91262M

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NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

2

SUPERVISOR'S USE ONLY

Te Pāngarau me te Tauanga, Kaupae 2, 2017

91262M Te whakahāngai tikanga tuanaki hei whakaoti rapanga

2.00i te ahiahi Rāmere 24 Whiringa-ā-rangi 2017
Whiwhinga: Rima

Paetae	Kaiaka	Kairangi
Te whakahāngai tikanga tuanaki hei whakaoti rapanga.	Te whakahāngai tikanga tuanaki mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai tikanga tuanaki mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tirohia mēnā kei a koe te Puka Tikanga Tātai L2–MATHMF.

Whakaaturia ngā mahinga KATOA.

Mēnā ka hiahia whārangi atu anō mō ō tuhinga, whakamahia ngā whārangi wātea kei muri o tēnei pukapuka, ka āta tohu ai i ngā tau tūmahi.

Me mātua whakaatu e koe te whakamahi tuanaki i ō tuhinga mō ngā tūmahi katoa i tēnei pepa.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–23 kei roto i tēnei pukapuka, ā, kāore tētahi o aua whārangi i te takoto kau.

HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU

TAPEKE

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATAHI

MĀ TE KAIMĀKA ANAKE

(a) Ka tohua he pānga f mā te $f(x) = x^5 + 3x^2 - 7x + 2$.

Whiriwhiria te rōnaki o te kauwhata o te pānga kei te pūwāhi $x = 1$.

(b) Whiriwhiria te whārite o te pātapa ki te kauwhata o te pānga

$f(x) = 6 + 14x - 2x^3$

i te pūwāhi (2,18) kei te kauwhata.

(c) Ka tuhia te nekehanga o tētahi ahanoa mai i te wā ka hipa i tētahi pūwāhi pūmau.
Ko te tere i muri i te t hēkona he v m s^{-1}, ka taea te whakatauira mā te pānga

$v(t) = 0.5t^2 - 2t + 1$

Whakamahia te tuanaki hei whiriwhiri e hia te roa ka eke te whakaterenga[1] ki te 2.8 m s^{-2}.

[1] whakahohorotanga

QUESTION ONE

ASSESSOR'S USE ONLY

(a) A function f is given by $f(x) = x^5 + 3x^2 - 7x + 2$.

Find the gradient of the graph of the function at the point where $x = 1$.

(b) Find the equation of the tangent to the graph of the function

$f(x) = 6 + 14x - 2x^3$

at the point (2,18) on the graph.

(c) The movement of an object is recorded from the time it passes a fixed point.
After t seconds it has a speed v m s^{-1}, which can be modelled by the function

$v(t) = 0.5t^2 - 2t + 1$

Use calculus to find how long it takes to reach an acceleration of 2.8 m s^{-2}.

(d) He 2 te rōnaki o te pātapa ki te kauwhata o te pānga $f(x) = 3x^2 - 4x$, ā, ka rere mā te pūwāhi $(5,a)$, ina he pūmau a a.

MĀ TE KAIMĀKA ANAKE

Whiriwhiria te uara o a.

(e) He pūwāhi huringa kei te pānga $f(x) = x^3 + ax^2 + bx + 2$ ina ko $x = -1$ me te $x = 3$.

Whiriwhiria ngā uara o te a me b.

ASSESSOR'S USE ONLY

(d) A tangent to the graph of the function $f(x) = 3x^2 - 4x$ has a gradient of 2, and passes through the point $(5,a)$, where a is a constant.

Find the value of a.

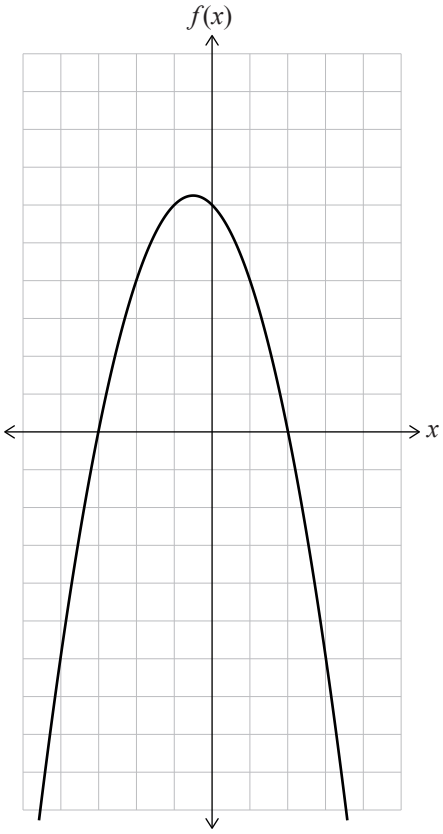
(e) The function $f(x) = x^3 + ax^2 + bx + 2$ has turning points when $x = -1$ and $x = 3$.

