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SUPERVISOR'S USE ONLY

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Level 2 Mathematics and Statistics, 2018

91262 Apply calculus methods in solving problems

9.30 a.m. Wednesday 14 November 2018
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You must show the use of calculus in answering all questions in this paper.

Check that this booklet has pages 2–15 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Achievement

TOTAL 14

ASSESSOR'S USE ONLY

QUESTION ONE

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(a) A function f is given by $f(x) = x^3 - 6x + 2$.

Find the gradient of the graph of the function at the point where $x = 4$.

$f'(x) = 2x - 6$

$= 2(4) - 6$ Gradient $= 2$

$= 2.$

n.

(b) A rectangle is expanding in area so that at all times its length is three times its width.

Find the rate of change of the area of the rectangle with respect to its width when the area of the rectangle is 75 cm^2.

$L = 3w$

$A = Lw$

$= (3w)w$

$= 3w^2$

$6w = A'$

$6w = 75$

$w = 12.5$ Rate of change is 12.5 cm^2

u

(c) The derivative of a function f is given by $f'(x) = -3x^2 + 12x$.

The graph of the function has a local minimum at the point (0,5).

Use calculus to find the value of the local maximum of the function.

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$-x^3 + 6x + c$

$5 = -(0)^3 + 6(0) + c$

$c = 5$ RAWW

$f(x) = -x^3 + 6x + 5$

$f'(x) = -3x^2 + 12x = 0$

$x = 4$

$= 0$

$y = -(4)^3 + 6(4) + 5$

$= -35$

max is -35 //

u

(d) Use calculus to find the values of x for which the graph of the function $f(x) = \frac{2}{3}x^3 + \frac{9}{2}x^2 - 5x - 18$ is increasing.

$f'(x) = 2x^2 + 9x - 5 = 0$

$x = 0.5$

$x = -5$

$\therefore$ when x is > 0.5 it is increasing

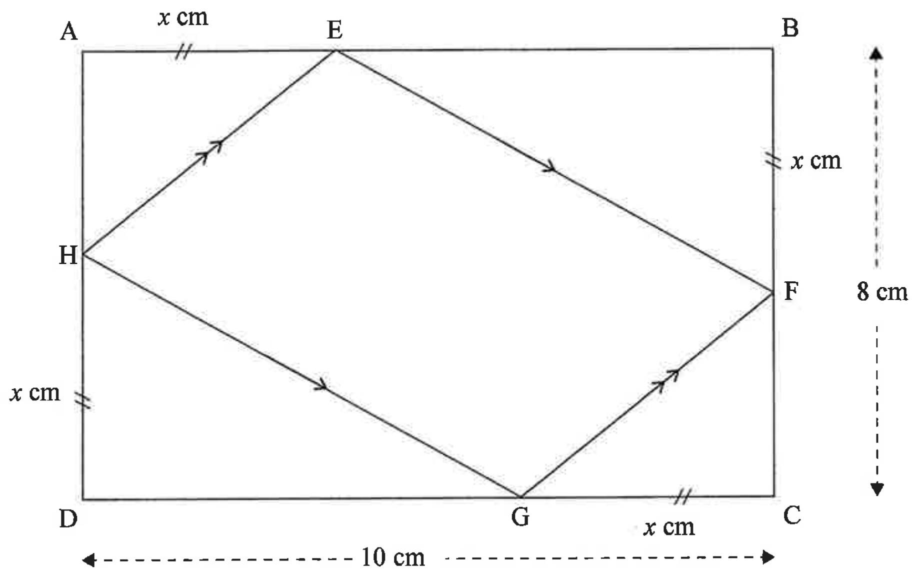
when x is > 5 it is decreasing //

r

(e) A rectangle ABCD measures 10 cm by 8 cm. A parallelogram EFGH can be drawn inside the rectangle, as shown in the diagram below.

Suppose that the distance from each corner of the rectangle to the next vertex of the parallelogram, in a clockwise direction, is x cm.

That is, AE = BF = CG = DH = x.



Use calculus to find the smallest possible area that the parallelogram can have.

Justify that your answer is a minimum.

