91262M

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NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

2

SUPERVISOR'S USE ONLY

Te Pāngarau me te Tauanga, Kaupae 2, 2018

91262M Te whakahāngai tikanga tuanaki hei whakaoti rapanga

9.30 i te ata Rāapa 14 Whiringa-ā-rangi 2018
Whiwhinga: Rima

Paetae	Kaiaka	Kairangi
Te whakahāngai tikanga tuanaki hei whakaoti rapanga.	Te whakahāngai tikanga tuanaki mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai tikanga tuanaki mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tirohia mēnā kei a koe te Puka Tikanga Tātai L2–MATHMF.

Whakaaturia ngā mahinga KATOA.

Mēnā ka hiahia whārangi atu anō koe mō ō tuhinga, whakamahia te (ngā) whārangi wātea kei muri o tēnei pukapuka, ka āta tohu ai i te tau tūmahi.

Me mātua whakaatu e koe te whakamahi tuanaki i ō tuhinga mō ngā tūmahi katoa i tēnei pepa.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–27 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATAHI

MĀ TE KAIMĀKA ANAKE

(a) Ka tohua he pānga f mā te $f(x) = x^3 - 6x + 2$.

Whiriwhiria te rōnaki o te kauwhata o te pānga kei te pūwāhi $x = 4$.

(b) Kei te whakaroha te horahanga o tētahi tapawhā hāngai kia toru whakareanga ake te roa i te whānui i ngā wā katoa.

Whiriwhiria te pāpātanga o te huri o te horahanga o te tapawhā hāngai e ai ki te whānui ina ko te horahanga o te tapawhā hāngai he 75 cm^2.

QUESTION ONE

ASSESSOR'S USE ONLY

(a) A function f is given by $f(x) = x^3 - 6x + 2$.

Find the gradient of the graph of the function at the point where $x = 4$.

(b) A rectangle is expanding in area so that at all times its length is three times its width.

Find the rate of change of the area of the rectangle with respect to its width when the area of the rectangle is 75 cm^2.

(c) Ko te pārōnaki o tētahi pānga f ko te $f'(x) = -3x^2 + 12x$.
Ko te kauwhata o te pānga he mōkito paetata i te pūwāhi (0,5).

Whakamahia te tuanaki hei whiriwhiri i te uara o te mōrahi paetata o te pānga.

MĀ TE KAIMĀKA ANAKE

(d) Whakamahia te tuanaki hei whiriwhiri i ngā uara o x e nui haere ake te kauwhata o te pānga $f(x) = \frac{2}{3}x^3 + \frac{9}{2}x^2 - 5x - 18$.

ASSESSOR'S USE ONLY

(c) The derivative of a function f is given by $f'(x) = -3x^2 + 12x$.
The graph of the function has a local minimum at the point (0,5).

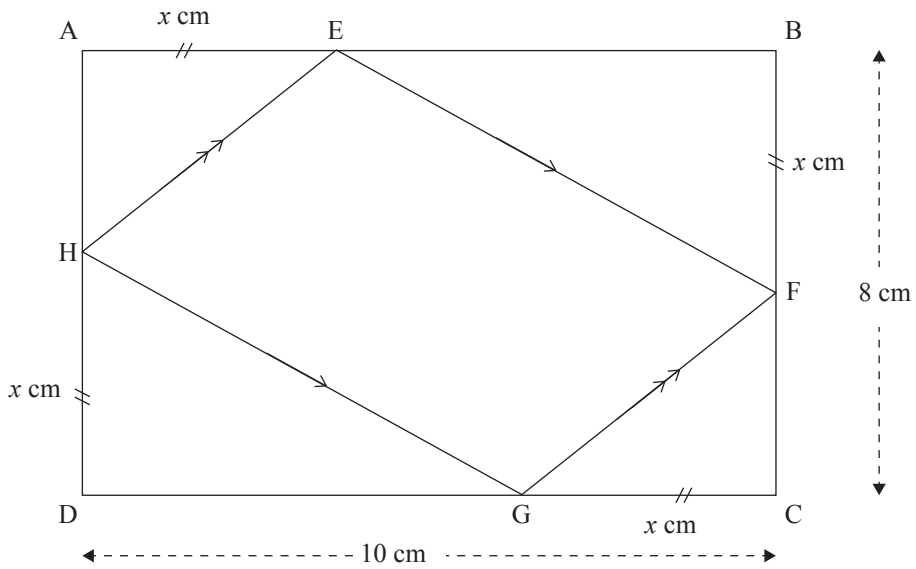
Use calculus to find the value of the local maximum of the function.

(d) Use calculus to find the values of x for which the graph of the function $f(x) = \frac{2}{3}x^3 + \frac{9}{2}x^2 - 5x - 18$ is increasing.

(e) He 10 cm mā te 8 cm te inenga o te tapawhā hāngai ABCD. Ka taea te tapawhā whakarara EFGH te tātuhi i roto i te tapawhā hāngai, e ai ki te hoahoa i raro.

Me kī ko te tawhiti mai i ia kokonga o te tapawhā hāngai ki te akitu whai ake o te tapawhā whakarara, i te ahunga whakatekaraka, he x cm.

Arā, AE = BF = CG = DH = x.



Whakamahia te tuanaki hei whiriwhiri i te horahanga iti rawa ka taea mō te tapawhā whakarara.

Parahautia he mōkito tō whakautu.

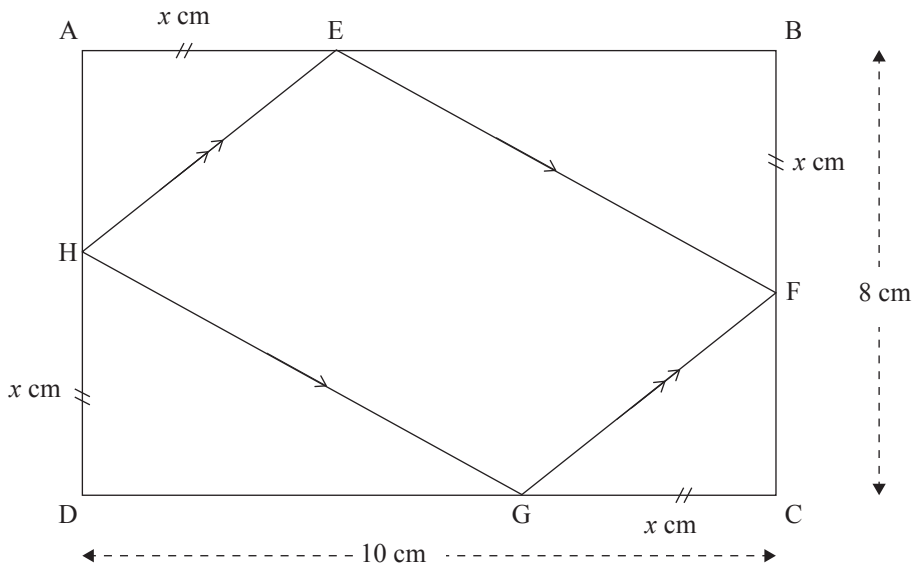
MĀ TE KAIMĀKA ANAKE

ASSESSOR'S USE ONLY

(e) A rectangle ABCD measures 10 cm by 8 cm. A parallelogram EFGH can be drawn inside the rectangle, as shown in the diagram below.