Find the values of a and b.

TŪMAHI TUARUA

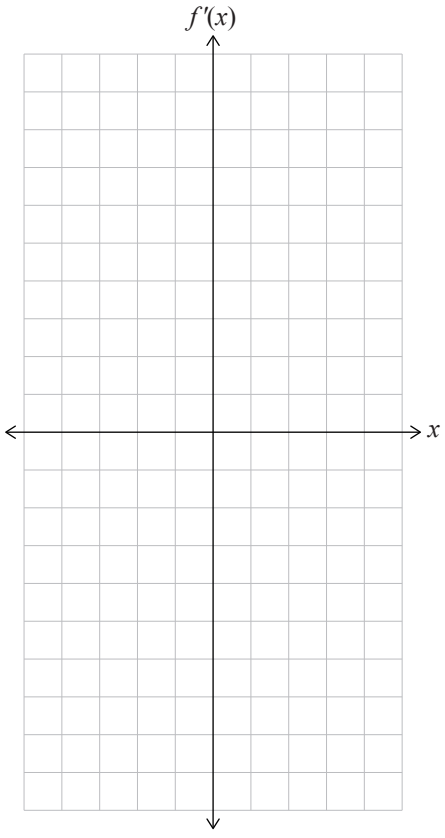
MĀ TE KAIMĀKA ANAKE

(a) E whakaatu ana te hoahoa o raro nei i te kauwhata o te pānga $y = f(x)$.



Tātuhia te kauwhata o te pānga rōnaki $y = f'(x)$ ki ngā tuaka o raro.

He ōrite te āwhata o ngā huinga tuaka e rua.

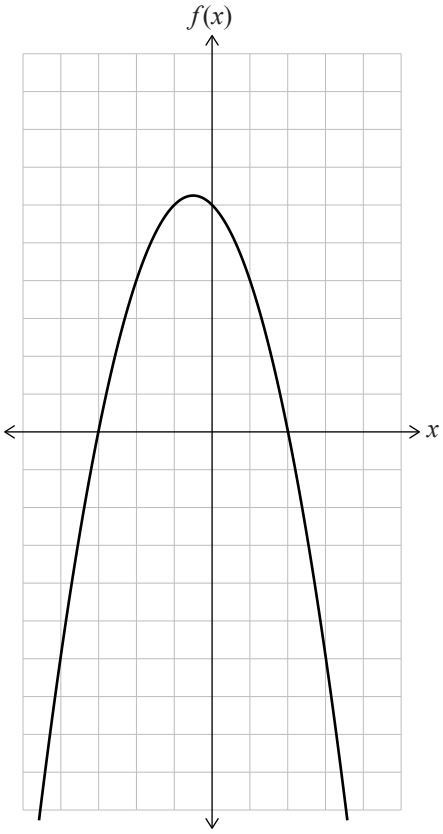


Ki te hiahia koe ki te tuhi anō i tēnei kauwhata, whakamahia te tukutuku i te whārangi 20.