$ABCD = 8 \times 10 = 80$

$EFGH = 80 - 2x(10-x) - 2x(8-x)$

$= 80 - 20x + 2x^2 - 16x + 2x^2$

$= 80 + 2x^2 - 36x.$

equation $= 2x^2 - 36x + 80$

minimum $= f'(x) = y' = 0$

$f(x) = 2x^2 - 36x + 80$

$f'(x) = 4x - 36$

$4x - 36 = 0$

$x = 9\text{cm}$ //

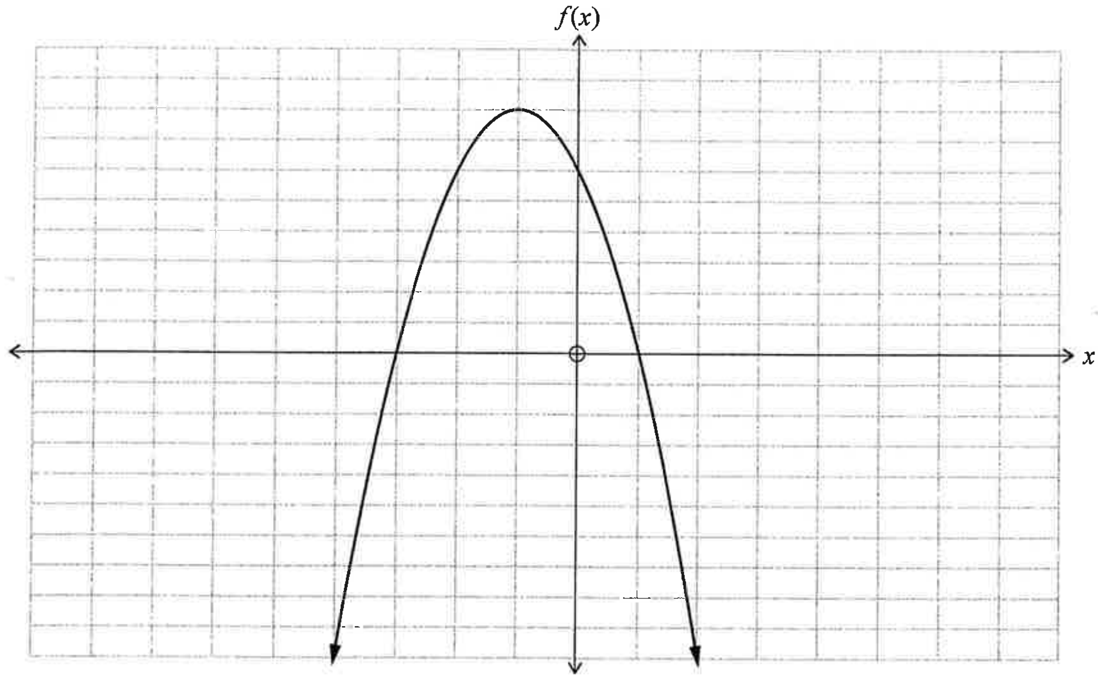
ASSESSOR'S USE ONLY

M5

ASSESSOR'S USE ONLY

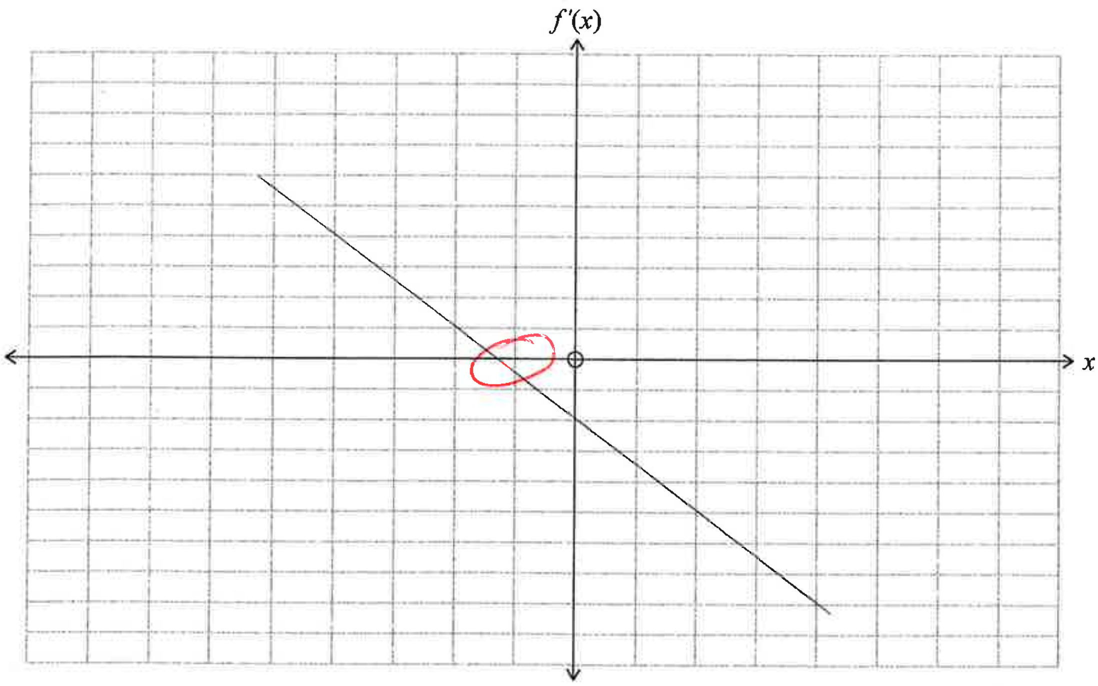
QUESTION TWO

(a) The graph of a function $y = f(x)$ is shown on the axes below.



