

$= x^2 - 2px + p^2 + q$

$p = 4 \quad q = -6$

(c) (i) Show that the solutions of the equation $x^2 + x - 56 = 0$ are four times the solutions of the equation $4x^2 + x - 14 = 0$.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-56)}}{2(1)}$

$= 7$ or $-8$

$x = \frac{-1 \pm \sqrt{1^2 - 4(4)(-14)}}{2(4)}$

$= -2$ or $1.75$

$\frac{7}{1.75} = 4$

$\frac{-8}{-2} = 4$ //

(ii) Find the relationship between the solutions of the equation $dx^2 + ex + f = 0$ and the solutions of the equation $x^2 + ex + df = 0$, where $d$, $e$, and $f$ are real numbers.

$x = \frac{-e \pm \sqrt{e^2 - 4(d)(f)}}{2(d)}$ and $x = \frac{-e \pm \sqrt{e^2 - 4(1)(d)f}}{2(1)}$

$x = \frac{-e \pm \sqrt{e^2 - 4df}}{2d}$ $\quad x = \frac{-e \pm \sqrt{e^2 - 4df}}{2}$

$\frac{-e \pm \sqrt{e^2 - 4df}}{2d} = \frac{-e \pm \sqrt{e^2 - 4df}}{2}$

ASSESSOR'S USE ONLY

(d) A quadratic equation of the form $ax^2 + bx + c = 0$ has solutions $-\frac{1}{2}$ and $\frac{2}{3}$.

Find a possible set of values for $a$, $b$, and $c$.

$x = -\frac{1}{2}$ or $x = \frac{2}{3}$

$2x = -1$ $\quad$ $3x = 2$

$(2x + 1)(3x - 2) = 0$

$6x^2 - 4x + 3x - 2 = 0$

$6x^2 - x - 2 = 0$

$a = 6 \quad b = -1 \quad c = -2$ //

(e) Find positive integer value(s) for $k$ so that the quadratic equation $2x^2 + 4kx + (2k^2 + 3k - 11) = 0$ has **real rational** solutions.

Justify your answer.

$a = 2 \quad b = 4k \quad c = 2k^2 + 3k - 11$

$b^2 - 4ac$

$(4k)^2 - 4(2)(2k^2 + 3k - 11) = 0$

$16k^2 - 8(2k^2 + 3k - 11) = 0$

$16k^2 - 16k^2 + 24k - \cancel{\#}\,88 = 0$

$\cancel{24k}\ 24k - 88 = 0$

$24k = 88$

A $\quad k = 3.67$ // A

N6

## QUESTION TWO

ASSESSOR'S USE ONLY

(a) Find the discriminant of the quadratic equation $x^2 = 10x + 3$.

$x^2 - 10x - 3 = 0$

$b^2 - 4ac = (10)^2 - 4 \times 3$

$= 88$

discriminant is 88 //

(b) Simplify $\frac{4\log(u^3)}{\log u}$.

~~4(log u + log u~~

$\cancel{\frac{4(\log u + \log u + \log u)}{\log u}}$

$\frac{\log u^{12}}{\log u} = \frac{\log u + \log u^{11}}{\log u}$

$= \log u^{11}$ //

(c) Marie buys a new car for \$24 990.

The car's value decreases continuously by 12% each year.

The value of the car, \$$P$, $t$ years after she first bought it, can be modelled by a function of the form $P = A(r)^t$.

How long will it take for the value of the car to halve?

$P = 24990 \times 0.88^t$

~~12495 = 24990~~

$12495 = 24990 \times 0.88^t$

$0.5 = 0.88^t$

$\log 0.5 = t \log 0.88$

$t = \frac{\log 0.5}{\log 0.88}$

$= 5.42\ldots$ years

It will take ~~5.42 years~~

about 5.4 years, or about 5 years and 5 months

for the value to halve //

ASSESSOR'S USE ONLY

(d) (i) Solve the equation $\log_8 x = \frac{2}{3}$.

$8^{\frac{2}{3}} = x$

$x = 4$

(ii) Solve the equation $6(\log_8 x)^2 + 2\log_8 x - 4 = 0$.

