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QUALIFY FOR THE FUTURE WORLD
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Level 2 Mathematics and Statistics, 2016

91261 Apply algebraic methods in solving problems

9.30 a.m. Thursday 24 November 2016
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply algebraic methods in solving problems.	Apply algebraic methods, using relational thinking, in solving problems.	Apply algebraic methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You are required to show algebraic working in this paper. Guess-and-check methods and correct answer(s) only will generally limit grades to Achievement.

Check that this booklet has pages 2–11 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Excellence

TOTAL 23

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QUESTION ONE

(a) Simplify $\left(\frac{3b}{c^2}\right)^{-4}$ leaving your answer with positive indices.

$\left(\frac{c^2}{3b}\right)^4 = \frac{c^8}{81b^4}$ //

(b) Write $x^2 - 8x + 10$ in the form $(x-p)^2 + q$.

$(x-4)^2 - 6$ //

(c) (i) Show that the solutions of the equation $x^2 + x - 56 = 0$ are four times the solutions of the equation $4x^2 + x - 14 = 0$.

$x^2 + x - 56 = 0 \quad (x-7)(x+8) = 0 \quad x = 7$ and -8

$4x^2 + x - 14 = 0 \quad x = \frac{-1 \pm \sqrt{1+224}}{8} = 1.75$ and -2

Thus the solutions of $x^2 + x - 56 = 0$ are 4 times the Solutions of $4x^2 + x - 14 = 0$ // $1.75 \times 4 = 7$

$-2 \times 4 = -8$

OR

$x^2 + x - 56 = 0 \quad x = \frac{-1 \pm \sqrt{1+4\times56}}{2}$ — ① ① is 4 times of ②

$4x^2 + x - 14 = 0 \quad x = \frac{-1 \pm \sqrt{1+4\times56}}{8}$ — ② /

(ii) Find the relationship between the solutions of the equation $dx^2 + ex + f = 0$ and the solutions of the equation $x^2 + ex + df = 0$, where d, e, and f are real numbers.

Solutions of $dx^2 + ex + f = 0 \quad x = \frac{-e \pm \sqrt{e^2 - 4df}}{2d}$

Solutions of $x^2 + ex + df = 0 \quad x = \frac{-e \pm \sqrt{e^2 - 4df}}{2}$

Thus the solutions of $x^2 + ex + df = 0$ is d times the solutions of $dx^2 + ex + f = 0$ //

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(d) A quadratic equation of the form $ax^2 + bx + c = 0$ has solutions $-\frac{1}{2}$ and $\frac{2}{3}$.

Find a possible set of values for a, b, and c.

$(x + \frac{1}{2})(x - \frac{2}{3}) = 0$ $\quad x^2 - \frac{1}{6}x - \frac{1}{3} = 0$

$\Rightarrow 6x^2 - x - 2 = 0$

possible set of values $a = 6$

" $b = -1$

$c = -2$ //

(e) Find positive integer value(s) for k so that ~~the quadratic~~ equation $2x^2 + 4kx + (2k^2 + 3k - 11) = 0$ has **real rational** solutions.

Justify your answer.

$\Delta\ b^2 - 4ac > 0$

$16k^2 - 4 \times 2(2k^2 + 3k - 11) > 0$

$16k^2 - 16k^2 - 24k + 88 > 0$

$24k < 88 \quad k < 3.67$ (2dp)

positive integer value of $k = 1, 2, 3$ //

E7

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QUESTION TWO

(a) Find the discriminant of the quadratic equation $x^2 = 10x + 3$.

~~$x^2 = 10x + 3$~~ $x^2 - 10x - 3 = 0$

discriminant $b^2 - 4ac$

$\Delta = 100 + 4 \times 1 \times 3 = \underline{112}$ ✓

(b) Simplify $\frac{4\log(u^3)}{\log u}$.

$\frac{\log u^{12}}{\log u} = \underline{\underline{12}}$ ✓✓

(c) Marie buys a new car for \$24 990. $\times 0.88^t$

The car's value decreases continuously by 12% each year.

The value of the car, \$$P$, t years after she first bought it, can be modelled by a function of the form $P = A(r)^t$.

How long will it take for the value of the car to halve?

$P = 24990 \times 0.88^t$

$\frac{24990}{2} = 24990 \times 0.88^t$

$0.5 = 0.88^t$

$t = \log 0.5 \div \log 0.88$

$= \underline{5.42 \text{ years}}$ (2dp) ✓✓

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(d) (i) Solve the equation $\log_8 x = \frac{2}{3}$.

