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91261

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 2 Mathematics and Statistics, 2018

91261 Apply algebraic methods in solving problems

9.30 a.m. Wednesday 14 November 2018
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply algebraic methods in solving problems.	Apply algebraic methods, using relational thinking, in solving problems.	Apply algebraic methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You are required to show algebraic working in this paper. Guess-and-check methods, and correct answer(s) only, will generally limit grades to Achievement.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Achievement

TOTAL 14

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QUESTION ONE

(a) Simplify fully $\left(25m^{16}\right)^{\frac{1}{2}}$

$25m^{\frac{16}{2}}$

$= 25m^8$

n

(b) Simplify fully $\left(\frac{4}{3a}\right)^{-2}$, leaving your answer with a positive index.

$\left(\frac{4}{3a}\right)^{-1\times 2}$

$\left(\frac{3a}{4}\right)^2 = \frac{9a^2}{16}$

u

(c) Write $4 - \frac{b+8c}{3c}$ as a single fraction in its simplest form.

$\frac{4}{1} - \frac{b+8c}{3c}$

$= \frac{4}{1} \times \left(\frac{3c}{3c}\right) - \frac{b+8c}{3c}$

$= \frac{12c}{3c} - \frac{b+8c}{3c}$

$= \frac{12c - b - 8c}{3c} = \frac{4c-b}{3c}$

r

(d) Factorise fully $4bx + 2xy - 6ab - 3ay$

$= 2x(2b+y) - 3a(2b+y)$

$= (2x-3a)(2b+y)$

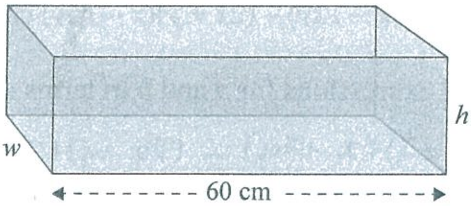
u

(e) A rectangular box has no lid.

The length of the base is 60 cm.

Its height is one quarter of the sum of its width and length.

The total area of the base **and** the four sides of the box is 7400 cm^2.



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Find the height of the box.

let length be x

$x = 60cm$

let width be W

to find area -

$60 \times W = 7400$

$W = \frac{7400}{60}$

$W = 123.33$

n

(f) $(3x + y)(x - 12y) - (2x + y)(x - 16y)$ can be written in the form $(a + b)^2$.

Find expressions for a and b in terms of x or y.

$= (3x+y)(x-12y) - (2x+y)(x-16y)$

$= 3x^2 - 36xy + xy - 12y^2 - 2x^2 + 32xy - xy + 16y^2$

$= \cancel{x^2 + 20xy + 4y^2}$ $\quad = x^2 - 4xy + 4y^2$

$\cancel{\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$ $\quad = (x - 2y)^2$

when expanded

$\cancel{-20 \pm \sqrt{20^2 - 4(2)(1)}}$ $\quad x^2 - 4xy + 4y^2$

$\cancel{x^2 + 20xy + 4y^2}$

$(3x+y)(x-12y) - (2x+y)(x-16y)$ can be written in the form $(x-2y)^2$.

$(x \times 2y)^2$

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M6

QUESTION TWO

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(a) Find x if $\log_x 243 = 5$

$x^5 = 243$

$\sqrt[5]{243} = 3 \qquad \underline{x = 3}$ //

u

(b) Find m if $\log_3(4m - 1) = 2$

$3^2 = 4m - 1$

$9 = 4m - 1$

$4m = 9 + 1$

$4m = 10$

$m = \frac{10}{4} \qquad \underline{m = 2.5}$ //

u

(c) Find an expression for x in terms of w if $\frac{3^{4x+1}}{9^x} = 27^{\frac{w}{3}}$