let $\log_8 x$ be $x$

$6x^2 + 2x - 4 = 0$

$x = \frac{2}{3}, -1$

3 −2 −4

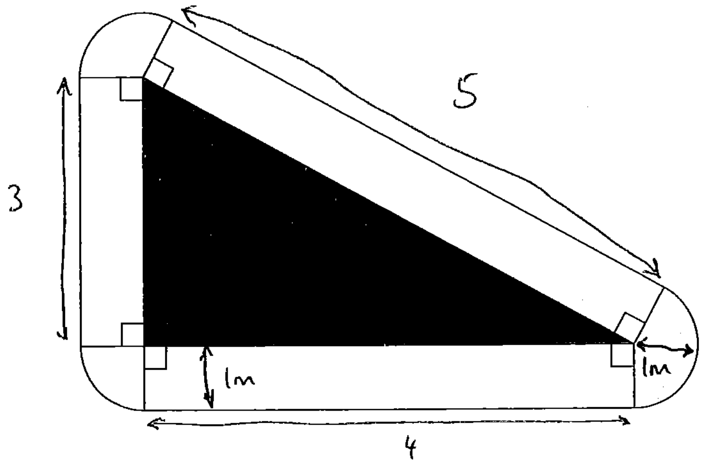
2 2 6

$(3x-2)(2x+2)$

$x = \frac{2}{3}, -1$

ASSESSOR'S USE ONLY

(e) The diagram below shows a triangular garden with a path around it.




The triangular garden has sides with lengths in the ratio 3:4:5.

The path is 1 m wide.

At each corner of the garden, the path is a sector (part) of a circle with a radius of 1 m.

The difference between **twice** the **total** area of the path and the area of the garden is $2\pi$ m$^2$.

Find the length of the longest side of the garden.

(Area of circle = $\pi r^2$)

$A = \pi r^2$ let area of path be $p$, area of garden be $g$

$A = \pi \times 1^2$ $2p - g = 2\pi$

$A = \pi$ $g = -2\pi + 2p$

The area of the path corners $p = \frac{2\pi + g}{2}$

add up to $\pi m^2 = 3.14 m^2$ (3sf)

~~p = 3.14~~

let area of non circular paths be q $q + 3.14 = \frac{2\pi + g}{2}$

$q = p - 3.14$ $2q + \cancel{6.28 = 6.28} + g$

$p = q + 3.14$ $2q = g$, $q = \frac{g}{2}$

$p = \frac{g}{2} + 3.14$

$\cancel{\frac{g}{2} + 3.14 = \frac{6.28 + g}{2}}$

$\cancel{g + 6.28}$

M6

## QUESTION THREE

ASSESSOR'S USE ONLY

(a) Where would the graph of $y = 12x^2 - x - 6$ cut the $x$-axis?

$0 = 12x^2 - x - 6$

$x = \frac{1 \pm \sqrt{1^2 - 4 \times 12 \times -6}}{24}$

$= 0.75, \ -\frac{2}{3}$

at $x = \frac{3}{4}$ and $x = \frac{-2}{3}$ //

(b) For what value(s) of $x$ does $\log_x(216) = 3$?

$x^3 = 216$

$x = \sqrt[3]{216}$

$x = 6$ when $\log_x(216) = 3$ //

(c) Rearrange the following formula to make $x$ the subject: $\frac{4x}{5} = \frac{y(x+3)}{2}$.

$2(4x) = 5(yx + 3y)$

$8x = 5yx + 15y$

$8x - 5yx = 15y$

$x(8 - 5y) = 15y$

$x = \frac{15y}{8 - 5y}$ //

Question Three continues on the following page.

ASSESSOR'S USE ONLY

(d) Solve the equation $9^{8n+6} = 27^{n^2-1} \times 3^{1-3n}$.

$(8n+6)\log 9 = \cancel{\log(27^{n^2-1} \times 3^{1-3n})}$

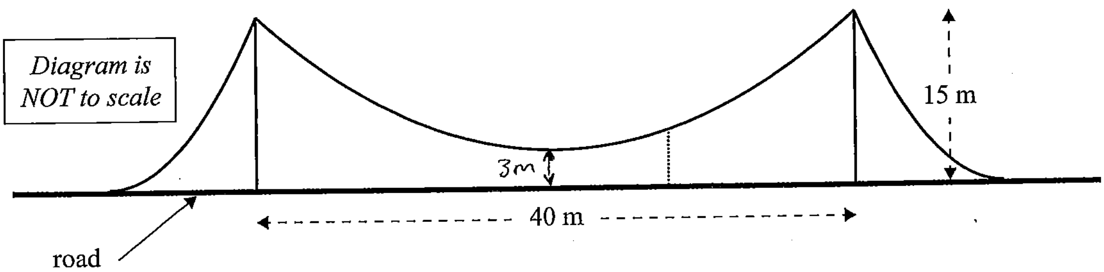
$(8n+6)\log 9 = (n^2-1)\log 27 + (1-3n)\log 3$ //

(e) A symmetrical bridge has its central cable in the shape of a parabola, as shown in the diagram below.

The towers supporting the cable are each 15 m high and 40 m apart.

At the point midway between the towers, the height of the cable above the road is 3 m.

A vertical post (shown dotted in the diagram) is placed 10 m from the centre of the bridge and just touches the cable.




(i) Use algebra to show that the post is 6 m high.