QUESTION TWO

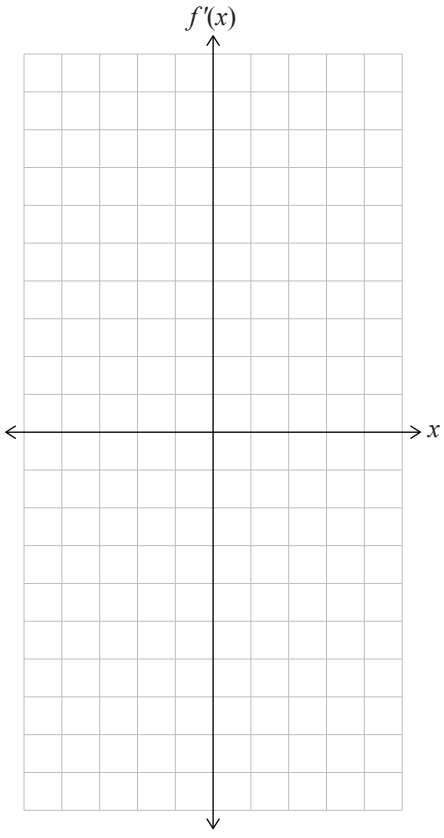
ASSESSOR'S USE ONLY

(a) The diagram below shows the graph of the function $y = f(x)$



Sketch the graph of the gradient function $y = f'(x)$ on the axes below.

Both sets of axes have the same scale.



If you need to redraw this graph, use the grid on page 21.

MĀ TE KAIMĀKA ANAKE

(b) Ko te kauwhata o tētahi pānga $f(x) = 2x^3 + bx^2 - 2$ he pūwāhi huringa tōna ina ko $x = -1$.

Whiriwhiria te uara o b.

(c) Whakamahi tuanaki hei whakaatu ko te rārangi $y = 15x - 12$ he pātapa ki te kauwhata o te pānga $f(x) = 4x^2 - x + 4$.

(d) Whakamahi tuanaki hei whiriwhiri i te uara o te k mēnā he pātapa te rārangi $y = 6x + k$ ki te kauwhata o te pānga $f(x) = x^2 + 2x - 1$.

He wāhi anō mō tō tuhinga mō tēnei tūmahi kei te whārangi 10.

ASSESSOR'S USE ONLY

(b) The graph of a function $f(x) = 2x^3 + bx^2 - 2$ has a turning point when $x = -1$.

Find the value of b.

(c) Use calculus to show that the line $y = 15x - 12$
is a tangent to the graph of the function $f(x) = 4x^2 - x + 4$.

(d) Use calculus to find the value of k if the line $y = 6x + k$ is a tangent to the graph of the function $f(x) = x^2 + 2x - 1$.

There is more space for your answer on page 11.

MĀ TE KAIMĀKA ANAKE

(e) Whakamahi tuanaki hei hāpono kua whai mōrahi paetata te kauwhata o te pānga

$$y = x^3(3 - x)$$

ina $x = \frac{9}{4}$.

Me parahau ko te pūwāhi huringa he mōrahi paetata.

ASSESSOR'S USE ONLY

(e) Use calculus to prove that the graph of the function

$y = x^3(3 - x)$

has a local maximum when $x = \frac{9}{4}$.

Justify that the turning point is a local maximum.

TŪMAHI TUATORU

MĀ TE KAIMĀKA ANAKE

(a) Ka tohua te kauwhata rōnaki o te pānga $f(x)$ mā te

$$f'(x) = 6x^2 - 2x + 4$$

E takoto ana te pūwāhi (1,3) ki te kauwhata.

Whiriwhiria te whārite mō te pānga $f(x)$.

