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QUALIFY FOR THE FUTURE WORLD
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Level 2 Mathematics and Statistics, 2016

91262 Apply calculus methods in solving problems

9.30 a.m. Thursday 24 November 2016
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You must show the use of calculus in answering all questions in this paper.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Excellence

TOTAL 23

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QUESTION ONE

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(a) A function f is given by $f(x) = 4x^3 - 7x^2 + 2x - 4$.

Find the gradient of the graph of the function at the point where $x = 2$.

$f'(x) = 12x^2 - 14x + 2$

$f'(2) = 12 \times 2^2 - 14 \times 2 + 2$

$f'(2) = 48 - 28 + 2$

$f'(2) = 22$

(b) The line $y = x + 3.25$ is a tangent to the graph of the function $f(x) = 3x^2 - 2x + 4$.

Use calculus to show that the line is a tangent to the curve, and that the point where this tangent touches the curve is (0.5,3.75).

$y - y_1 = m(x - x_1)$ $\therefore m = 1$

$f'(x) = 6x - 2$

$1 = 6x - 2$

$3 = 6x$

$x = 0.5$

$y_1 = f(0.5) = 3 \times 0.5^2 - 2 \times 0.5 + 4$

$y_1 = 0.75 - 1 + 4$

$y_1 = 3.75$

$\therefore \boxed{y - y_1 = m(x - x_1)}$

$y - 3.75 = 1(x - 0.5)$

$y = x - 0.5 + 3.75$

$y = x + 3.25$

(c) The function $f(x) = 2x^3 + kx^2 + 5$ has a minimum turning point when $x = 1$.

What are the coordinates of the **maximum** turning point?

$f'(x) = 6x^2 + 2kx$

$f'(1) = 0 = 6 \times 1 + 2k \times 1$

$0 = 6 + 2k$

$-6 = 2k$

$k = -3$

$\therefore f(x) = 2x^3 - 3x^2 + 5$

$f'(x) = 6x^2 - 6x$

$0 = 6x^2 - 6x$

$6x(x-1) = 0$

$\therefore x = 0$ or $x = 1$

Maximum point.

when $x = 0$

$f''(x) = 12x - 6$ $\quad -6 < 0 \therefore$ Max turning pt.

$f''(0) = 0 - 6 = -6$

$f(0) = 2 \times 0 - 3 \times 0 + 5$

$f(0) = 5$

$\therefore$ Maximum turning point :

$(x, y) = (0, 5)$

(d) The equation of a function $y = f(x)$ has gradient function of the form $f'(x) = 2x - a$, where a is a constant.

$f'(x) = (3, 0)$

The point (3,4) is the turning point on the graph of the function.

Find the equation of the function.

$f(x) = x^2 - ax + c$

~~insert (3,4) → f(3) = ...~~

$(3, 0) \quad f'(3) = 0 = 2 \times 3 - a$

$6 - a = 0$

$a = 6$

$\therefore f'(x) = 2x - 6$

$f(x) = x^2 - 6x + c$

insert $(3, 4) \rightarrow f(3) = 4 = 3^2 - 6 \times 3 + c$

$4 = 9 - 18 + c$

$c = 13$

$\therefore f(x) = x^2 + 6x + 13$

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(e) Find the local minimum value of the function $y = x^3(x - 4)$.

Justify your answer.

$y = x^4 - 4x^3$

$y' = 4x^3 - 12x^2$

$0 = 4x^3 - 12x^2$

$0 = x^3 - 3x^2$

$0 = x^2(x - 3)$

$\therefore x = 0, x = 0, x = 3$

$y'' = 12x^2 - 24x$

$y''(3) = 12 \times 9 - 24 \times 3$

$= 108 - 72$

$= 36$

$36 > 0 \therefore$ Minimum turning point

$\therefore y(3) = 3^3(3 - 4)$

$= 27 \times -1$

$= -27$ ← Minimum value.

minimum turning point .. (3, −27)