Suppose that the distance from each corner of the rectangle to the next vertex of the parallelogram, in a clockwise direction, is x cm.

That is, AE = BF = CG = DH = x.



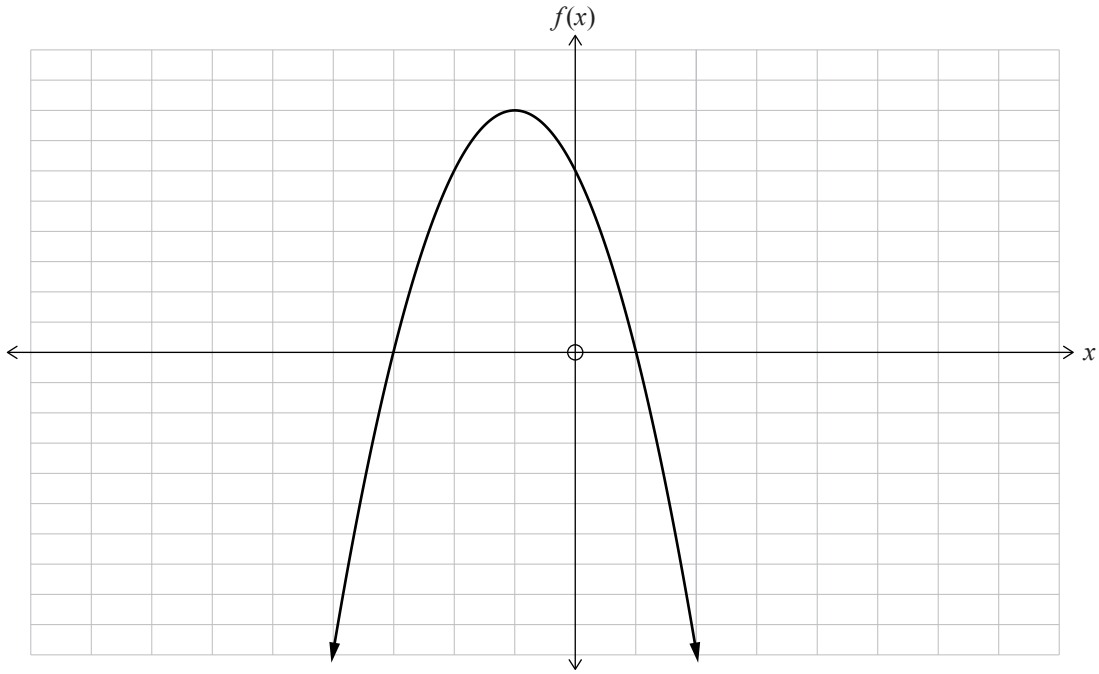
Use calculus to find the smallest possible area that the parallelogram can have.

Justify that your answer is a minimum.

TŪMAHI TUARUA

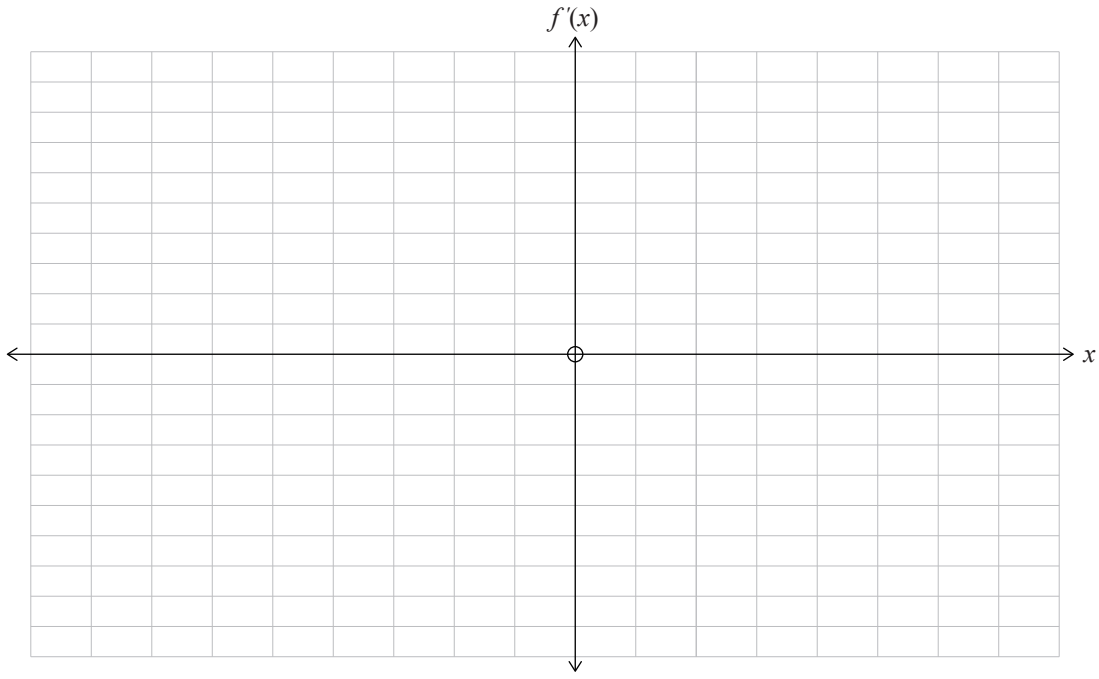
MĀ TE KAIMĀKA ANAKE

(a) E whakaatuhia ana te kauwhata o te pānga $y = f(x)$ ki ngā tuaka i raro nei.



Tātuhia te kauwhata o te pānga rōnaki $y = f'(x)$ ki ngā tuaka o raro.

He ōrite te āwhata o ngā huinga tuaka e rua.

