2

91261

912610

SUPERVISOR'S USE ONLY

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 2 Mathematics and Statistics, 2016

91261 Apply algebraic methods in solving problems

9.30 a.m. Thursday 24 November 2016
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply algebraic methods in solving problems.	Apply algebraic methods, using relational thinking, in solving problems.	Apply algebraic methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You are required to show algebraic working in this paper. Guess-and-check methods and correct answer(s) only will generally limit grades to Achievement.

Check that this booklet has pages 2–11 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Achievement

TOTAL 11

ASSESSOR'S USE ONLY

ASSESSOR'S USE ONLY

QUESTION ONE

(a) Simplify $\left(\frac{3b}{c^2}\right)^{-4}$ leaving your answer with positive indices.

$\left(\frac{c^2}{3b}\right)^4 = \frac{c^8}{27b^4}$ //

$(x-4)(x-4) = x^2 - 4x - 4x + 16$

(b) Write $x^2 - 8x + 10$ in the form $(x-p)^2 + q$.

$(x-4)^2 + (-6)$

$= (x-4)^2 - 6$ //

(c) (i) Show that the solutions of the equation $x^2 + x - 56 = 0$ are four times the solutions of the equation $4x^2 + x - 14 = 0$.

$x^2 + x - 56 = 0$. $\quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$(x+8)(x-7) = 0$ $\quad x = 1.75$

$x = -8$ or $x = 7$. $\quad x = -2$

$-8 \div -2 = 4$ $\quad 7 \div 1.75 = 4$ //

(ii) Find the relationship between the solutions of the equation $dx^2 + ex + f = 0$ and the solutions of the equation $x^2 + ex + df = 0$, where d, e, and f are real numbers.

$dx^2 + ex + f = x^2 + ex + df$.

$d = 1$, $f = 1$ //

ASSESSOR'S USE ONLY

(d) A quadratic equation of the form $ax^2 + bx + c = 0$ has solutions $-\frac{1}{2}$ and $\frac{2}{3}$.

Find a possible set of values for a, b, and c.

$x = \frac{-1}{2}, \frac{2}{3}$

$(x + \frac{1}{2})(x - \frac{2}{3}) = 0$

$x^2 + \frac{1}{2}x - \frac{2}{3}x - \frac{1}{3} = 0$

$x^2 - \frac{1}{6}x - \frac{1}{3} = 0$

$a = 1, \quad b = -\frac{1}{6}, \quad c = \frac{-1}{3}$

(e) Find positive integer value(s) for k so that the quadratic equation $2x^2 + 4kx + (2k^2 + 3k - 11) = 0$ has **real rational** solutions.

Justify your answer.

$b^2 - 4ac \geq 0$

$4k^2 - 4 \times 2 \times (2k^2 + 3k - 11) \geq 0$

$4k^2 - 8 \times (2k^2 + 3k - 11) \geq 0$

$4k^2 - 16k^2 - 24k + 88 \geq 0$

$-12k^2 - 24k + 88 \geq 0$

$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$k = \frac{24 \pm \sqrt{24^2 - 4 \times (-12) \times 88}}{2 \times (-12)}$

$= \frac{24 \pm \sqrt{4800}}{-24}$

$= -3.89$ or 1.89 MS

positive, therefore, $k = 1.89$

MS

QUESTION TWO

ASSESSOR'S USE ONLY

(a) Find the discriminant of the quadratic equation $x^2 = 10x + 3$.

$x^2 - 10x - 3 = 0$.

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

$x = \frac{10 \pm \sqrt{100 - 4 \times 1 \times (-3)}}{2} = \frac{10 \pm \sqrt{100+12}}{2} = 10.29$ or -0.29.

(b) Simplify $\frac{4\log(u^3)}{\log u}$.

$\frac{\log(u^3)^4}{\log u} = \frac{\log u^{12}}{\log u} = \log u^{11}$. //

(c) Marie buys a new car for $24 990.

The car's value decreases continuously by 12% each year.

The value of the car, $P, t years after she first bought it, can be modelled by a function of the form $P = A(r)^t$.

How long will it take for the value of the car to halve?

$24990 \div 2 = 12495$.

$12495 = A(1-12\%)^t$

$12495 = 24990(1-12\%)^t$.