E8

QUESTION TWO

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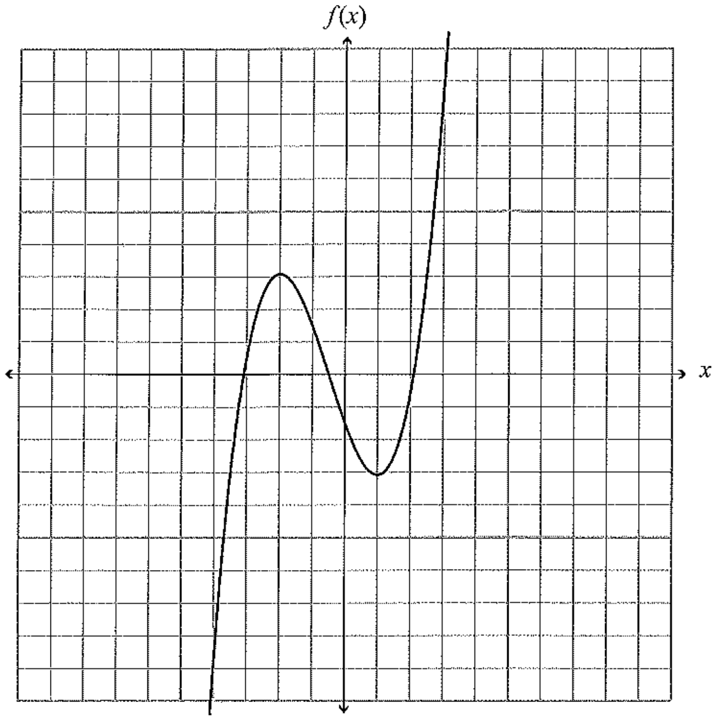
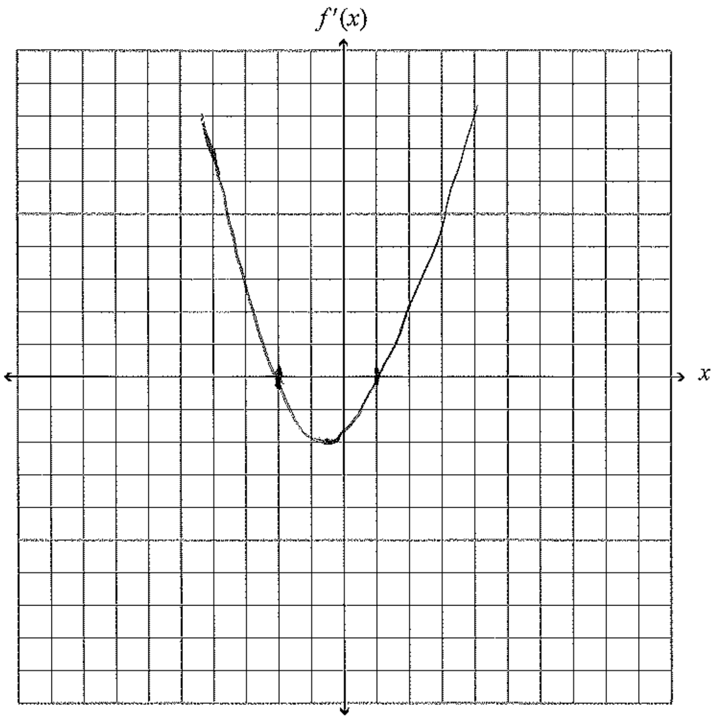
(a) The diagram below shows the graph of the function $y = f(x)$.



On the axes below sketch the gradient function $y = f'(x)$.



If you need to redraw this graph, use the grid on page 11.

(b) The line $y = ax + b$ is a tangent to the graph of the function $y = 2x^2 - 3x + 1$ at the point (3,2).

Find the values of a and b.

$y' = 4x - 3$

$m = y'(3) = 4 \times 3 - 3$

$m = 12 - 3$

$m = 9$

$\therefore y - y_1 = m(x - x_1)$

$y - 2 = 9(x - 3)$

$y = 9x - 27 + 2$

$y = 9x - 25$

$\therefore a = 9, b = -25$

(c) A function f is given by $f(x) = 2 - 4x + 5x^2 + ax^3$.

The gradient of the graph of the function at the point where $x = 1$ is 3.

Find the value of a.

$f'(x) = -4 + 10x - 3ax^2$

$f'(1) = 3 = -4 + 10 - 3a$

$-3 = 6 - 3a$

$3a = 3$

$a = 1$

$\therefore f(x) = 2 - 4x + 5x^2 + x^3$

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(d) A chemical is slowly leaking onto a floor.

The chemical spreads out from the point where it lands in a shape that can be modelled by a circle of radius r cm.

$10 = 0.1t + 2$

At a time t seconds after the chemical leak is noticed, r is given by $0.1t + 2$.

$8 = 0.1t \quad t = 80$

Use calculus to find the rate of change of area of the circle, with respect to time, when its radius is 10 cm.