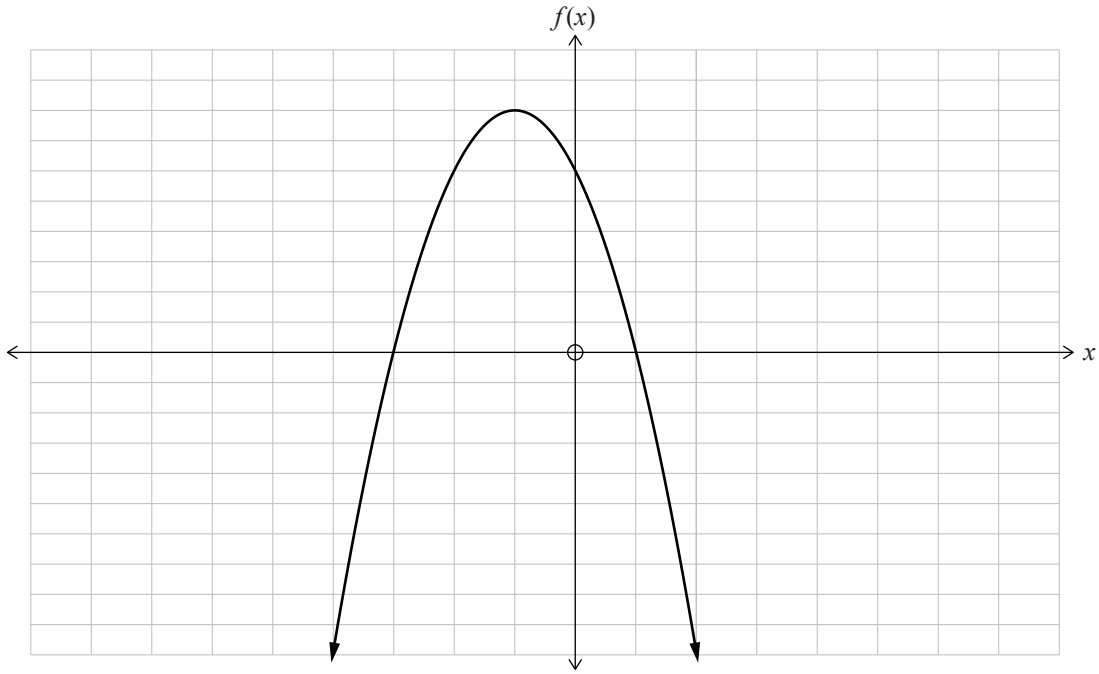


Ki te hiahia koe ki te tuhi anō i tēnei mahinga, whakamahia ngā tukutuku i te whārangi 22.

QUESTION TWO

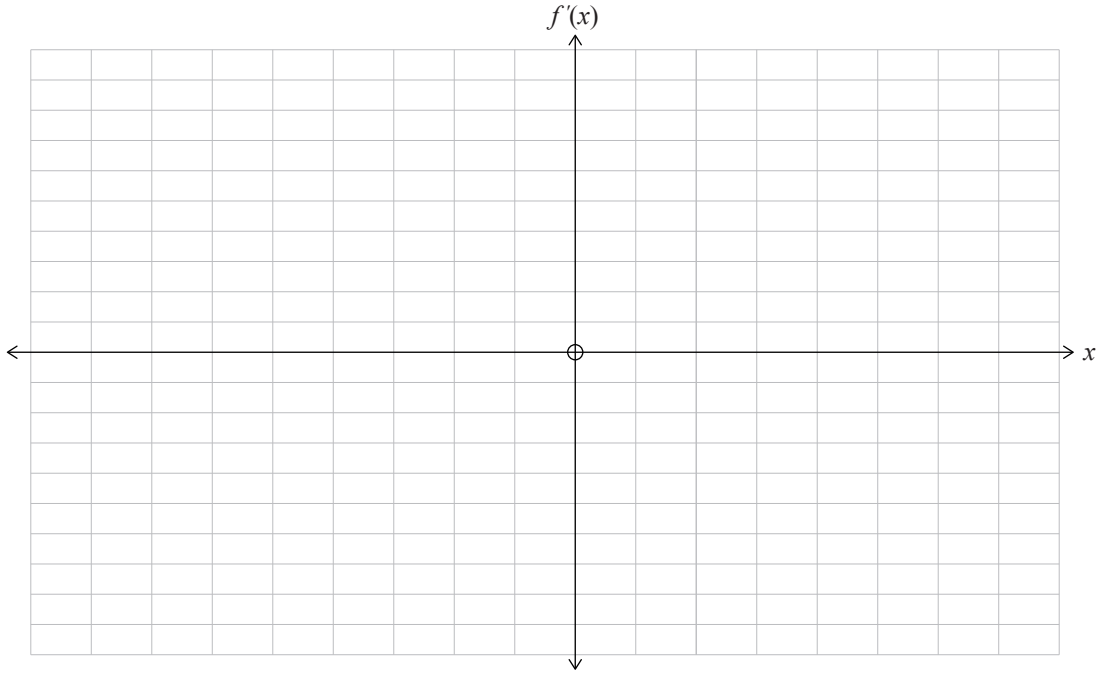
ASSESSOR'S USE ONLY

(a) The graph of a function $y = f(x)$ is shown on the axes below.



Sketch the graph of the gradient function $y = f'(x)$ on the axes below.

Both sets of axes have the same scale.



If you need to redo this question part, use the grids on page 23.

MĀ TE KAIMĀKA ANAKE

(b) Ka whakawhanatia tētahi tākirirangi ki te takiwā, ā, ko te t hēkona mai i te whakarewatanga, te teitei, h mita, i runga ake i te papa ko te

$$h(t) = 39.2t - 4.9t^2.$$

He aha te teitei mōrahi ka eke te tākirirangi?

(c) Kei te whakahaere a Adam i tana matatopa. E torotika ana te rere, ā, i te t hēkona i muri i te hipanga i tētahi rākau ko te whakaterenga a m s^{-2} , ko te

$$a(t) = 6 - 12t.$$

E rua hēkona i muri i te hipanga o te matatopa i te rākau, he 20 m s^{-1} te tere.

E hia te tawhiti o te matatopa mai i te rākau i te ekenga o te tere ki te 20 m s^{-1} ?

ASSESSOR'S USE ONLY

(b) A skyrocket is projected into the air so that t seconds after it is launched, its height, h metres, above the ground is given by

$$h(t) = 39.2t - 4.9t^2.$$

What is the maximum height that the skyrocket will reach?

(c) Adam is operating his drone. It is moving in a straight line and t seconds after passing a tree its acceleration, a m s^{-2} , is given by