QUESTION THREE

ASSESSOR'S USE ONLY

(a) The gradient graph of a function $f(x)$ is given by

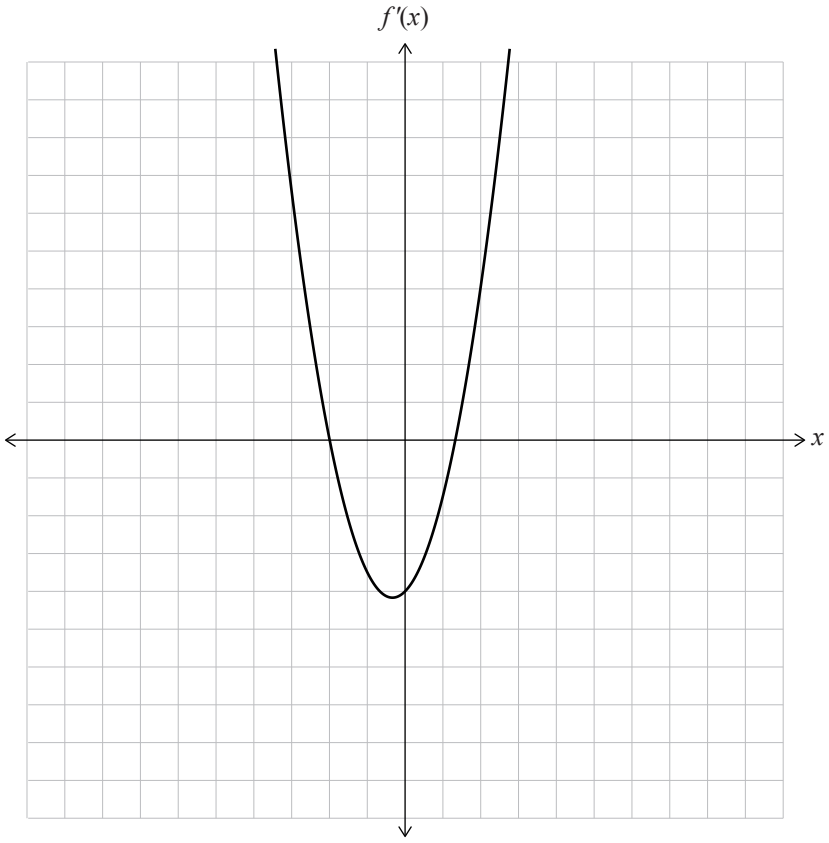
$$f'(x) = 6x^2 - 2x + 4$$

The point (1,3) lies on the graph.

Find the equation of the function $f(x)$.

(b) E whakaatu ana te hoahoa i raro i te kauwhata o tētahi pānga rōnaki $y = f'(x)$.

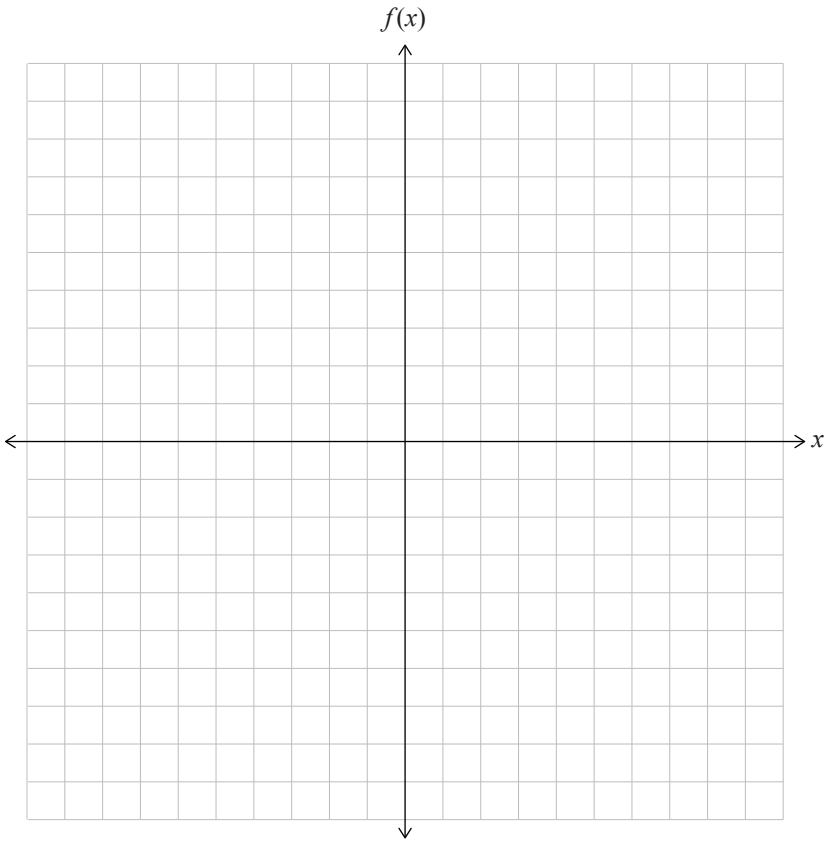
MĀ TE KAIMĀKA ANAKE



E takoto ana te pūwāhi (0,0) ki te kauwhata o te pānga $y = f(x)$.

Ki ngā tuaka o raro, tātuhia te pānga $f(x)$.