(Area of circle = πr^2)

$A = \pi r^2 \rightarrow A = \pi(0.1t + 2)^2 = \pi(0.01t^2 + 0.4t + 4)$

$A' = 0.02\pi t + 0.4\pi$

when $r = 10$ — when $t = 80$

$r = 0.1t + 2$ — $A' = 0.02\pi \times 80 + 0.4\pi$

$10 = 0.1t + 2$ — $= 6.28$ cm²/s (cm² per second)

$8 = 0.1t$

$t = 80$

(e) A function is defined by $y = 3x^3 - 4a^2x + 5$ where a is a positive number.

Find the range of values of x in terms of a for which the function is decreasing.

$y' = 9x^2 - 4a^2$

decreasing function = negative gradient

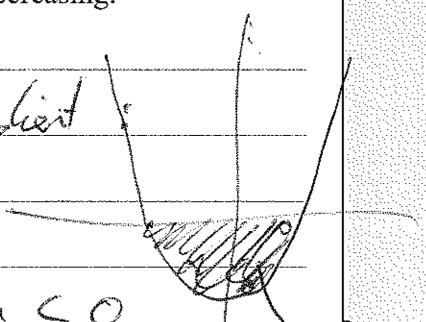
$9x^2 - 4a^2 < 0$

$9x^2 < 4a^2$

$x^2 < 0.444a^2 \longrightarrow x^2 - 0.444a < 0$

$0.667a > x > -0.667a$

(3sf)



$x^2 - \frac{4}{9}a^2 < 0$
(below x axis)
$y < 0$

QUESTION THREE

ASSESSOR'S USE ONLY

(a) The gradient function for a curve is given by $\frac{dy}{dx} = 3x^2 - 5$.

The curve passes through the point (1,0).

Find the equation of the curve.