$x = 8^{\frac{2}{3}} = \sqrt[3]{8^2} = 4$ //

(ii) Solve the equation $6(\log_8 x)^2 + 2\log_8 x - 4 = 0$.

Assume $\log_8 x = a$

$6a^2 + 2a - 4 = 0$

$a = -1$ and $\frac{2}{3}$

$\log_8 x = -1$ $x = \frac{1}{8}$

and $\log_8 x = \frac{2}{3}$ $x = 4$

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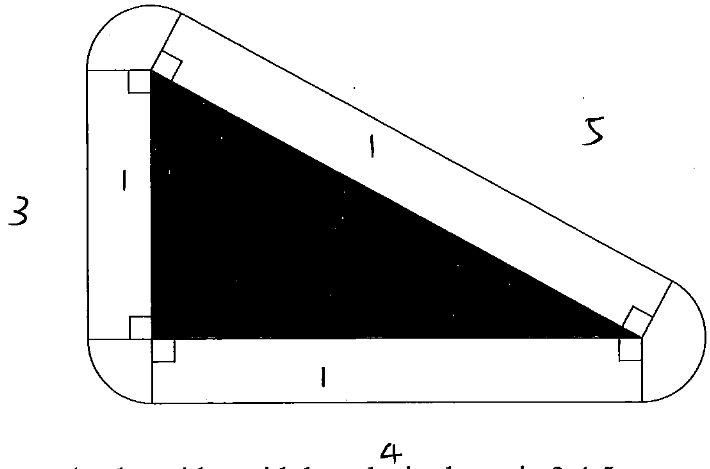
(e) The diagram below shows a triangular garden with a path around it.



The triangular garden has sides with lengths in the ratio 3:4:5.

The path is 1 m wide.

At each corner of the garden, the path is a sector (part) of a circle with a radius of 1 m.

The difference between **twice** the **total** area of the path and the area of the garden is 2π m^2.

Find the length of the longest side of the garden.

(Area of circle = πr^2)

~~Total Area of path : 3x1 + 4x1 + 5x1 = 12 m²~~

~~Area of garden : 3x4 ÷ 2 = 6 m²~~

$\because$ ratio = 3 : 4 : 5

$\therefore$ Set the length is $3x$, $4x$ and $5x$

$A_{path} = 12x + \pi$

$A_{garden} = 3x \times 4x \div 2 = 6x^2$

$2(12x+\pi) - 6x^2 = 2\pi$

$24x + 2\pi - 6x^2 = 2\pi$

$\cancel{24x - 6x = 0}$

$24x - 6x^2 = 0$

$-6x^2 + 24x = 0$

$-6x(x-4) = 0$

$x = 0$ or $x = 4$

$\because$ It's distance

$\therefore$ It can't be 0

$\therefore x = 4$ longest = 5×4 = 20 m //

Eg

QUESTION THREE

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(a) Where would the graph of $y = 12x^2 - x - 6$ cut the x-axis? $y=0$

when ~~y=0~~ $x = \frac{3}{4}$ and $-\frac{2}{3}$ //

(b) For what value(s) of x does $\log_x(216) = 3$?

$x = \sqrt[3]{216} = 6$ //

(c) Rearrange the following formula to make x the subject: $\frac{4x}{5} = \frac{y(x+3)}{2}$. $\times 10$

$8x = 5y(x+3)$

$8x = 5yx + 15y$

$8x - 5yx = 15y$

$x(8-5y) = 15y$

$x = \frac{15y}{8-5y}$ //

Question Three continues on the following page.

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(d) Solve the equation $9^{8n+6} = 27^{n^2-1} \times 3^{1-3n}$.