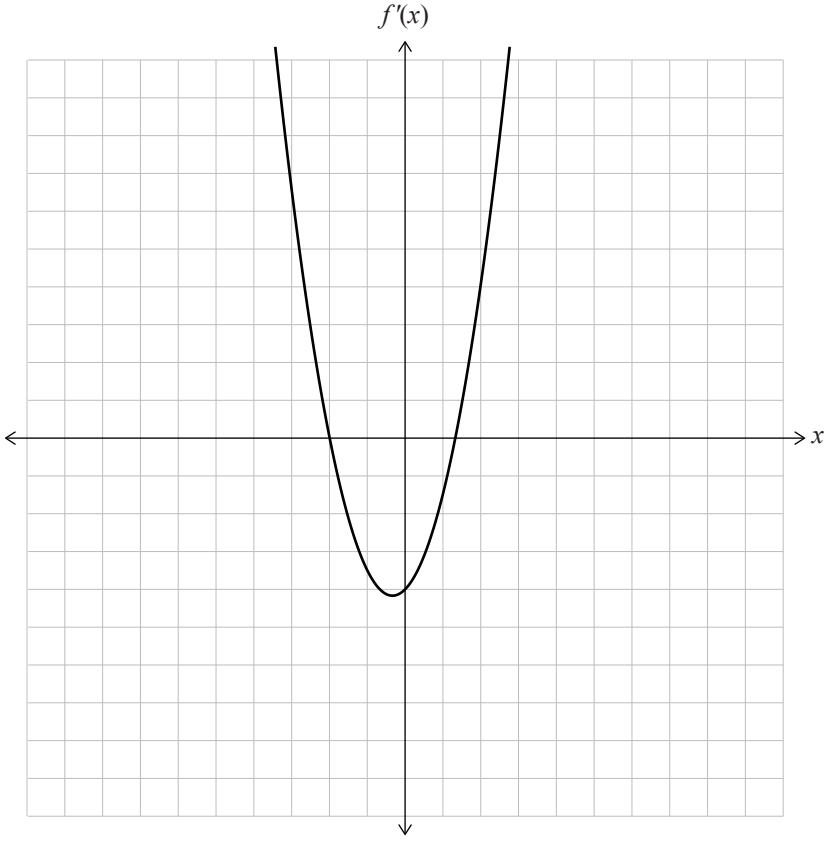
He ōrite te āwhata o ngā huinga tuaka e rua.



Ki te hiahia koe ki te tuhi anō i tēnei kauwhata, whakamahia te tukutuku i te whārangi 20.

ASSESSOR'S USE ONLY

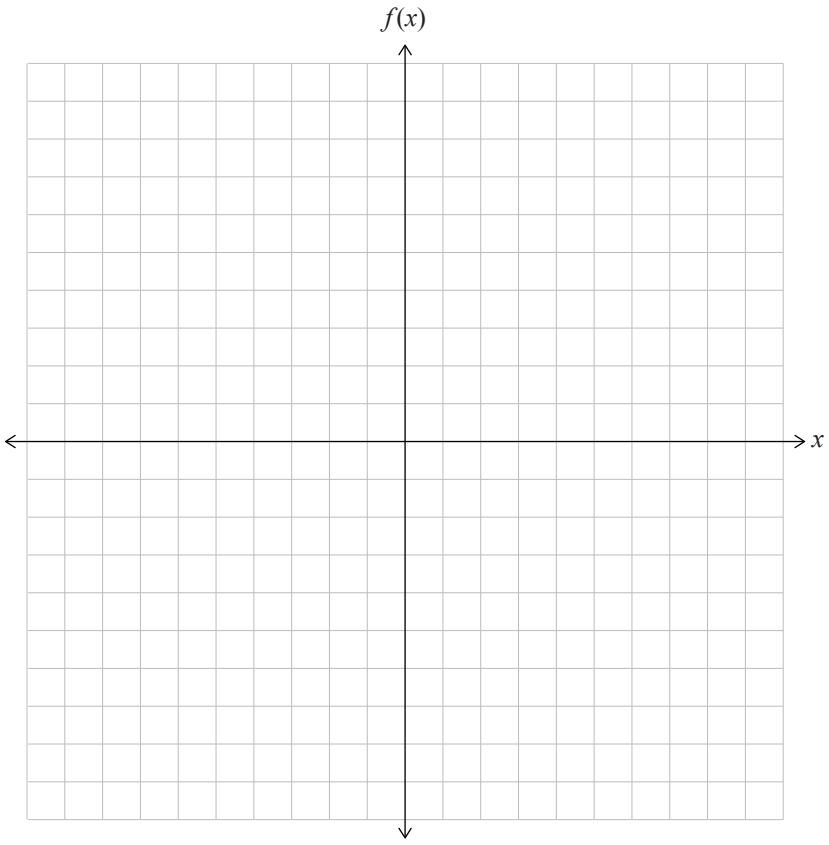
(b) The diagram below shows the graph of a gradient function $y = f'(x)$.