$3^{4x+1} = 27^{\frac{w}{3}} \cancel{\times 9^x}$

$\frac{9x}{\cancel{3^{4x+1}}} = \frac{1}{27^{\frac{w}{3}}}$

$9x = \frac{1}{27^{\frac{w}{3}}} \times \cancel{3^{4x+1}}$

$9x = \frac{\cancel{3^{4x+1}}}{\cancel{27^{\frac{w}{3}}}}$

$9x = \cancel{\frac{4x+1\log 3}{\frac{w}{3}\log 27}}$

$\cancel{9x = \frac{4x+1}{\frac{w}{3}} - 0.954}$

$\cancel{9x \times \frac{w}{3} = 4x + 1 - 0.954}$

$\cancel{9x \times \frac{w}{3} = 4x - 0.046}$

$\cancel{9x \times w = 4x - 0.046 \times 3}$

$\cancel{9x \times w = 4x - 0.138}$

$\cancel{9xw - 4x = -0.138}$

$\frac{9^x}{3^{4x+1}} = \frac{1}{27^{\frac{w}{3}}}$

$9^x = \frac{1}{27^{\frac{w}{3}}} \times 3^{4x+1}$

$9^x = \frac{3^{4x+1}}{27^{\frac{w}{3}}}$

$27^{\frac{w}{3}} = \frac{3^{4x+1}}{9^x}$

$27^{\frac{w}{3}} = \frac{4x+1\log 3}{x\log 9}$

$27^{\frac{w}{3}} = \frac{4x-1}{x} - 0.48$

$\frac{4x+1}{x} = 27^{\frac{w}{3}} + 0.48$

$4x + 1 = (27^{\frac{w}{3}} + 0.48) \times x$

$1 = (27^{\frac{w}{3}} + 0.48) \times x - 4x$

$\frac{1}{27^{\frac{w}{3}} + 0.48} = x - 4x$

$\frac{1}{27^{\frac{w}{3}} + 0.48} = -3x$

$x = \frac{1}{27^{\frac{w}{3}} + 0.48} \div (-3)$ //

n

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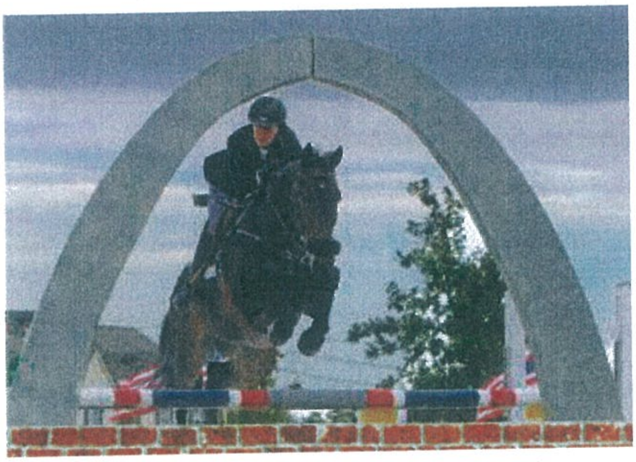
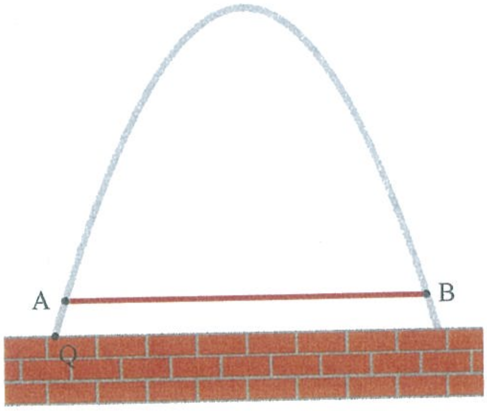
(d) An equestrian jump has a parabolic arch mounted on a wall. Horses and riders jump through the arch.





Source: http://luxequestrian.com/slideshow/incredible-jumps-brody-robertson

The arch rises 2.43 metres **above the wall**.

The arch can be modelled by a function of the form $h(x) = kx(3.6 - x)$, where k is a constant, h metres is the height above the wall, and x metres is the horizontal distance from Q.

A rail AB can be placed above the wall and attached at each end to the arch. For one competition, the rail is placed 0.5 metres above the wall.

How long is the rail AB?

$h(x) = Kx(3.6 - x)$

$0.5(x) = Kx(3.6 - x)$

$0.5x = 3.6Kx - Kx^2$

$-Kx^2 + 3.6Kx - 0.5x = 0$

2.43

$\cancel{0.5(x) = 0.5(x)}$

$2.43(x) = 0.5x(3.6 - x)$

$2.43x = 1.8x - 0.5x^2$

$0 = 1.8x - 0.5x^2 - 2.43x$

$0 = -0.5x^2 - 2.43x + 1.8x$

$0 = \cancel{-0.63} - 0.5x^2 - 0.63x$

$-0.5x^2 - 0.63x = 0$

$-0.5x^2 = 0.63x$

$x^2 = \frac{0.63x}{-0.5}$

$x = 1.26$ rail AB is 1.26m long.