$(3^2)^{8n+6} = (3^3)^{n^2-1} \times 3^{1-3n}$

$3^{16n+12} = 3^{3n^2-3} \times 3^{1-3n}$

$16n+12 = 3n^2-3+1-3n$

$3n^2-19n-14=0$

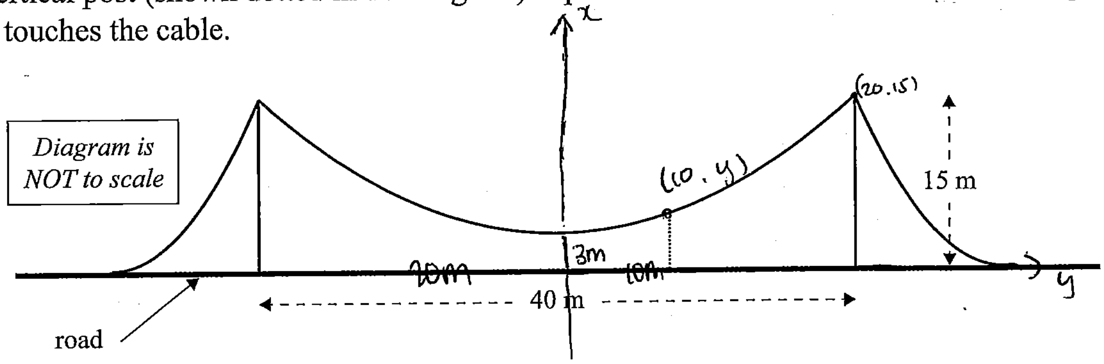
$n = 7$ and $-\frac{2}{3}$

(e) A symmetrical bridge has its central cable in the shape of a parabola, as shown in the diagram below.

The towers supporting the cable are each 15 m high and 40 m apart.

At the point midway between the towers, the height of the cable above the road is 3 m.

A vertical post (shown dotted in the diagram) is placed 10 m from the centre of the bridge and just touches the cable.



(i) Use algebra to show that the post is 6 m high.

$y = kx^2 + 3$ Sub(20, 15)

$15 = k \times 400 + 3$ $k = 0.03$ $y = 0.03x^2 + 3$

Sub $x = 10$

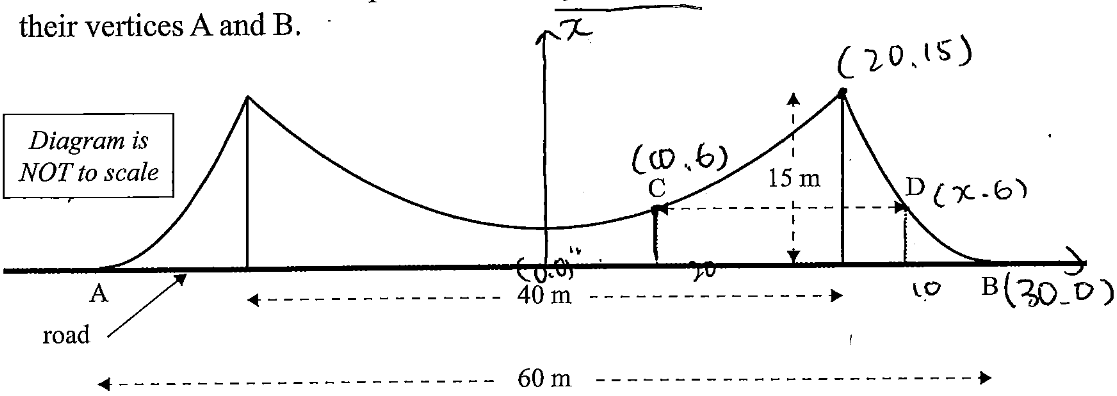
$y = 0.03 \times 100 + 3$

$= 6$

$\therefore$ the post which is 10 m away from the centre is 6 m high

(ii) The length of the bridge AB is 60 m.

The outside cables are also parabolic and symmetrical in shape, and touch the road at their vertices A and B.



Find the distance, CD, between the two parabolas at a height of 6 m above the road (the distance CD is shown in the diagram).

2^{nd} parabola $\quad y = k(x-30)^2 \quad$ sub (20, 15)

$15 = 100k \quad k = 0.15$

$y = 0.15x^2 - 9x + 135$

sub $y = 6$ $\quad 6 = 0.15x^2 - 9x + 135$

$0.15x^2 - 9x + 129 = 0$

$x = 36.325$ or 23.675 (3dp)

x should be 23.675 according to the graph above.

$\overline{CD} = 23.675 - 10 = 13.675$ m

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E8

Annotated Exemplar Template

Excellence exemplar 2016

Subject:	Mathematics	Standard:	91261	Total score:	23

Q	Grade score	Annotation
1	E7	1a Power correctly applied and expression simplified 1ci Solutions to both equations found and relation stated 1cii Solutions to both equations found and relation stated 1d Quadratic formed and correct values for a, b and c stated 1e Correct substitution into discriminant, k expressed as an inequality. Correct positive integers found but k= 2 not eliminated (k = 2 generates irrational roots).
2	E8	2b Correct application of log rule and simplification 2c Equation correctly formed and solved using logs with answer in context 2dii Quadratic formed and solved, log rules applied to find both solutions for x 2e Ratios correctly applied, quadratic formed to represent difference, solved and answer for the longest side given in context.
3	E8	3c Terms with x gathered to one side, equation given with x as subject 3d Equation written with common base, quadratic established using powers, correct values of n stated 3ei Equation formed and evidence of y = 6 when x = 10 3eii Equation for second parabola formed and solution for y = 6 used to determine length CD.