The point (0,0) is on the graph of the function $y = f(x)$.

On the axes below sketch the function $f(x)$.

Both sets of axes have the same scale.



If you need to redraw this graph, use the grid on page 21.

MĀ TE KAIMĀKA ANAKE

(c) Ka taea tētahi ahanoa te neke ki ngā ahunga e rua ki tētahi ara tōtika, ā, he whakaterenga aumou tōna o te -4 cm s^{-2}.

Kua tohua tētahi pūwāhi P pūmau ki te ara.

Ina tīmata te hopu i te nekehanga o te ahanoa, ko te ahanoa:

- he 12 cm mai i P
- kei te neke atu i P, ā,
- he 6 cm s^{-1} te tere.

(i) Mā te whakamahi tuanaki, whiriwhiria te tere o te ahanoa i te 5 hēkona mai i te tīmatanga o te hopu i te nekehanga.

(ii) He aha te tawhiti mōrahi o te ahanoa mai i te pūwāhi P?

Me parahau koinei te tawhiti mōrahi.

Ka haere tonu te Tūmahi Tuatoru i te whārangi 18.

ASSESSOR'S USE ONLY

(c) An object can move in either direction on a straight track and has a constant acceleration of -4 cm s^{-2}.

A fixed point P is marked on the track.

When a recording of the object's motion begins, the object:

- is 12 cm from P
- is moving away from P, and
- has a velocity of 6 cm s^{-1}.

(i) Using calculus, find the speed of the object 5 seconds after its motion began being recorded.

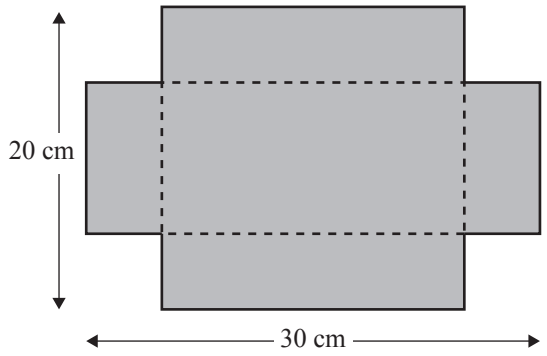
(ii) What is the maximum distance of the object from the point P?

Justify that this is the maximum distance.

Question Three continues on page 19.

(d) Whiriwhiria te rōrahi mōrahi o tētahi pouaka tuwhera (arā, he pouaka whai tūāpapa me ngā taha engari kāore he taupoki) ka taea te mahi mai i tētahi pepamārō tapawhā hāngai e 20 cm mā te 30 cm, mā te tango i ngā tapawhā rite kokonga me te tākai ki ngā rārangi iraira.

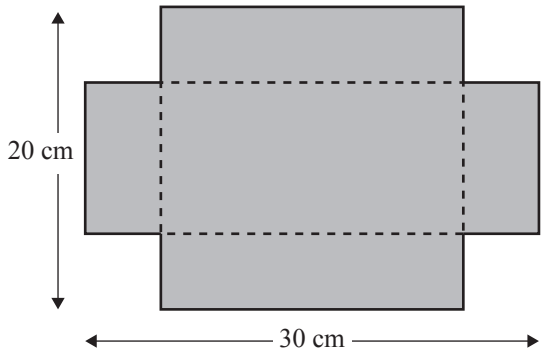
Parahau koinei te rōrahi mōrahi.



MĀ TE KAIMĀKA ANAKE

(d) Find the maximum volume of an open box (i.e. a box with a base and sides, but no lid) that can be made from a rectangular piece of cardboard measuring 20 cm by 30 cm, by removing the corner squares and folding along the dotted lines.