$y = ax(x-40)$

$-12 = a \times 20(20-40)$

$-12 = 20a \times -20$

$a = 0.03$

so eqn of cable is

$y = 0.03x(x-40)$

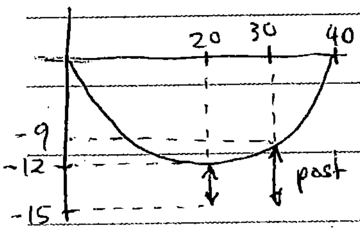
at vertical post, $x = 30$

$y = 0.03 \times 30(30-40)$

$y = -9$

since road is at $-15$,

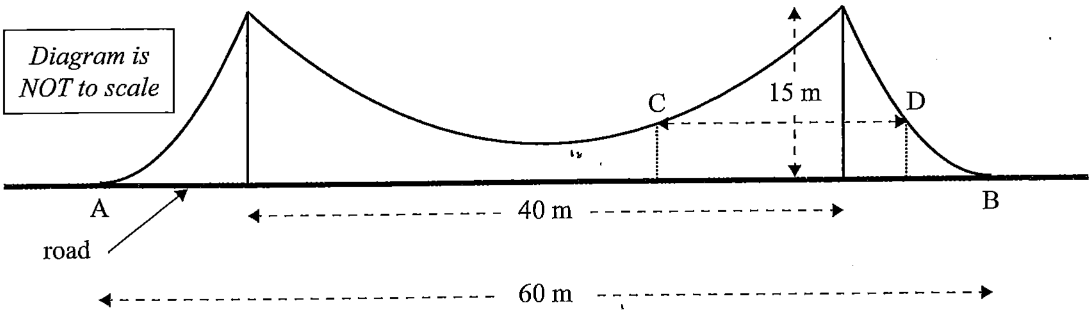
the height of the post is $15 - 9 = 6$ m high. //



ASSESSOR'S USE ONLY

(ii) The length of the bridge AB is 60 m.

The outside cables are also parabolic and symmetrical in shape, and touch the road at their vertices A and B.




Find the distance, CD, between the two parabolas at a height of 6 m above the road (the distance CD is shown in the diagram).

Equation of centre cable, as previously calculated, is

$y = 0.03x(x-40)$ and $C$ is at point $(30, -9)$

Eqn of outside cable

$y = ax(x-20)$

$-15 = a \times 10 \times (10-20)$

$-15 = 10a \times -10$

$a = 0.15$

so eqn is $y = 0.15x(x-20)$

D is at $y = -9$

$-9 = 0.15x(x-20)$

$0 = 0.15x^2 - 3x + 9$

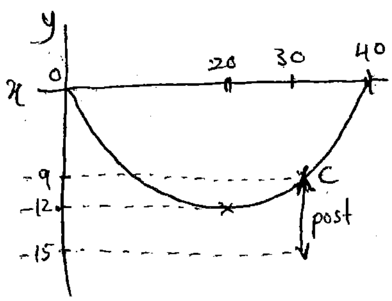
$x = \frac{3 \pm \sqrt{3^2 - 4 \times 0.15 \times 9}}{2 \times 0.15} = 3.675\ldots$ or $16.324\ldots$

so dist between C and tower is 10 m

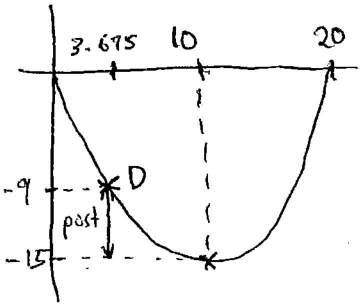
dist between D and tower is 3.675 m (3 dp)

total distance between C and D is 13.675 m (3 dp)

The central cable




The outside cables




E7

# Annotated Exemplar Template

**Merit exemplar 2016**

| **Subject:** | **Mathematics** | **Standard:** | **91261** | **Total score:** | **19** |
|---|---|---|---|---|---|

| **Q** | **Grade score** | **Annotation** |
|---|---|---|
| 1 | M6 | 1a Power correctly applied and expression simplified<br>1ci Solutions to both equations found and relationship shown numerically<br>1cii Only one equation formed correctly<br>1d Quadratic formed and correct values for a, b and c stated<br>1e Correct substitution into discriminant, no inequality in solution for k. No integer values which provide real rational solutions. |
| 2 | M6 | 2a Incorrectly calculated discriminant<br>2b Incorrect application of log rules<br>2c Equation correctly formed and solved using logs with answer in context<br>2dii Correct quadratic with solutions, solutions to original equation not found<br>2e No application of ratios, no quadratic formed for difference of areas. |
| 3 | E7 | 3c Terms with x gathered to one side, equation given with x as subject<br>3d Correct application of log rules, no quadratic formed.<br>3ei Equation formed and evidence of correct post height<br>3eii Equation for second parabola formed and solution for y = -9 used to determine length CD. |