Sketch the graph of the gradient function $y = f'(x)$ on the axes below.

Both sets of axes have the same scale.



n

If you need to redo this question part, use the grids on page 12.

ASSESSOR'S USE ONLY

(b) A skyrocket is projected into the air so that t seconds after it is launched, its height, h metres, above the ground is given by

$$h(t) = 39.2t - 4.9t^2.$$

What is the maximum height that the skyrocket will reach?

$h(t) = 39.2t - 4.9t^2$

$= t(39.2 - 4.9t)$

$0 = t(39.2 - 4.9t)$ A

$t = 0$ or $t = 8$

Max at $t = 4$

RAWW

$h(4) = 39.2 \times 4 - 4.9(4)^2 = 78.2\text{ m}$ max height

n

(c) Adam is operating his drone. It is moving in a straight line and t seconds after passing a tree its acceleration, a m s^{-2}, is given by

$$a(t) = 6 - 12t.$$

Two seconds after the drone passed the tree, its velocity was 20 m s^{-1}.

How far was the drone from the tree when its velocity was 20 m s^{-1}?

$a(t) = -12t + 6$

$\int a(t) = -6t^2 + 6t + c$

$\int a = v \quad \therefore \quad 20 = -6(2)^2 + 6(2) + c \quad \therefore c = 32$