~~t 24~~ $12495 = \cancel{24990 \times 0}\ 24990(0.88)^t$

$\log 12495 = t\log 24990(0.88)$

$t = \frac{\log 12495}{\log 21991.2}$

$= 0.94$

$1 + 0.94 = 1.94$ year. //

ASSESSOR'S USE ONLY

(d) (i) Solve the equation $\log_8 x = \frac{2}{3}$.

$8^{\frac{2}{3}} = x$

$x = \frac{64}{3}$

A

(ii) Solve the equation $6(\log_8 x)^2 + 2\log_8 x - 4 = 0$.

$\log_8 x^{12} + 2\log_8 x^2 - 4 = 0$

$\log_8 x^{12} + \log_8 x^2 - 4 = 0$

$12 \log_8 x + 2\log_8 x = 4$

$\cancel{8}\ 14 \log_8 x = 4$

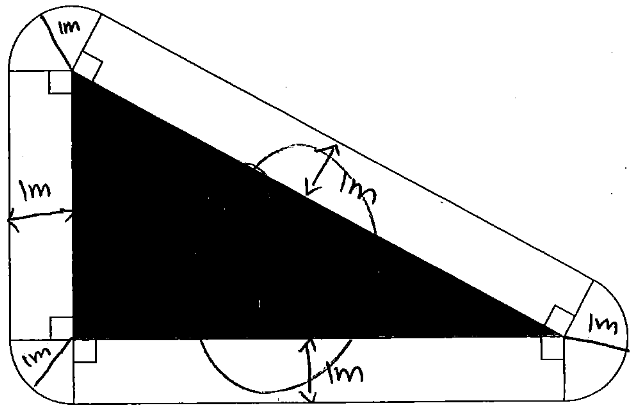
$\log_8 x = \frac{2}{7}$

$8^{\frac{2}{7}} = x$

$x = \frac{64}{7}$

(e) The diagram below shows a triangular garden with a path around it.

ASSESSOR'S USE ONLY



The triangular garden has sides with lengths in the ratio 3:4:5.

The path is 1 m wide.

At each corner of the garden, the path is a sector (part) of a circle with a radius of 1 m.

The difference between **twice** the **total** area of the path and the area of the garden is 2π m^2.

Find the length of the longest side of the garden.

(Area of circle $= \pi r^2$)

Total Path Area $= (3 \times 1) + (\frac{1}{4}(1)^2)$

$= 3 + \frac{1}{4}$

$= 3.25$

Area of Garden $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times (x+1)(x)$

$= \frac{1}{2}(x^2 + x)$

$= \frac{1}{2}x^2 + \frac{1}{2}x$

$2(3.25) - \frac{1}{2}x^2 + \frac{1}{2}x = 2\pi$

$6.5 - \frac{1}{2}x^2 + \frac{1}{2}x = 2\pi$

$= 2\pi$

$-0.5x^2 + 0.5x - 6.5 = 2\pi$

A5

N2

QUESTION THREE

ASSESSOR'S USE ONLY

(a) Where would the graph of $y = 12x^2 - x - 6$ cut the x-axis?

$b^2 - 4ac = 1 - 4(12)(-6) = 1 + 288 = 289.$

2 real roots, cut the x - axis. //

(b) For what value(s) of x does $\log_x(216) = 3$?

$x^3 = 216$

$x = 6.$ //

(c) Rearrange the following formula to make x the subject: $\frac{4x}{5} = \frac{y(x+3)}{2}$.

$2 \times 4x = y(x+3) \times 5$

~~$8x = (yx + 3y)5$~~

~~$8x = 5yx + 15y$~~

~~$8x - 5yx = 15y$~~

~~$\frac{8x}{(x+3)} = 5y$~~

$8x = 5yx + 15y$

$8x - 5yx = 15y.$

$x(8 - 5y) = 15y.$ //

Question Three continues on the following page.

ASSESSOR'S USE ONLY

(d) Solve the equation $9^{8n+6} = 27^{n^2-1} \times 3^{1-3n}$.