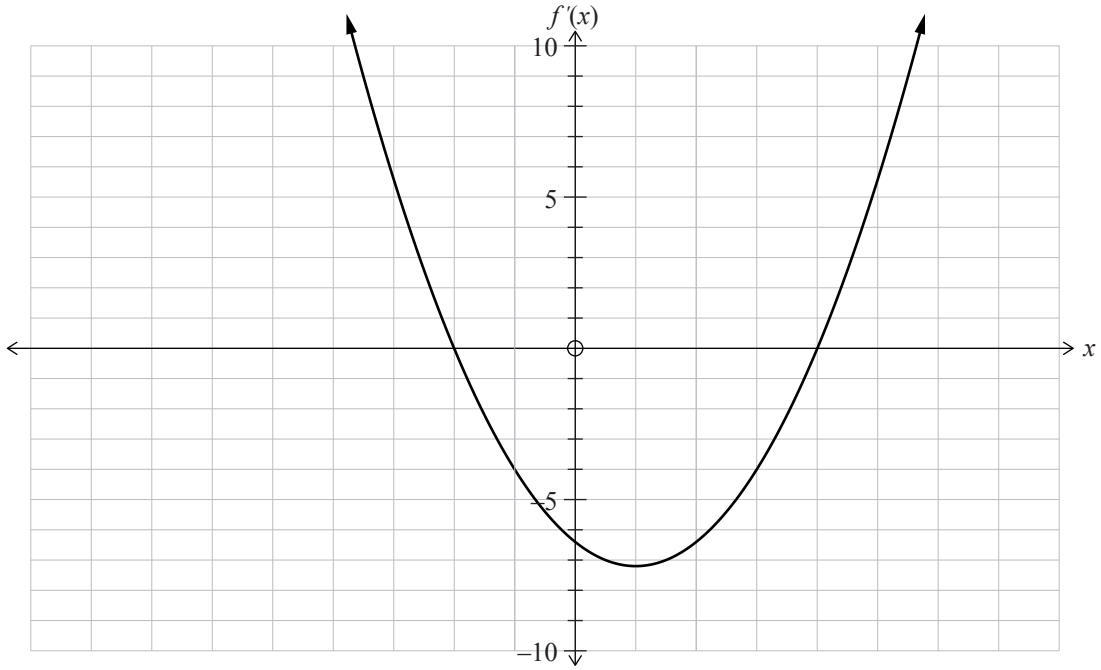
$$a(t) = 6 - 12t.$$

Two seconds after the drone passed the tree, its velocity was 20 m s^{-1}.

How far was the drone from the tree when its velocity was 20 m s^{-1} ?

MĀ TE KAIMĀKA ANAKE

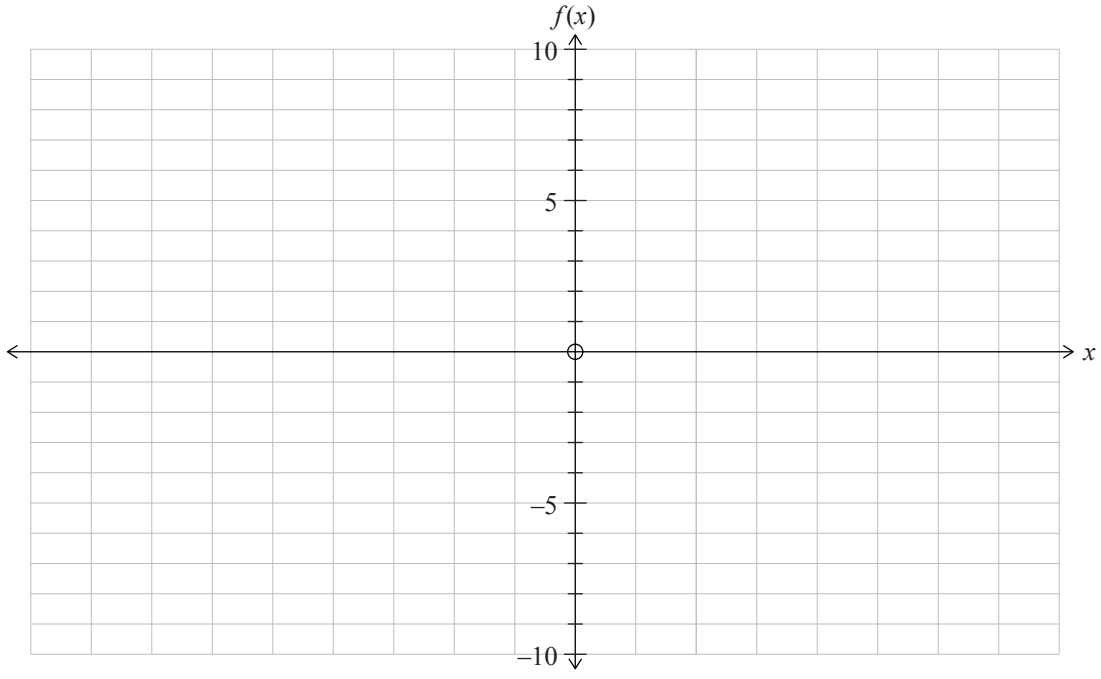
(d) E whakaatu ana te hoahoa o raro i te kauwhata o te pānga rōnaki $y = f'(x)$ mō te pānga $y = f(x)$.



Ka hipa te kauwhata o te pānga $y = f(x)$ i te (0,3).

Ki ngā tuaka o raro, tātuhia te kauwhata o te pānga f.

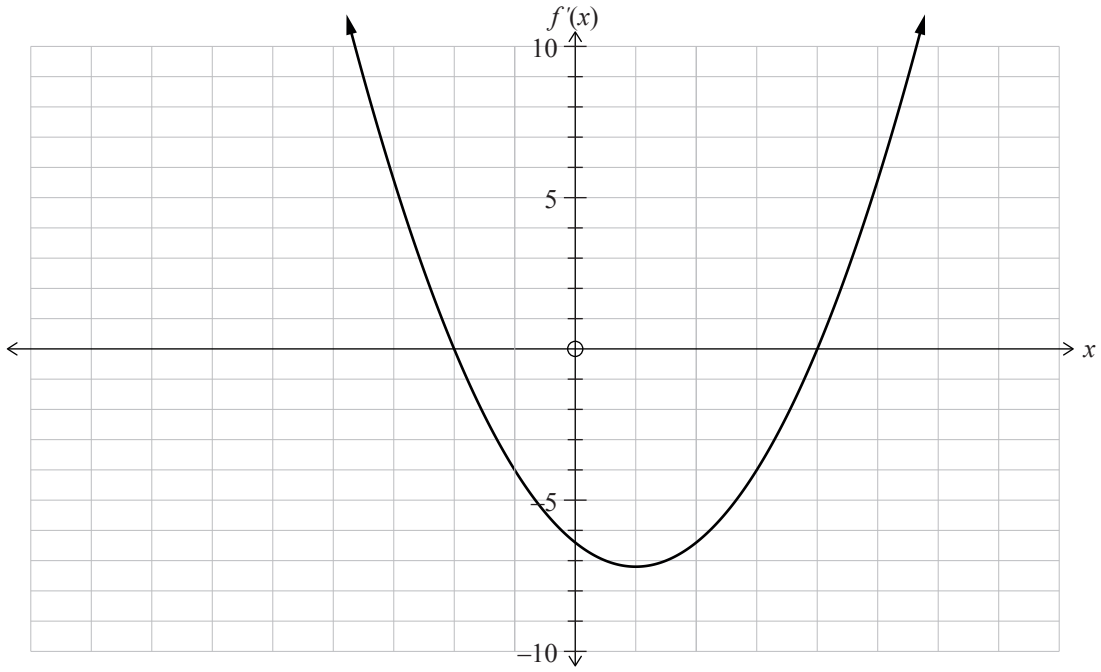
He ōrite te āwhata o ngā huinga tuaka e rua.



Ki te hiahia koe ki te tuhi anō i tēnei mahinga, whakamahia ngā tukutuku i te whārangi 24.

