No part of the candidate evidence in this exemplar material may be presented in an external assessment for the purpose of gaining credits towards an NCEA qualification.

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91261

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SUPERVISOR'S USE ONLY

Level 2 Mathematics and Statistics, 2017

91261 Apply algebraic methods in solving problems

2.00 p.m. Friday 24 November 2017
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply algebraic methods in solving problems.	Apply algebraic methods, using relational thinking, in solving problems.	Apply algebraic methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You are required to show algebraic working in this paper. Guess-and-check methods, and correct answer(s) only, will generally limit grades to Achievement.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Excellence

TOTAL 24

ASSESSOR'S USE ONLY

QUESTION ONE

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(a) Simplify the following, leaving your answer with positive indices:

(i) $3(4x)^{-2}$

$= 3 \frac{1}{(4x)^2} = \frac{3}{16x^2}$

(ii) $\left(\frac{16x^4}{x^6}\right)^{\frac{3}{2}}$

$= \frac{16^{\frac{3}{2}} x^{(4 \times \frac{3}{2})}}{x^{(6 \times \frac{3}{2})}} = \frac{64x^6}{x^9} = \frac{64}{x^3}$

(b) Fully simplify the expression $\frac{2x^2 - 50}{9x^2 - 39x - 30}$.

$= \frac{2(x^2 - 25)}{3(3x^2 - 13x - 10)} = \frac{2(x-5)(x+5)}{3[3x(x-5) + 2\cancel{x}(x-5)]}$

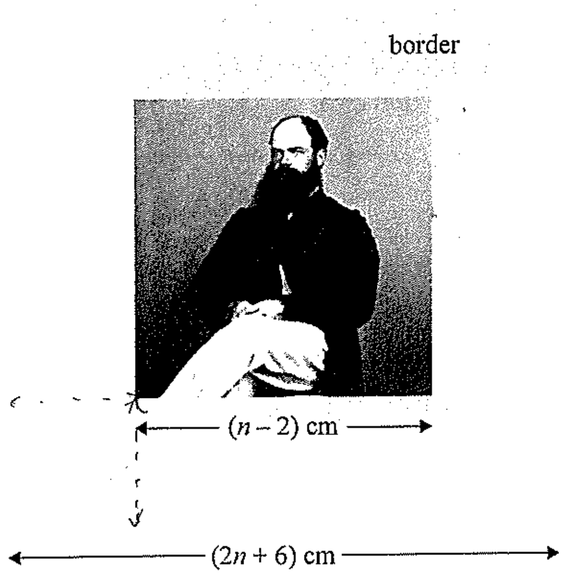
$= \frac{2(x-5)(x+5)}{3(3x+2)(x-5)}$

$= \frac{2(x+5)}{3(3x+2)}$

$x \neq 5$, $x \neq \cancel{\frac{+3}{2}}\ \frac{-2}{3}$

ASSESSOR'S USE ONLY

(c) David has mounted a square photo on a square piece of card as shown below.



The border around the photo is of constant width.

The photo has sides of length $(n-2)$ cm while the card has sides of $(2n+6)$ cm.

If the total area of the border is 200 cm^2, find the width of the border.

$A_{card} = (2n+6)^2 = 4n^2 + 24n + 36.$

$A_{photo} = (n-2)^2 = n^2 - 4n + 4.$

$A_{border} = (4n^2 + 24n + 36) - (n^2 - 4n + 4)$

$= 4n^2 + 24n + 36 - n^2 + 4n - 4.$

$= 3n^2 + 28n + 32 = 200$

$\cancel{= (3n+4)(n+8)}$

$\Rightarrow 3n^2 + 28n - \overset{168}{\cancel{120}} = 0$

$\Rightarrow n = \dfrac{-14 + 10\sqrt{7}}{3} = 4.15$

or $n = \dfrac{-14 - 10\sqrt{7}}{3} = -13.49$

Because $n > 2 \Rightarrow n = 4.15.$

$\rightarrow$ Width of the border $= \dfrac{(2n+6)-(n-2)}{2} = 6.075$ cm

(d) A teacher has hired a school bus for \$560 for a day trip with students.

The cost of hiring the bus is to be shared equally between the students.

At the last moment three of the students were unable to go.

As a result, the cost to each of those who did go was increased by \$1.50.

How many students finally went on the trip?

Justify your answer.

let the ~~f~~ number of final students went on the trip $= a \ (a>0)$

$\rightarrow$ Number of initial students ~~of~~ went on the trip $= a+3$.

Initial cost per student $= \dfrac{560}{a+3}$.

final cost per student $= \dfrac{560}{a}$.

$$\frac{560}{a+3} + 1.50 = \frac{560}{a}.$$

$560a + 1.5a(a+3) = 560(a+3)$.

$1.5a^2 + 4.5a = 1680$

$1.5a^2 + 4.5a - 1680 = 0$

$(a-32)(a+35) = 0$

$a = 32$ or $a = -35$ but $a>0$

so $a = 32$

There are 32 students finally went on the trip

ASSESSOR'S USE ONLY

E8

QUESTION TWO

ASSESSOR'S USE ONLY

(a) Solve the following equation for x:

$\log_2 x = 10$

$x = 2^{10} = 1024.$

(b) Solve the following equation for x:

$\log_x 49 = 2$

Justify your answer.

$49 = x^2$

$x = \sqrt{49}$

$x = \pm 7$ but x is a base so $x > 0$

$x = 7.$

(c) Find the value of $\log_{\sqrt{5}}\left(\frac{1}{125}\right)$.

