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SUPERVISOR'S USE ONLY

91261

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NZQA

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# Level 2 Mathematics and Statistics, 2017

## 91261 Apply algebraic methods in solving problems

2.00 p.m. Friday 24 November 2017
Credits: Four

| Achievement | Achievement with Merit | Achievement with Excellence |
|---|---|---|
| Apply algebraic methods in solving problems. | Apply algebraic methods, using relational thinking, in solving problems. | Apply algebraic methods, using extended abstract thinking, in solving problems. |

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

**You should attempt ALL the questions in this booklet.**

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

**You are required to show algebraic working in this paper. Guess-and-check methods, and correct answer(s) only, will generally limit grades to Achievement.**

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

**YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.**

Merit

TOTAL 19

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## QUESTION ONE

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(a) Simplify the following, leaving your answer with positive indices:

(i) $3(4x)^{-2}$

$= 3\left(\frac{1}{4x}\right)^2$

$= 3\,\frac{1}{16x^2}$

$= \frac{3}{16x^2}$ //

(ii) $\left(\frac{16x^4}{x^6}\right)^{\frac{3}{2}}$

$= \left(\frac{16}{x^2}\right)^{\frac{3}{2}}$

$= \frac{64}{x^3}$ //

(b) Fully simplify the expression $\frac{2x^2-50}{9x^2-39x-30}$.

$= \frac{2(x^2-25)}{3(3x^2-13x-10)}$

$= \frac{2(x+5)(x-5)}{3(3x+2)(x-5)}$

$= \frac{2x+10}{9x+6}$ //

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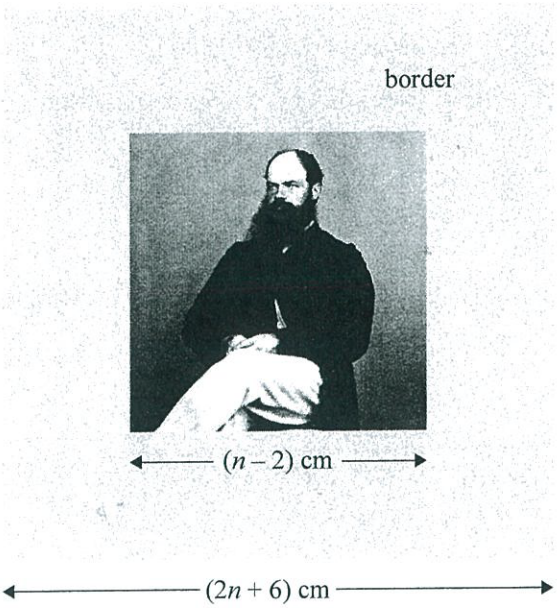
(c) David has mounted a square photo on a square piece of card as shown below.




The border around the photo is of constant width.

The photo has sides of length $(n-2)$ cm while the card has sides of $(2n+6)$ cm.

If the total area of the border is 200 cm$^2$, find the width of the border.

The area of the photo is $(n-2)^2 = n^2-4n+4$

~~We let the width of the border is $x$~~

~~$200 = (2n+6)x - (n-2)^2$~~

~~$= 2nx+6x - n^2-4n+4$~~

$(2n+6)^2-(n-2)^2 = 200$

$4n^2+24n+36-n^2+4n-4 = 200$

$3n^2+28n+32 = 200$

$3n^2+28n = 168$

$3n^2+28n-168 = 0$

$b^2-4ac = 784+2016$

$= 2800$

$n = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$n_1 = \frac{-28+\sqrt{2800}}{2\times3}$

$= \frac{-28+53}{6}$

$= 4.2$ cm

$n_2 = \frac{-28-\sqrt{2800}}{2\times3}$

$= \frac{-28-53}{6}$

$= -13.5$ cm

$\therefore n = 4.2$

$\therefore$ the width of the border is $2n+6$

$= 2\times4.2+6$

$= 14.4$ cm.

(d) A teacher has hired a school bus for $560 for a day trip with students.
The cost of hiring the bus is to be shared equally between the students.
At the last moment, three of the students were unable to go.
As a result, the cost to each of those who did go was increased by $1.50.

How many students finally went on the trip?
Justify your answer.

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Let make number of student is $x$. The much of each student is $n$. $n=\frac{560}{x}$

$$\begin{cases} xn=560 \\ 560-(x-3)n=1.5(x-3) \end{cases}$$

$560-xn+3n=1.5x-4.5$

$560-560+3n=1.5x-4.5$

$1.5x-3n=4.5$

~~$1.5x=$~~

$1.5x-3\times\frac{560}{x}=4.5$

$1.5x-\frac{1680}{x}=4.5$

$1.5x^2-1680=4.5x$

$x^2-1120=3x$

$x^2-3x-1120=0$

$(x-35)(x+32)=0$

$x=35$ or $x=-32$

$\therefore x=35$

$\therefore$ there are 35 students in the class.