ASSESSOR'S USE ONLY

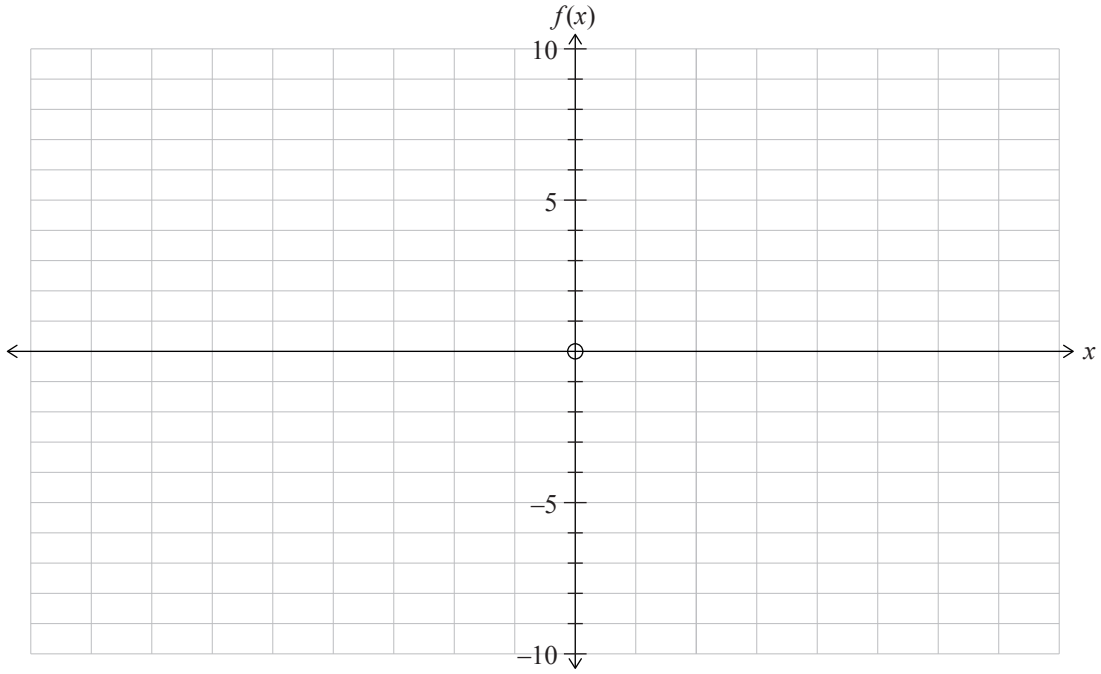
(d) The diagram below shows the graph of the gradient function $y = f'(x)$ of a function $y = f(x)$.



The graph of the function $y = f(x)$ passes through (0,3).

On the axes below sketch the graph of the function f.

Both sets of axes have the same scale.



If you need to redo this question part, use the grids on page 25.

(e) He pūwāhi huringa tō te kauwhata o te pānga $y = x^3 - 6x^2 + kx - 5$ i $x = 3$.

MĀ TE KAIMĀKA ANAKE

Whakamahia ngā tikanga tuanaki hei whiriwhiri i ngā taunga o ngā pūwāhi huringa e rua. Whakatauhia te āhua o ia pūwāhi huringa me te parahau i tō tuhinga.

(e) The graph of the function $y = x^3 - 6x^2 + kx - 5$ has a turning point at $x = 3$.

Use calculus methods to find the coordinates of both turning points.

Determine the nature of each turning point, justifying your answer.

ASSESSOR'S USE ONLY

TŪMAHI TUATORU

MĀ TE KAIMĀKA ANAKE

(a) Ko te pānga rōnaki o tētahi ānau, ko te $\frac{dy}{dx} = -5x^4 + 6$.

Ka hipa te ānau i te pūwāhi (1,7).

Whiriwhiria te whārite mō y.

(b) Me kī, i te tīmatanga o tētahi rā, 1000 ngā tāngata i te tauhokohoko i tētahi mākete, ā, i te t rā mai i te tīmatanga o taua rā, ka taea te maha o ngā kaihokohoko N, te whakatauira mā te

$$N(t) = 1000 + 400t + 100t^2.$$

E hia ngā rā e eke ai te pāpātanga o te panoni o te maha o ngā kaihokohoko ki te 14 400 i te rā?

QUESTION THREE

ASSESSOR'S USE ONLY

(a) The gradient function of a curve is given by $\frac{dy}{dx} = -5x^4 + 6$.

The curve passes through (1,7).

Find the equation for y.

(b) Suppose that, at the start of a particular day, 1000 people were trading in a market, and that t days after the start of that day, the number of traders, N, can be modelled by

$$N(t) = 1000 + 400t + 100t^2.$$

How many days will it take for the rate of change of the number of traders to be 14 400 per day?

(c) Kei te hokona e tētahi kura ana tīkiti mō tana whakaari.

Ko ngā moni whiwhi, $\$R$, mai i te hoko tīkiti o te $\$p$ mō ia tīkiti, ka taea te whakatauira mā te pānga

$$R(p) = 40p\,(29 - 2p)$$

Whakamahia te tuanaki hei whiriwhiri i te moni whiwhi mōrahi ka taea (mā te whakamahi i tēnei tauira).

MĀ TE KAIMĀKA ANAKE

(c) A school is selling tickets for its drama production.

The revenue, $\$R$, from selling tickets for a price of $\$p$ each, can be modelled by the function

$$R(p) = 40p(29 - 2p)$$

Use calculus to find the maximum possible revenue (using this model).

ASSESSOR'S USE ONLY

(d) He pātapa ki te kauwhata o te pānga $y = -\frac{1}{3}x^3 + \mathrm{k}x + 4$

i tētahi pūwāhi P, he –7 te rōnaki, ā, ka haukoti anō i te kauwhata i (–6,64).

Whakamahia te tuanaki hei whiriwhiri i ngā taunga o te pūwāhi P.

MĀ TE KAIMĀKA ANAKE

(d) A tangent to the graph of the function $y = -\frac{1}{3}x^3 + kx + 4$ at a certain point P, has gradient of –7 and intersects the graph again at (–6,64).

Use calculus to find the co-ordinates of the point P.