$(3^2)^{8n+6} = (3^3)^{n^2-1} \times (3)^{1-3n}$

$3^{\cancel{16n+12}} = 3^{\cancel{(3n)^2-3}} \times 3^{1-3n}$

$16n+12 = ((3n)^2-3) \times 1-3n$

$16n+12 = (9n^2-3)(1-3n)$

$16n+12 = 9n^2 - 27n^3 - 3 + 9n$

$27n^3 - 9n^2 + 7n + 15 = 0$

$n = -0.635$ (3dp) //

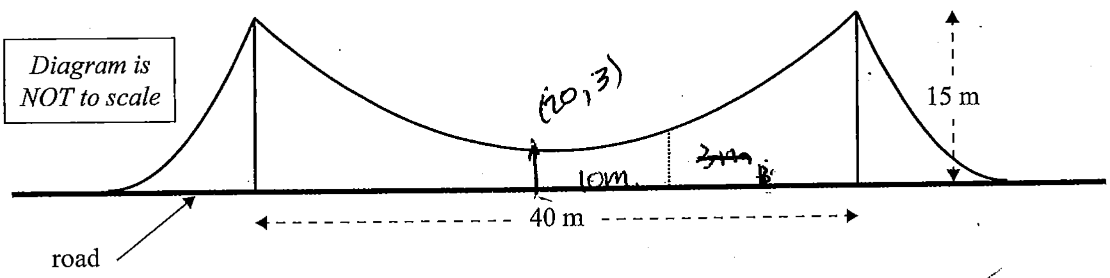
//

(e) A symmetrical bridge has its central cable in the shape of a parabola, as shown in the diagram below.

The towers supporting the cable are each 15 m high and 40 m apart.

At the point midway between the towers, the height of the cable above the road is 3 m.

A vertical post (shown dotted in the diagram) is placed 10 m from the centre of the bridge and just touches the cable.



(i) Use algebra to show that the post is 6 m high.

$y = k(x-a)^2 + b$

$y = k(x-20)^2 + 3$

At (20, 3) $\cancel{3 = k(20-20)^2 + 3}$

$k = 0$

$y = (x-20)^2 + 3$

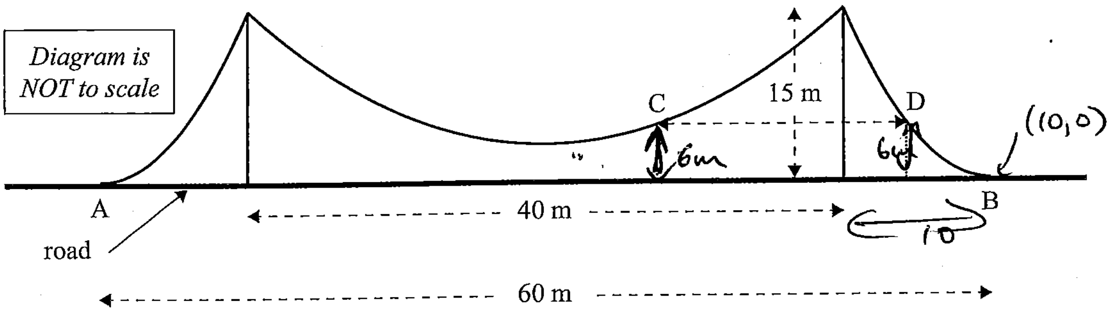
when $x = 30$

$y = 103$ //

ASSESSOR'S USE ONLY

(ii) The length of the bridge AB is 60 m.

The outside cables are also parabolic and symmetrical in shape, and touch the road at their vertices A and B.



Find the distance, CD, between the two parabolas at a height of 6 m above the road (the distance CD is shown in the diagram).

① $y = (x-20)^2 + 3$ ② $y = (x-10)^2$

DISTANCE CD: $(x-20)^2 + 3 = (x-10)^2$

A4

Annotated Exemplar Template

Achieved exemplar 2016

Subject:	Mathematics	Standard:	91261	Total score:	11

Q	Grade score	Annotation
1	M5	1a Correct application of negative power, incorrect final answer 1ci Solutions to both equations found and relationship shown numerically 1cii No solution for either equation 1d Correct quadratic, incorrect value for b 1e Incorrect substitution in discriminant, no inequality in the consistent solution for k.
2	N2	2a Discriminant not clearly given 2b Partially correct application of logs, incorrect simplification 2c Correct equation formed, incorrectly solved 2di Wrong solution 2dii Incorrect method, no substitution made nor quadratic formed 2e Incorrect application of ratio and area of path, quadratic formed for difference of areas but not rearranged correctly nor solved.
3	A4	3a x intercepts not found 3c Terms correctly gathered to one side, no final answer 3d Correct application of common base, powers multiplied instead of addition 3ei Point chosen for substitution was not suitable to determine k. 3eii No constant values determined