91261

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Level 2 Mathematics and Statistics, 2018

91261 Apply algebraic methods in solving problems

9.30 a.m. Wednesday 14 November 2018
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply algebraic methods in solving problems.	Apply algebraic methods, using relational thinking, in solving problems.	Apply algebraic methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You are required to show algebraic working in this paper. Guess-and-check methods, and correct answer(s) only, will generally limit grades to Achievement.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Excellence

TOTAL 22

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QUESTION ONE

(a) Simplify fully $\left(25m^{16}\right)^{\frac{1}{2}}$

$= \sqrt{25m^{16}}$

$= 5m^{8}$ //

u

(b) Simplify fully $\left(\frac{4}{3a}\right)^{-2}$, leaving your answer with a positive index.

$= \left(\frac{3a}{4}\right)^{2} = \frac{9a^{2}}{16}$ //

u

(c) Write $4 - \frac{b+8c}{3c}$ as a single fraction in its simplest form.

$= \frac{4(3c) \quad -b - 8c}{3c}$

$= \frac{12c - 8c - b}{3c}$

$= \frac{4c - b}{3c}$ //

r

(d) Factorise fully $4bx + 2xy - 6ab - 3ay$

$= 2x(2b+y) - 3a(2b+y)$

$= (2x-3a)(2b+y)$ //

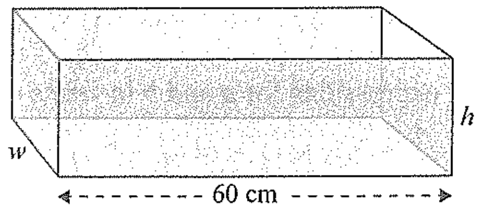
u

(e) A rectangular box has no lid.

The length of the base is 60 cm.

Its height is one quarter of the sum of its width and length.

The total area of the base **and** the four sides of the box is 7400 cm^2.



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Find the height of the box.

$A = 2(wh) + 2(60h) + 60w$

$h = \frac{1}{4}(60 + w)$

$A = 2wh + 120h + 60w$

$A = 2w(15 + \frac{1}{4}w) + 120(15 + \frac{1}{4}w) + 60w$

$A = 30w + \frac{1}{2}w^2 + 1800 + 30w + 60w$

$A = \frac{1}{2}w^2 + 120w + 1800$

$7400 = \frac{1}{2}w^2 + 120w + 1800$

$0 = \frac{1}{2}w^2 + 120w - 5600$

$w = \frac{-120 \pm \sqrt{14400 - 4(\frac{1}{2})(-5600)}}{1}$

$w = -120 \pm 160$

$w = 40$ or $w = -280$

However $w \neq -280$ as it cannot be negative

$\therefore$ $w = 40$ cm

$h = 15 + \frac{w}{4}$

$h = 15 + 10$

$h = 25$ cm

t

(f) $(3x + y)(x - 12y) - (2x + y)(x - 16y)$ can be written in the form $(a + b)^2$.

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Find expressions for a and b in terms of x or y.

$= (3x^2 - 36xy + xy - 12y^2) - (2x^2 - 32xy + xy - 16y^2)$

$= (3x^2 - 35xy - 12y^2) - (2x^2 - 31xy - 16y^2)$

$= 3x^2 - 35xy - 12y^2 - 2x^2 + 31xy + 16y^2$

$= x^2 - 4xy + 4y^2$

$\cancel{= x^2 - 2x - 2y + 4y^2}$ $= \cancel{4x}\ (x - 2y)(x - 2y)$

$\cancel{= x(x - 2) - 2y(}$ $= (x - 2y)^2$

$a = x$

$b = -2y$

t

E8

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QUESTION TWO

(a) Find x if $\log_x 243 = 5$

$x^5 = 243 \qquad x^5 = 3^5 \qquad \underline{x = 3}$ //

u

(b) Find m if $\log_3(4m - 1) = 2$

$3^2 = (4m - 1)$

$9 = 4m - 1$

$4m = 10$

$\underline{m = 2.5}$ //

u

(c) Find an expression for x in terms of w if $\frac{3^{4x+1}}{9^x} = 27^{\frac{w}{3}}$

$\frac{3^{4x+1}}{3^{2x}} = 3^w$

$3^{4x+1} = 3^w \times 3^{2x}$

$3^{4x+1} = 3^{w+2x}$

$4x + 1 = w + 2x$

$2x = w - 1$

$\underline{x = \frac{w-1}{2}}$ //

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(d) An equestrian jump has a parabolic arch mounted on a wall. Horses and riders jump through the arch.