Justify that this is the maximum volume.

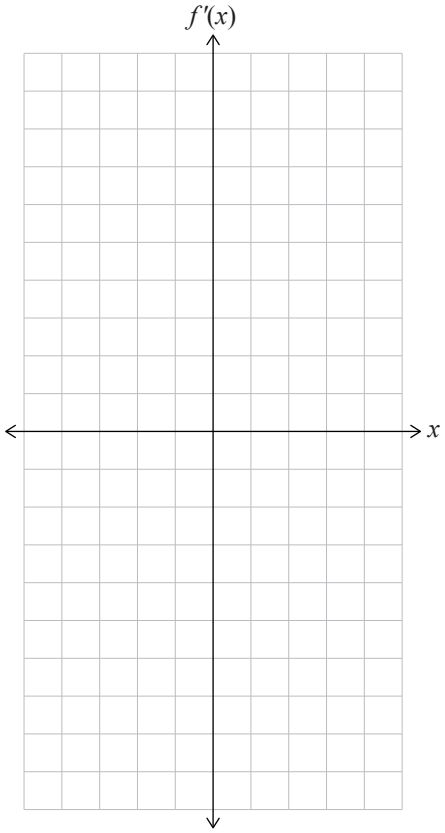


ASSESSOR'S USE ONLY

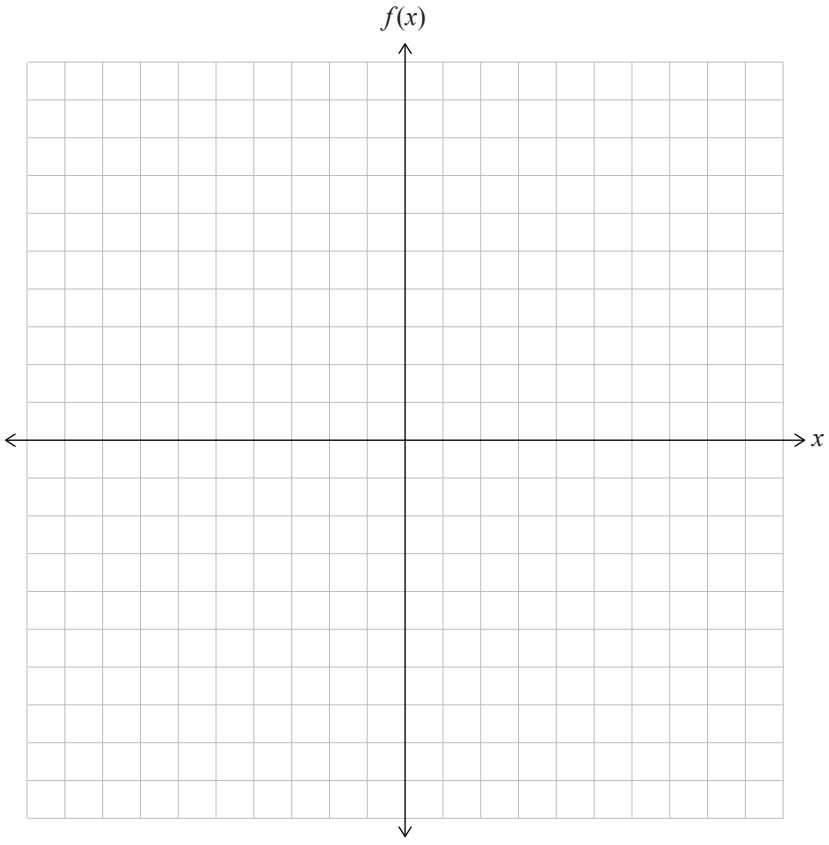
NGĀ TUKUTUKU TĀPIRI

MĀ TE KAIMĀKA ANAKE

Ki te hiahia koe ki te tuhi anō i tō kauwhata mai i te Tūmahi Tuarua (a), tuhia ki te tukutuku i raro.
Kia mārama te tohu ko tēhea te tuhinga ka hiahia koe kia mākahia.