$\frac{x^2}{x} = \frac{0.63}{0.5}$

n

ASSESSOR'S USE ONLY

(e) Interest is compounded on a principal investment, $\$P$, **at the end of each year**.

If the total amount of the investment after n years is $\$A$ then $A = P\left(1+\frac{r}{100}\right)^n$

where r % is the compound interest rate per year.

(i) Anushka invests \$20 000 at an interest rate of 3.85% (so $A = P(1.0385)^n$).

How many years will it take for her investment to be worth \$25 000?

$25,000 = (1.0385 \times 20,000)^n$

$\log 25,000 = n \log \cancel{1.0385}$ 20770

$\cancel{\frac{\log 25,000}{\log 1.0385} = A}$

$\frac{\log 25,000}{\log 20770} = n$

$n = 1.02$

~~A = 263.06~~ it will take one year for her investment to be worth \$25,000.

n

(ii) Semisi invests his money at a different interest rate than Anushka's investment. His investment will double in value after twelve years.

What is the interest rate for Semisi's investment?

~~25,000~~

n

A3

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QUESTION THREE

(a) Solve each of the following equations for x:

(i) $12x^2 - 5x = 2$

$12x^2 - 5x - 2 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{5 \pm \sqrt{25 - 4(12)(-2)}}{24} = \frac{5 \pm \sqrt{25 + 96}}{24} = \frac{5 \pm 11}{24}$

$x = \frac{16}{24} = 0.67$ or $x = \frac{-6}{24} = -0.25$ u

(ii) $x + 1 - \frac{3}{x} = 0$

$x - \frac{3}{x} = -1$

n

(b) Show that the graph of the function $y = 2x^2 - 5x + 6$ does not cross the x-axis.

You must use algebra to support your explanation.

$b^2 - 4ac < 0$

$(-5)^2 - 4(2)(6) < 0$

$25 - 48 < 0$

$-23 < 0$

y must be -23 in order for it to not cross the x axis. ~~so~~ ~~the~~ -23 is less than 0.

r

Question Three continues on the following page.

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(c) The equation $3x^2 + kx - 12 = 0$ has two real solutions.

If one of the solutions is $x = 3$, find the other solution.

$\cancel{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

Substituting 3 in place of x

$3(3)^2 + (K \times 3) - 12 = 0$

$27 + 3K - 12 = 0$

$\cancel{15 = 3K}$ $15 + 3K = 0$

$3K = -15$

$\cancel{K = -5}$

Other solution must be -5.

u

(d) Show that the roots of the equation $x^2 + 2(k + 1)x - (k^2 + 2k + 5) = 0$, where k is a constant, can never be equal for any real number k.

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$x^2 + 2x(k+1) - (k^2 + 2x + 5) = 0 \rightarrow x^2 + 2x(k+1) - k^2 - 2x - 5 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$-2x(k+1) \pm \sqrt{[2x(k+1)^2] - 4(\cancel{4})(}$

$x^2 + 2x(k+1) - (k^2 + 2x + 5) = 0$

$x^2 + 2xk + 2x - k^2 - 2x - 5\cancel{k^2} = 0$

$x^2 + 2xk - 6k^2 = 0$

$2xk - 6k^2 = -x^2 \quad = 2kx - 6k^2 = 1$

$\cancel{-6k^2 = x^2 - 2xk} \quad -6k^2 + 2kx - 1 = 0$

$\cancel{b^2 - 4ac}$

$\cancel{(2xk)^2 - 4(\cancel{1}}$ $\quad -b \pm \sqrt{b^2 - 4ac}$

$b^2 - 4ac \quad -2k \pm$

$2k^2 - 4(1)(-6)$

$2k^2 + 24 = 0$

$2k^2 = -24$

$k^2 = -\frac{24}{2}$

$k^2 = -12$

?

MS

Achievement Exemplar 2018

Subject	Mathematics	**Standard**	91261	**Total score**	14

Q	**Grade score**	**Annotation**
1	M6	1a Has not taken the square root of 25. 1e Neither the expression for the height nor the area of the base and four sides has been given. 1f There is a correct expansion of the original expression, and then a correct factorisation, but a and b have not been identified as required.
2	A3	2c Neither side has been reduced to powers of 3. 2d k = 0.75 has not been found so no further progress is possible. 2eii There has been an incorrect substitution.
3	M5	3c k = -5 has been found, but no further progress to find the other root. 3d Incorrect substitution into the discriminant, although there is recognition that equal roots have a discriminant equal to 0.