$y = x^3 - 5x + c$

insert $(1,0) \rightarrow 0 = 1^3 - 5 + c$

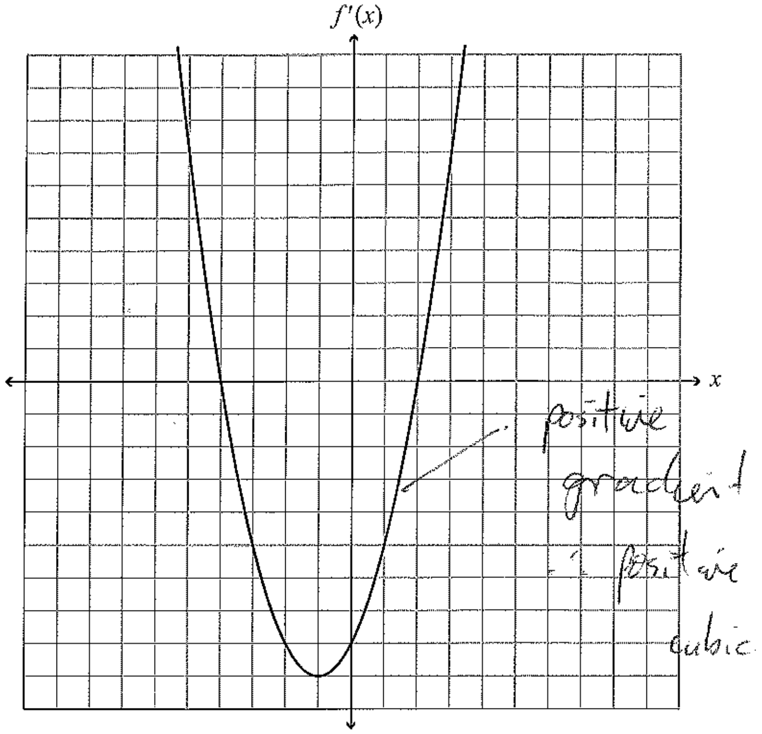
$5 - 1 = c$

$c = 4$

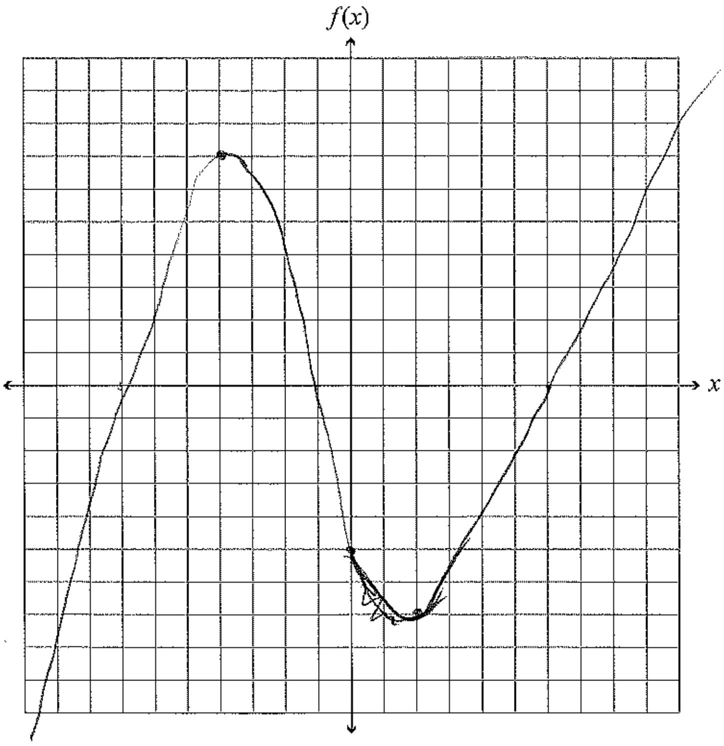
$\therefore y = x^3 - 5x + 4$

(b) The diagram below shows the graph of the gradient function $y = f'(x)$ of a function $y = f(x)$.

ASSESSOR'S USE ONLY



On the axes below sketch the graph of the function $y = f(x)$.



If you need to redraw this graph, use the grid on page 11.

(c) Meg is riding her motocross bike.

When she passes a fixed point P on the track, she has a speed, v, of 5 m s^{-1}, and her acceleration, a, is 0.6 m s^{-2}.

(i) If she were to continue to accelerate at this rate, what is her speed when she has been riding for 10 seconds after passing P?

$a = 0.6$

$v = 0.6t + c$

$\therefore 5 = 0.6 \times 0 + c \rightarrow \therefore c = 5$

$\therefore v = 0.6t + 5$

when $t = 10$

$v = 0.6 \times 10 + 5$

$= 6 + 5$

$= 11$ m/s or 11 ms^{-1}

(ii) How far will she have travelled from P when she reaches a speed of 8 m s^{-1}?

$d = 0.3t^2 + 5t + c$

when $d = 0$, $t = 0$

$0 = 0.3 \times 0 + 5 \times 0 + c \rightarrow \therefore c = 0$

$\therefore d = 0.3t^2 + 5t$

when $v = 8$

$8 = 0.6t + 5$

$3 = 0.6t$

$t = 5$

$\therefore$ when $t = 5$:

$d = 0.3 \times 5^2 + 5 \times 5$

$= 7.5 + 25$

$= 32.5$ m

when she reaches a speed of 8 ms^{-1}, she ~~will be~~ would have traveled 32.5 m

Question Three continues on the following page.

(iii) Meg's friend Leo was riding with her, but he begins to decelerate when they reach a speed of 8 m s^{-1}.

If he decelerates at 0.2 m s^{-2}, how far past the point P will he be when he reaches a speed of 6 m s^{-1}?

$a = -0.2$

$V = -0.2t + c$

when $t = 5$, $V = 8$

$\therefore 8 = -0.2 \times 5 + c$

$c = 8 + 1$

$c = 9$

$\therefore V = -0.2t + 9$

when $V = 6$:

$6 = -0.2t + 9$

$3 = 0.2t$

$t = 15$

$d = -0.1t^2 + 9t + c$

when $d = 0$, $t = 0$

$\therefore 0 = 0 + 0 + c \Rightarrow c = 0$

$\therefore \boxed{d = -0.1t^2 + 9t}$

when $V = 6$, $t = 15$

$\therefore d = -0.1 \times 15^2 + 9 \times 15$

$d = -22.5 + 135$

$d = 112.5$m

due to Leo decellerating at a rate of 0.2 ms^{-2} when he reaches a speed of 6 ms^{-1} he would have travelled 112.5m

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E7

Excellence exemplar 2016

Subject:	Mathematics	Standard:	91262	Total score:	23

Q	Grade score	Annotation
1	E8	1(b) Point/gradient formula accurately used 1(c) co-ordinates given. Two x values evaluated 1(d) Transfer error accepted due to appropriate working shown 1(e) Full answer given
2	E8	2(b) a and b written fully 2(c) Correct solution found. 1ms ignored 2(d) Correct algebraic expansion, accurate differentiation, solution accurate 2(e) Answer wriiten around the wrong way with -ve on RHS, but still correct solution (decimals accepted)
3	E7	3(c)(i) Velocity formula with evidence of +c evaluated 3(c)(ii) Distance formula with +c, then c=0 demonstrated. Time evaluated. Solution to the problem given, with units (not required) 3(c)(iii) Alternate method accurate to t = 15. C not equal to zero: should have c = -10 so does not satisfy “r” criteria. Wrong solution to problem.