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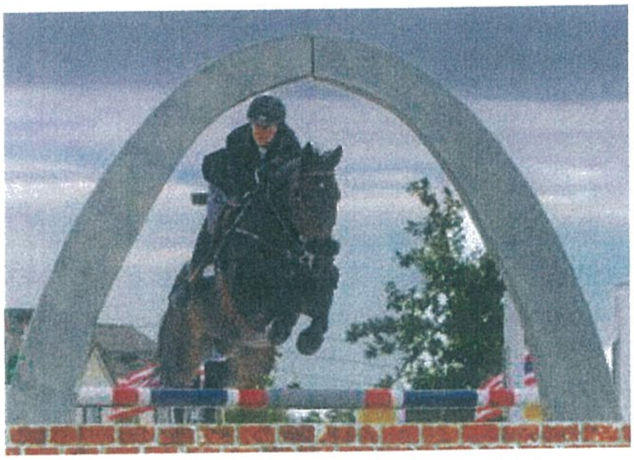
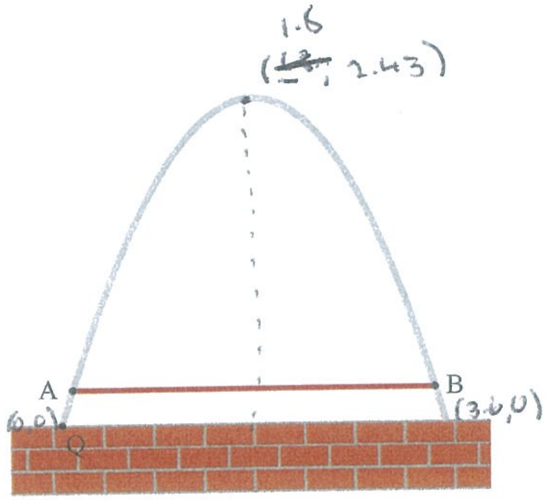


Source: http://luxequestrian.com/slideshow/incredible-jumps-brody-robertson



The arch rises 2.43 metres **above the wall.**

The arch can be modelled by a function of the form $h(x) = kx(3.6 - x)$, where k is a constant, h metres is the height above the wall, and x metres is the horizontal distance from Q.

A rail AB can be placed above the wall and attached at each end to the arch. For one competition, the rail is placed 0.5 metres above the wall.

How long is the rail AB?

$\cancel{h(x) = kx(3.6 - x)}$ $\qquad \cancel{2.43} = 3.6kx - kx^2$

$\cancel{0 = h(0)}$ $\qquad \cancel{2.43}$

$\cancel{h(x) = 3.6kx - kx^2}$

$\cancel{\frac{dh}{dx} = 3.6k - 2kx = 0}$

$\cancel{3.6k = 2kx}$

$\cancel{3.6 = 2x}$

$\cancel{x = 1.8}$

$h(x) = 3.6kx - kx^2$

$0 = 3.6kx - kx^2$

$kx^2 = 3.6kx$

$x^2 = 3.6x$

$x = 3.6$ or $x = 0$

$(3.6, 0)$

$h(x) = -0.75x^2 + 2.7x$

$k = 0.75$

$0.5 = -0.75x^2 + 2.7x$

$0 = -0.75x^2 + 2.7x - 0.5$

$x = \dfrac{-2.7 \pm \sqrt{(2.7)^2 - 4(-0.75)(-0.5)}}{-1.5}$

$x = \dfrac{-2.7 \pm \sqrt{5.79}}{-1.5}$ → $x = 0.1958$ m (4sf) or $x = 3.404$ m (4sf)

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Length AB = ~~Distance between~~ 3.404 - 0.1958
= 3.21m (3sF)

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(e) Interest is compounded on a principal investment, $\$P$, **at the end of each year.**

If the total amount of the investment after n years is $\$A$ then $A = P\left(1+\frac{r}{100}\right)^n$

where r% is the compound interest rate per year.

(i) Anushka invests \$20 000 at an interest rate of 3.85% (so $A = P(1.0385)^n$).

How many years will it take for her investment to be worth \$25 000?

$25000 = 20000(1.0385)^n$

$1.25 = 1.0385^n$

$\frac{\log 1.25}{\log 1.0385} = n$

$n = 5.91$

It will take 6 years //

r

(ii) Semisi invests his money at a different interest rate than Anushka's investment. His investment will double in value after twelve years.