ASSESSOR'S USE ONLY

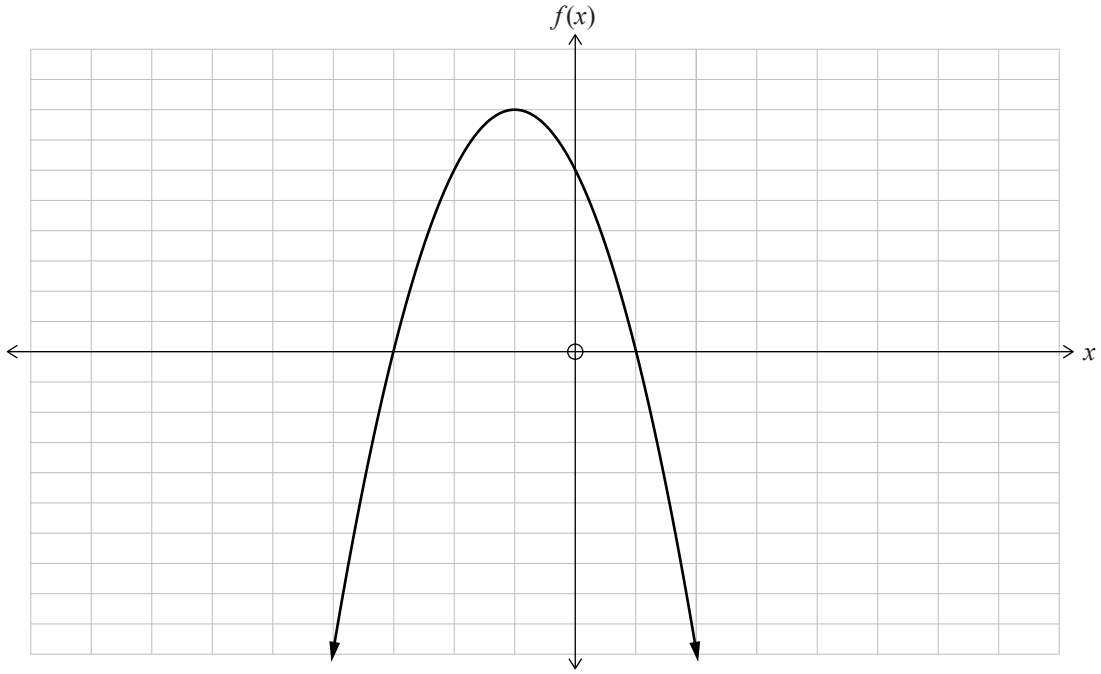
NGĀ TUKUTUKU TĀPIRI

MĀ TE KAIMĀKA ANAKE

Ki te hiahia koe ki te tātuhi anō i tō urupare ki te Tūmahi Tuarua (a), whakamahia te tukutuku i raro nei. Kia mārama te tohu ko tēhea te tuhinga ka hiahia koe kia mākahia.

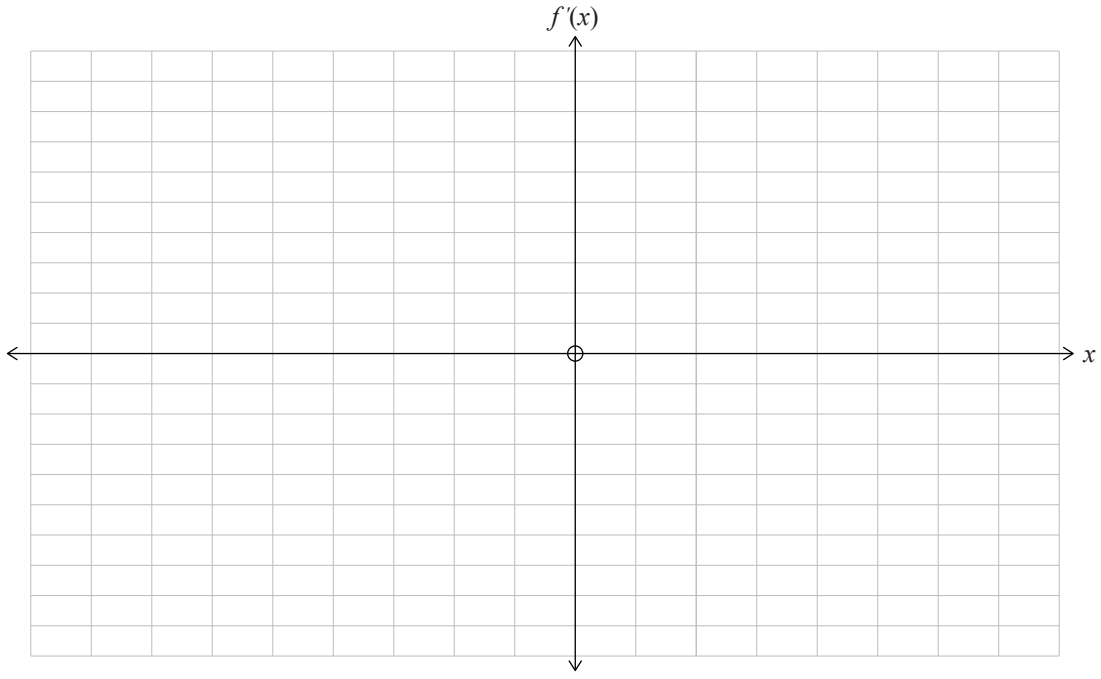
TŪMAHI TUARUA

(a) E whakaatuhia ana te kauwhata o te pānga $y = f(x)$ ki ngā tuaka i raro nei.



Tātuhia te kauwhata o te pānga rōnaki $y = f'(x)$ ki ngā tuaka o raro.

He ōrite te āwhata o ngā huinga tuaka e rua.



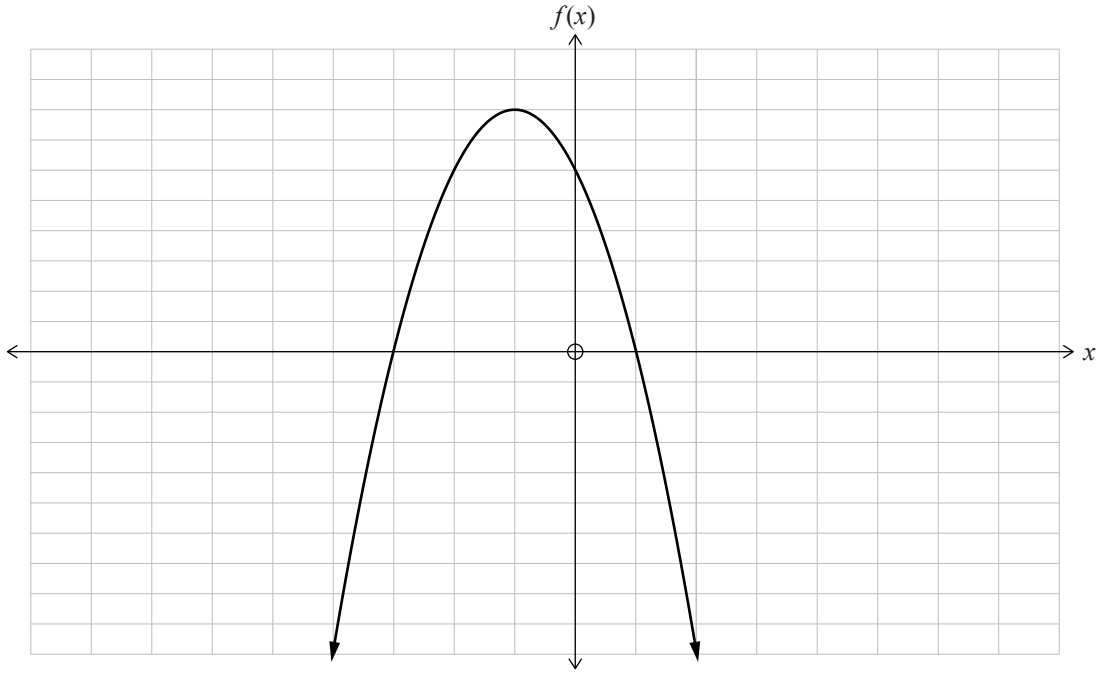
ASSESSOR'S USE ONLY

SPARE GRIDS

If you need to redo Question Two (a), use the grid below. Make sure it is clear which answer you want marked.