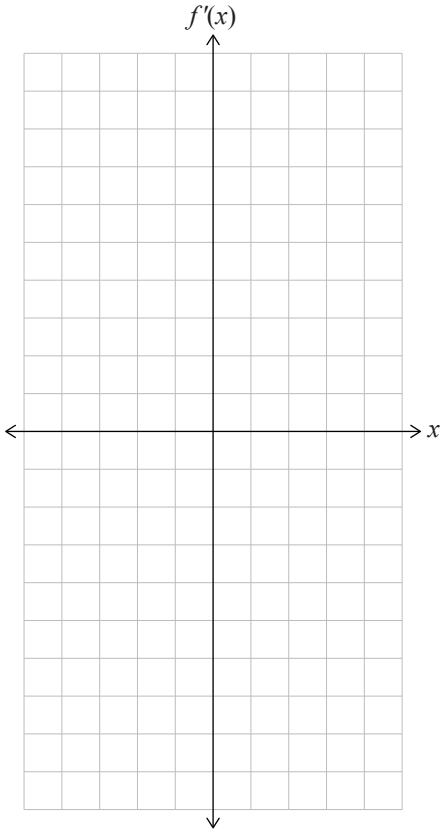
Ki te hiahia koe ki te tuhi anō i tō kauwhata mai i te Tūmahi Tuatoru (b), tuhia ki te tukutuku o raro.
Kia mārama te tohu ko tēhea te tuhinga ka hiahia koe kia mākahia.



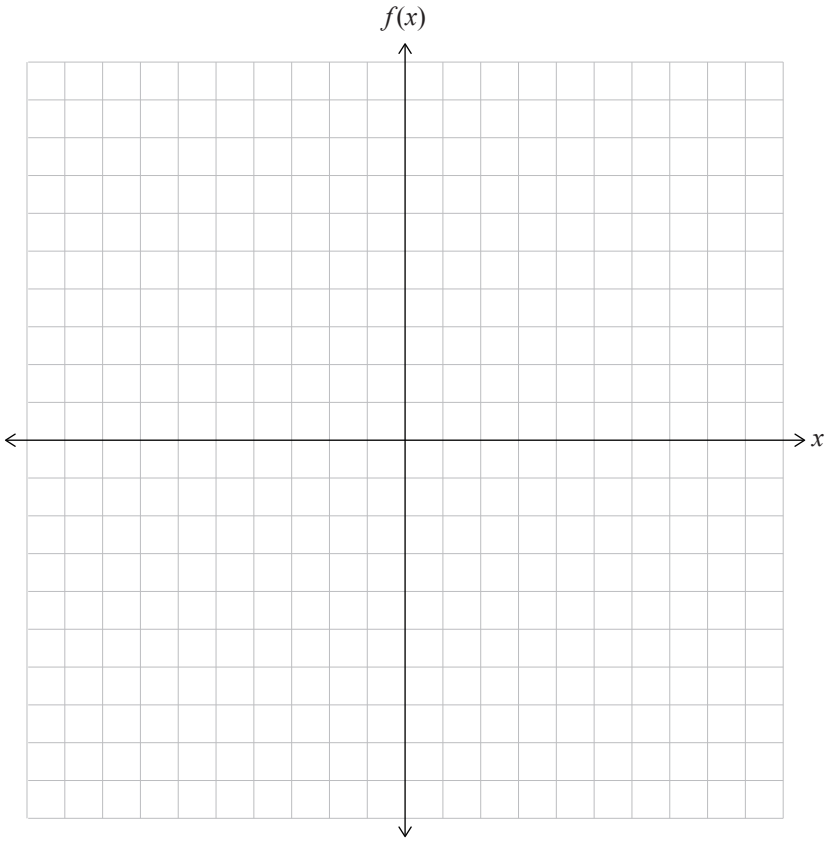
SPARE GRIDS

ASSESSOR'S USE ONLY

If you need to redraw your graph from Question Two (a), draw it on the grid below. Make sure it is clear which answer you want marked.



If you need to redraw your graph from Question Three (b), draw it on the grid below. Make sure it is clear which answer you want marked.



He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

English translation of the wording on the front cover

Level 2 Mathematics and Statistics, 2017

91262 Apply calculus methods in solving problems

2.00 p.m. Friday 24 November 2017
Credits: Five

91262M

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You must show the use of calculus in answering all questions in this paper.

Check that this booklet has pages 2–23 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.