$35-3=32$

$\therefore$ there are 32 students finally went on the trip.

E7

## QUESTION TWO

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(a) Solve the following equation for $x$:

$\log_2 x = 10$

$x = 2^{10}$

$x = 1024$

(b) Solve the following equation for $x$:

$\log_x 49 = 2$

Justify your answer.

$49 = x^2$

$x = 7$

(c) Find the value of $\log_{\sqrt{5}}\left(\frac{1}{125}\right)$.

$\frac{1}{125} = \sqrt{5}^x$

$x = -6$

$\therefore \log_{\sqrt{5}}\left(\frac{1}{125}\right) = -6$

A1

(d) A computer depreciates continuously in value from \$4699 to \$1500 over a period of 4.25 years.

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The value, \$$y$, of the computer $t$ years after its value was \$4699 can be modelled by a function of the form

$y = Ar^t$, where r is a constant.

Find the computer's value after six years.

$1500 = 4699\, r^{4.25}$

$r^{4.25} = 0.32$

$r = 1$

(e) Make $p$ the subject of the formula:

$$81^{\left(\frac{px}{q}-3\right)} = 243$$

$81^{\frac{px}{q}} \times \frac{1}{81^3} = 81 \times 3$

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MS

## QUESTION THREE

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(a) The quadratic equation $4x^2 + bx - 5 = 0$ has solutions $-\frac{1}{2}$ and $\frac{5}{2}$.

Find the value of $b$.

If it has two solutions

$\therefore 4\times(-\frac{1}{2})^2+(-\frac{1}{2})b-5=0$

$4\times\frac{1}{4}-\frac{b}{2}-5=0$

$-\frac{b}{2}=5$

$\frac{b}{2}=-5$

$b=-10$ //

(b) For what value(s) of $m$ does the equation $6x^2 - mx = -3$ have two equal roots?

If have two equal roots $6x^2-mx-3=0$

$\therefore b^2-4ac>0$

$(-m)^2-4\times6\times(-3)>0$

$m^2+72>0$

$m^2>72$

$m>8.5$ //

(c) Find the value(s) for $k$ for which the expression $kx^2 - 12x + 5k$ is always greater than zero.

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$kx^2 - 12x + 5k$

$b^2 - 4ac \leq 0$

$144 - 20k^2 \leq 0$

$144 \geq 20k^2$

$7.2 \geq k^2$

$k \leq 2.68.$

**Question Three continues on the following page.**

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(d) Write $\frac{9}{x^2-9}+\frac{3}{2x+6}$ as a single fraction in its simplest form.

$= \frac{9}{(x+3)(x-3)} + \frac{3}{2(x+3)}$

$= \frac{18}{2(x+3)(x-3)} + \frac{3(x-3)}{2(x+3)(x-3)}$

$= \frac{18+3x-9}{2(x+3)(x-3)}$

$= \frac{3x+9}{2(x+3)(x-3)}$

$= \frac{3(x+3)}{2(x+3)(x-3)}$

$= \frac{3}{2x-6}$ //

(e) Find the value(s) of $m$ for which the equation $2^{mx-3} = 8^{x^2}$ has exactly one solution.

$2^{mx-3} = 8^{x^2}$

$2^{mx-3} = 2^{3x^2}$

$\therefore mx-3 = 3x^2$

$3x^2 - mx + 3 = 0$

$b^2 - 4ac = 0$

$(-m)^2 - 4 \times 3 \times 3 = 0$

$m^2 = 36$

$m = \pm 6$

$m = 6$ or $m = -6$ //

E7

| Subject: | Mathematics | Standard: | 91261 | Total score: | 19 |
|---|---|---|---|---|---|

| Q | Grade score | Annotation |
|---|---|---|
| 1 | E7 | 1b Expansion of brackets for final answer acceptable.<br>1c Failure to subtract n – 2 and divide width by 2 a common error. |
| 2 | M5 | 2b Has not shown that 7 is the only possible solution from $\pm 7$ .<br>2d Cannot find the root or use logs to find the value of r.<br>2e Little real progress. |
| 3 | E7 | 3a A valid method but has dropped off the 4 x ¼.<br>3b Must have $\Delta = 0$ to get any credit for the question.<br>3c $\Delta \geq 0$ shows that there is little understanding of non-real roots<br>3e Has correctly used an index approach to reduce to a quadratic . |