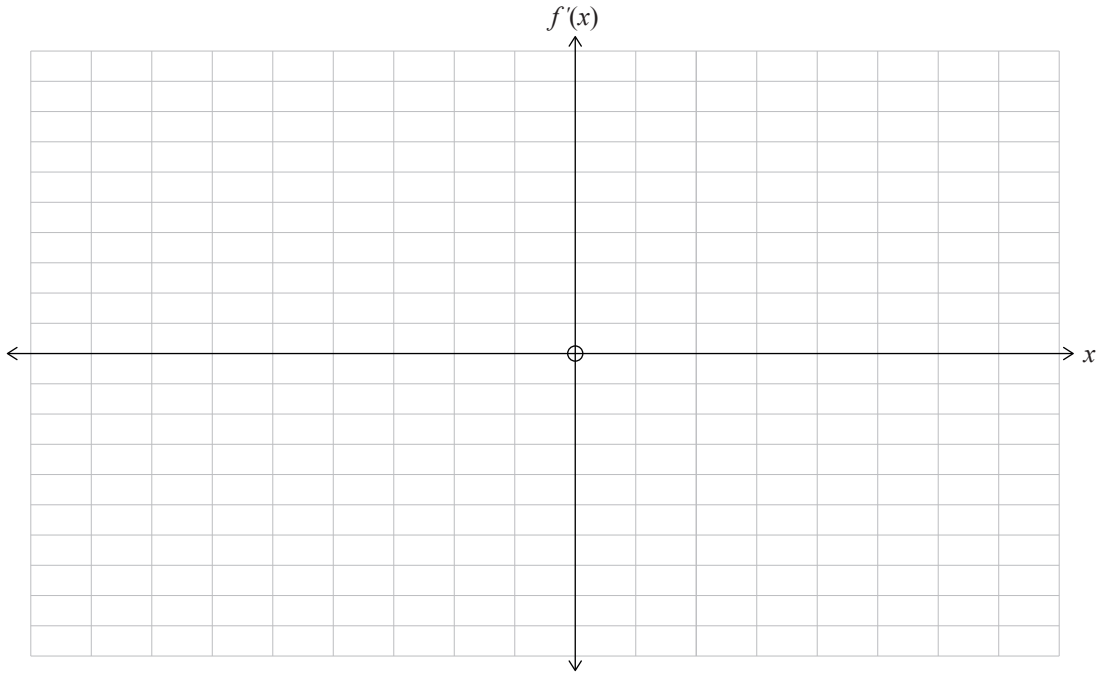
QUESTION TWO

(a) The graph of a function $y = f(x)$ is shown on the axes below.



Sketch the graph of the gradient function $y = f'(x)$ on the axes below.

Both sets of axes have the same scale.

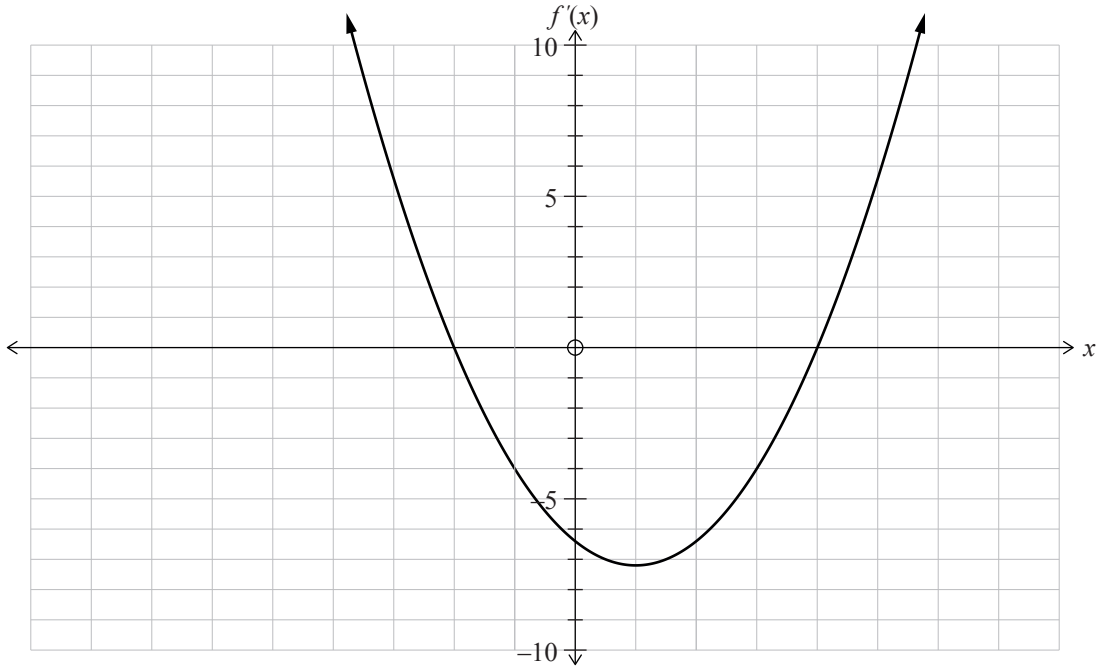


Ki te hiahia koe ki te tātuhi anō i tō urupare ki te Tūmahi Tuarua (d), whakamahia te tukutuku i raro nei. Kia mārama te tohu ko tēhea te tuhinga ka hiahia koe kia mākahia.

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUARUA

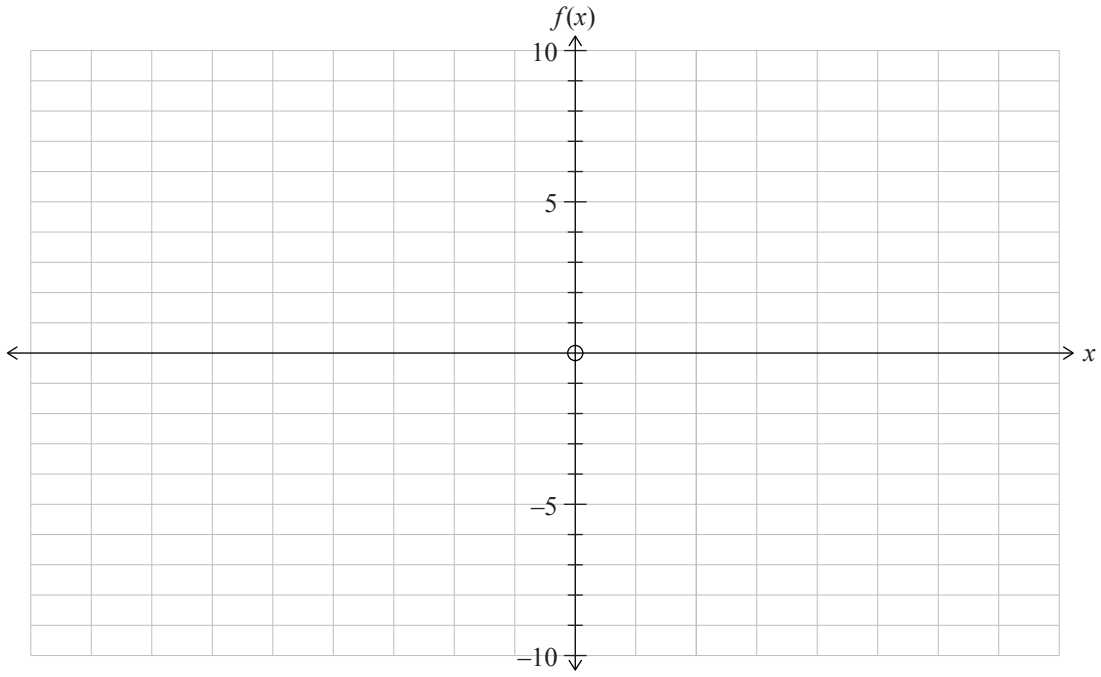
(d) E whakaatu ana te hoahoa o raro i te kauwhata o te pānga rōnaki $y = f'(x)$ mō te pānga $y = f(x)$.