$\log_{\sqrt{5}}\left(\frac{1}{125}\right) = a$

$\frac{1}{125} = (\sqrt{5})^a.$

$\frac{1}{5^3} = (\sqrt{5})^a$

$\frac{1}{(\sqrt{5})^6} = (\sqrt{5})^a.$

$(\sqrt{5})^{-6} = (\sqrt{5})^a$

$a = -6$

(d) A computer depreciates continuously in value from \$4699 to \$1500 over a period of 4.25 years.

The value, \$$y$, of the computer t years after its value was \$4699 can be modelled by a function of the form

$y = Ar^t$, where r is a constant.

Find the computer's value after six years.

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$y = Ar^t \Rightarrow 1500 = 4699r^{4.25}$

$\Rightarrow \frac{1500}{4699} = r^{4.25}$

$\cancel{\log\left(\frac{1500}{4699}\right) = \log\left(r^{4.25}\right)}$

$\cancel{\log\left(1500/4699\right) = 4.25 \log r}$

$\Rightarrow r = \sqrt[4.25]{1500/4699} = 0.76$

$y = 4699 \times 0.76^t$

$t = 6 \Rightarrow y = 4699 \times 0.76^6$

$= 905.5$

After 6 years Computer's value is \$905.5

(e) Make p the subject of the formula:

$$81^{\left(\frac{px}{q}-3\right)} = 243$$

$\cancel{81}\ (3^4)^{\left(\frac{px}{q} - 3\right)} = 3^5. \qquad (q \neq 0).$

$3^{4\left(\frac{px}{q} - 3\right)} = 3^5.$

$4\left(\frac{px}{q} - 3\right) = 5.$

$\frac{4px - 12q}{q} = \frac{5q}{q}.$

$4px - 12q = 5q.$

$4px = 17q.$

$p = \frac{17q}{4x}. \qquad (x \neq 0).$

E8

QUESTION THREE

ASSESSOR'S USE ONLY

(a) The quadratic equation $4x^2 + bx - 5 = 0$ has solutions $-\frac{1}{2}$ and $\frac{5}{2}$.

Find the value of b.

$x + \frac{1}{2} = 0 \rightarrow 2x + 1 = 0$

$x - \frac{5}{2} = 0 \rightarrow 2x - 5 = 0$

$(2x+1)(2x-5) = 0.$

$4x^2 - 10x + 2x - 5 = 0.$

$\rightarrow 4x^2 - 8x - 5 = 0$

$b = -8.$

(b) For what value(s) of m does the equation $6x^2 - mx = -3$ have two equal roots?

$6x^2 - mx + 3 = 0$

$\Delta = b^2 - 4ac = (-m)^2 - 4 \times 3 \times 6$

$= m^2 - 72$

Equation has 2 equal roots $\rightarrow \Delta = 0 \rightarrow m^2 - 72 = 0.$

$\rightarrow m = \pm\sqrt{72}$

$\approx \pm 8.485$

(c) Find the value(s) for k for which the expression $kx^2 - 12x + 5k$ is always greater than zero.

ASSESSOR'S USE ONLY

$kx^2 - 12x + 5k > 0$

$kx^2 - 12x + 5k = 0$ has no root.

$\Delta = b^2 - 4ac = (-12)^2 - 4 \cdot k \cdot 5k$

$= 144 - 20k^2 < 0$

$20k^2 < 144$

$k < \frac{-6\sqrt{5}}{5}$ or $k > \frac{6\sqrt{5}}{5}$.

~~for $kx^2 - 12x + 5k > 0$~~ $\Rightarrow k < -2.6$

~~$k > 0 \rightarrow k > \frac{6\sqrt{5}}{5}$~~ or $k > 2.7$

$k\left(x^2 - \frac{12}{k}x + 5\right) > 0$

$\rightarrow k\left(x^2 - 2x\right.$

Question Three continues on the following page.

(d) Write $\frac{9}{x^2-9}+\frac{3}{2x+6}$ as a single fraction in its simplest form.

$$\frac{9}{(x-3)(x+3)} + \frac{3}{2(x+3)} = \frac{9\times 2}{2(x-3)(x+3)} + \frac{3(x-3)}{2(x-3)(x+3)}$$

$$= \frac{18+3x-9}{2(x-3)(x+3)} = \frac{9+3x}{2(x-3)(x+3)}$$

$$= \frac{3(x+3)}{2(x-3)(x+3)} = \frac{3}{2(x-3)}$$

$x \neq \pm 3$

(e) Find the value(s) of m for which the equation $2^{mx-3} = 8^{x^2}$ has exactly one solution.

$2^{mx-3} = 8^{x^2}$

$2^{mx-3} = 2^{3x^2}$

$mx - 3 = 3x^2$

$3x^2 - mx + 3 = 0$

$\Delta = m^2 - 4\times 3\times 3 = 0$ as equation has 1 solution

$m^2 - 36 = 0$

$m^2 = 36$

$m = \pm 6$

E8

Subject:	Mathematics	Standard:	91261	Total score:	24

Q	Grade score	Annotation
1	E8	1ai 0.1875 would have been acceptable for 3/16. 1b The restrictions $x \neq 5, -2/3$ were not required. 1d Solution by G.C. rather than factorising was acceptable.
2	E8	2b Recognition that there cannot be a negative x value for log gains 'r'. 2d The variable r was more commonly found by solving by logs. 2e This is an index approach. Most candidates solved by logs.
3	E8	3a Could also be solved by substituting either solution into the equation. 3b Necessary to state or show $\Delta = 0$ for equal roots. 3c Has shown that the graph will always be above the x-axis and $\Delta < 0$. Not required to show that because $k > 0$, then $k > 2.68$. 3e Could also have taken log of both sides.