What is the interest rate for Semisi's investment?

$40{,}000 = 20{,}000\left(1 + \frac{r}{100}\right)^{12}$

$2 = \left(1 + \frac{r}{100}\right)^{12}$

$1.059 = 1 + \frac{r}{100}$

$0.059 = \frac{r}{100}$

$r = 5.95\%$ per year //

t

E8

QUESTION THREE

ASSESSOR'S USE ONLY

(a) Solve each of the following equations for x:

(i) $12x^2 - 5x = 2$

$12x^2 - 5x - 2 = 0$

$12x^2 - 8x + 3x - 2 = 0$

$4x(3x-2) + 1(3x-2) = 0$

$(4x+1)(3x-2) = 0$

$x = \frac{-1}{4}$ or $x = \frac{2}{3}$

u

(ii) $x + 1 - \frac{3}{x} = 0$

$x + 1 = \frac{3}{x}$

$x^2 + x = 3$

$x^2 + x - 3 = 0$

~~(x-1)(x+2)~~ $x = \frac{-1 \pm \sqrt{1 - 4(1)(-3)}}{2}$

$x = \frac{-1 \pm \sqrt{13}}{2}$

$x = -2.302$ or $x = 1.303$

u

(b) Show that the graph of the function $y = 2x^2 - 5x + 6$ does not cross the x-axis.

You must use algebra to support your explanation.

~~$y = 2x^2 +$~~ $y = 2x^2 - 5x + 6$

If doesnt touch x-axis, no real roots. so $\Delta < 0$

$\Delta = b^2 - 4ac = 25 - 4(2)(6)$

$25 - 48 < 0$

$-23 < 0$

As $\Delta < 0$, there are no real roots so the graph doesn't cross the x axis

r

Question Three continues on the following page.

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(c) The equation $3x^2 + kx - 12 = 0$ has two real solutions.

If one of the solutions is $x = 3$, find the other solution.

~~Two real solutions ∴ Δ > 0~~

~~$\Delta = b^2 - 4ac$~~

$3(3)^2 + k(3) - 12 = 0$

$27 + 3k - 12 = 0$

$3k = -15$

$k = -5$

$3x^2 - 5x - 12 = 0$

$x = \dfrac{5 \pm \sqrt{25 - 4(3)(-12)}}{6}$

$x = \dfrac{5 \pm 13}{6}$

$x = 3$ or $x = -\frac{4}{3}$

(d) Show that the roots of the equation $x^2 + 2(k + 1)x - (k^2 + 2k + 5) = 0$, where k is a constant, can never be equal for any real number k.

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If roots cannot be equal, $\Delta > 0$

$\Delta = b^2 - 4ac = (2(k+1))^2 - 4(1)(-k^2 - 2k - 5)$

$(2k+2)^2 - 4(-k^2 - 2k - 5) > 0$

$4k^2 + 8k + 4 + 4k^2 + 8k + 20 > 0$

$8k^2 + 16k + 20 > 0$

$k = \dfrac{-16 \pm \sqrt{256 - 4(8)(20)}}{16}$

$k = \dfrac{-16 \pm \sqrt{-384}}{16}$

~~k has no~~ Δ of this equation is negative. This means there are no real roots, which means the graph $8k^2 + 16k + 20$ must always be > 0. Therefore, for any real value of k in the original equation, the Δ will be > 0 and therefore there will be 2 different real roots.

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M6

Excellence Exemplar 2018

Subject	Mathematics	Standard	91261	Total score	22

Q	Grade score	Annotation
1	E8	1e Has correctly combined the algebraic fraction by subtracting both b and 8c. 1e Has correctly formed a quadratic equation and then solved for h. 1f Has correctly identified a and b after expanding the original expression and then factorising the perfect square.
2	E8	2c Has correctly reduced expression to powers of 3 and then manipulated result to make x the subject. 2d k=0.75 is critical for solution of this problem. The candidate has then formed a quadratic, solved, and then correctly found the difference of the two values. 2ei Has provided 6 as the answer, recognising that the interest is calculated at the end of each year. 2eii Any two values for 2P and P were acceptable in working out the interest rate.
3	M6	3a Graphic calculator or factorisation were valid methods. 3b Graphic calculator or quadratic formula were valid methods. 3d A minor numerical error was ignored but there is no recognition of equal roots having a discriminant equal to 0. The best approach was to show that no values for k could be found as the discriminant of the resulting quadratic was less than 0.