Ka hipa te kauwhata o te pānga $y = f(x)$ i te (0,3).

Ki ngā tuaka o raro, tātuhia te kauwhata o te pānga f.

He ōrite te āwhata o ngā huinga tuaka e rua.

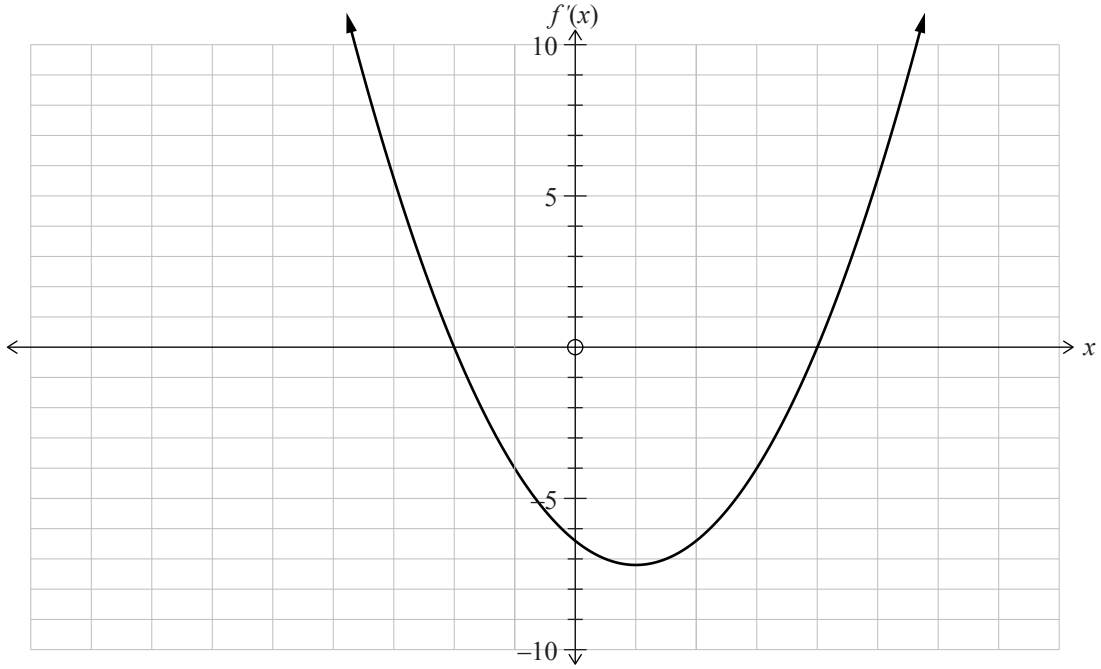


If you need to redo Question Two (d), use the grid below. Make sure it is clear which answer you want marked.

ASSESSOR'S USE ONLY

QUESTION TWO

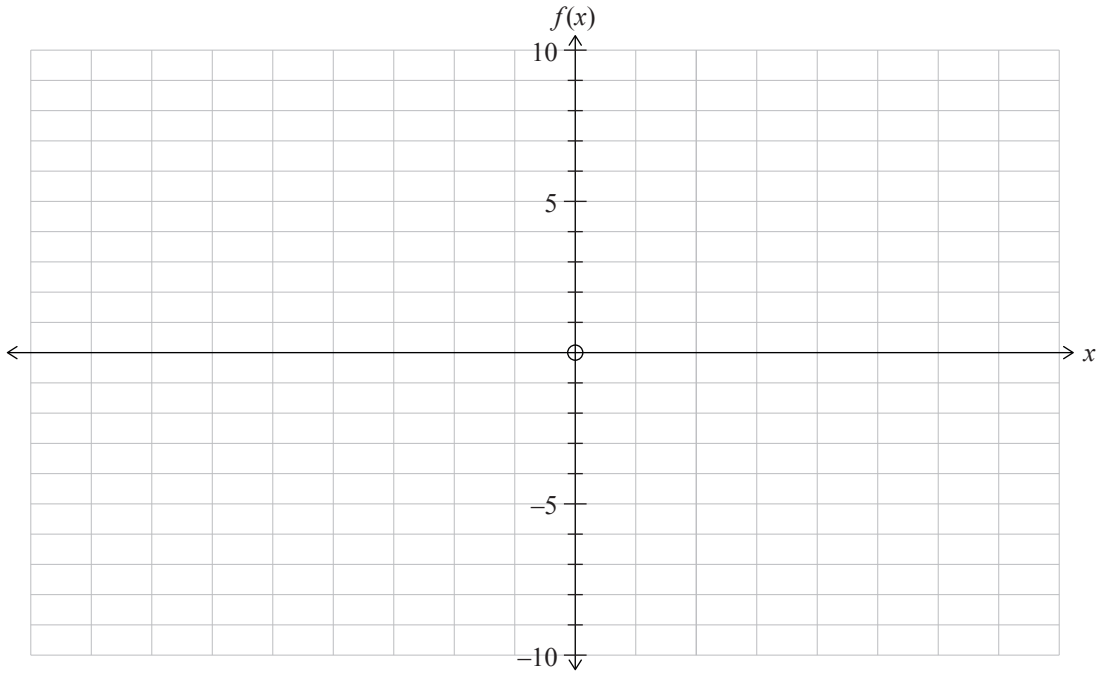
(d) The diagram below shows the graph of the gradient function $y = f'(x)$ of a function $y = f(x)$.



The graph of the function $y = f(x)$ passes through (0,3).

On the axes below sketch the graph of the function f.

Both sets of axes have the same scale.



He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

English translation of the wording on the front cover

Level 2 Mathematics and Statistics, 2018

91262 Apply calculus methods in solving problems

9.30 a.m. Wednesday 14 November 2018
Credits: Five

91262M

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You must show the use of calculus in answering all questions in this paper.

Check that this booklet has pages 2–28 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.