$v = -6t^2 + 6t + 32$

$20 = -6t^2 + 6t + 32$

$0 = -6t^2 + 6t + 12$

$t = 2$ or $t = -1$ so t must be 2

$s' = v \quad \therefore \int v(t) = -2t^3 + 3t^2 + 32t + c$

$c = 0$ when $v = 0$ and $t = 0$

$s = 2t^3 + 3t^2 + 32t$ A

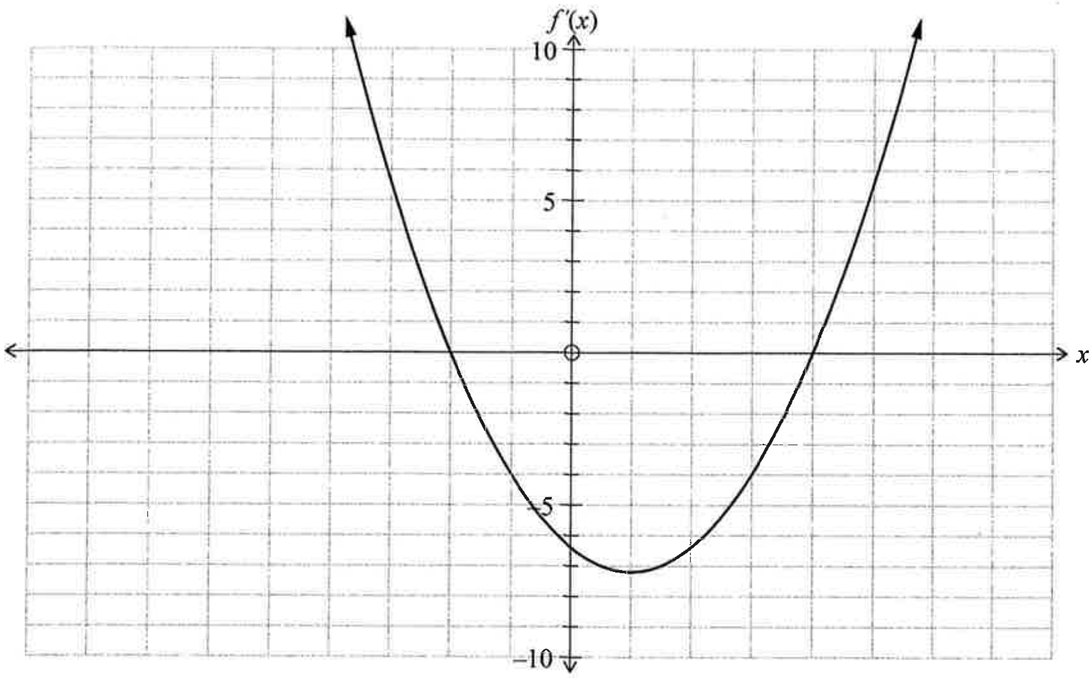
$= 2(2)^3 + 3(2)^2 + 32(2)$ A

$s = 92$ A

r

ASSESSOR'S USE ONLY

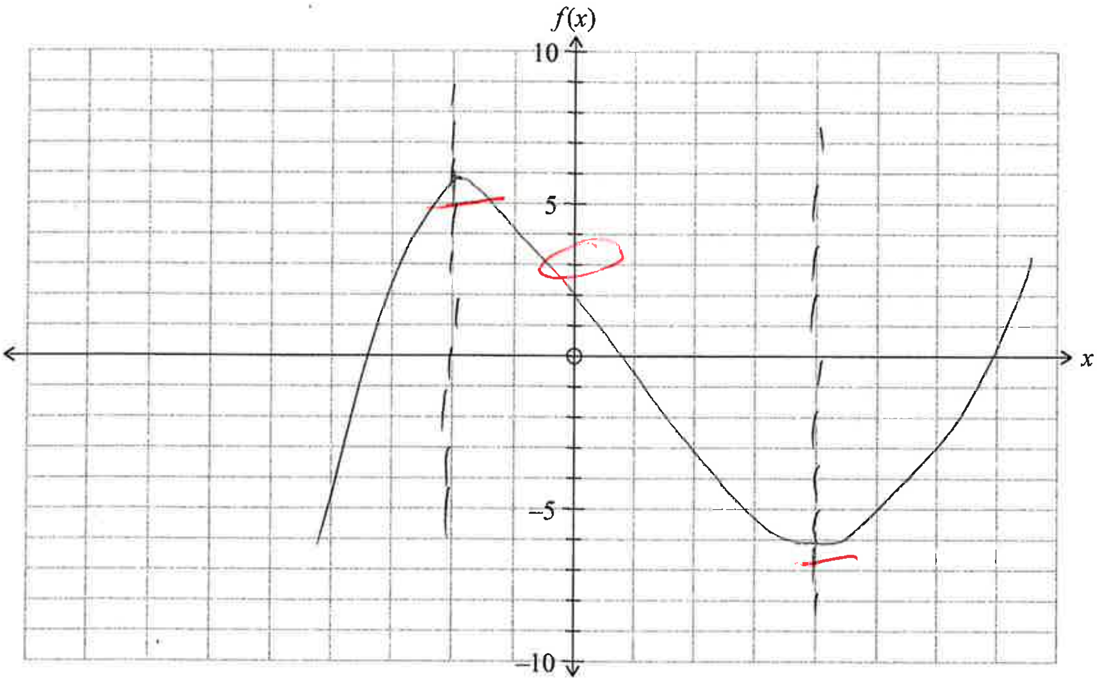
(d) The diagram below shows the graph of the gradient function $y = f'(x)$ of a function $y = f(x)$.



The graph of the function $y = f(x)$ passes through (0,3).

On the axes below sketch the graph of the function f.

Both sets of axes have the same scale.



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If you need to redo this question part, use the grids on page 13.

(e) The graph of the function $y = x^3 - 6x^2 + kx - 5$ has a turning point at $x = 3$.

Use calculus methods to find the coordinates of both turning points.

Determine the nature of each turning point, justifying your answer.

ASSESSOR'S USE ONLY

$f(x) = x^3 - 6x^2 + kx - 5$

$f'(x) = 3x^2 - 12x + k$

At a turning point gradient is 0

$0 = 3x^2 - 12x + k$

$0 = 3(3)^2 - 12(3) + k$

$k = 9$

u

$f(x) = x^3 - 6x^2 + 9x - 5$

$f(3) = 3^3 - 6(3)^2 + 9(3) - 5$

$= 27 - 54 + 27 - 5$

$= -5$ A

Point is $(3, -5)$ A

M5

QUESTION THREE

ASSESSOR'S USE ONLY

(a) The gradient function of a curve is given by $\frac{dy}{dx} = -5x^4 + 6$.

The curve passes through (1,7).

Find the equation for y.

$y = -x^5 + 6x + c$

$7 = -x^5 + 6x + c$

$7 = -1 + 6 + c$

$c = 2$

$y = x^5 + 6x + 2$

T

u.

(b) Suppose that, at the start of a particular day, 1000 people were trading in a market, and that t days after the start of that day, the number of traders, N, can be modelled by

$N(t) = 1000 + 400t + 100t^2$.

How many days will it take for the rate of change of the number of traders to be 14 400 per day?

$N'(t) = 200t + 400$

$14400 = 200t + 400$

$14000 = 200t$

$t = 70$

u.

Question Three continues on the following page.

ASSESSOR'S USE ONLY

(c) A school is selling tickets for its drama production.

The revenue, $\$R$, from selling tickets for a price of $\$p$ each, can be modelled by the function

$$R(p) = 40p(29 - 2p)$$

Use calculus to find the maximum possible revenue (using this model).

$R(p) = 1160p - 80p^2$

$R'(p) = -160p + 1160 = 0$

$p = 7.25$

$R(7.25) = 1160 \times 7.25 - 80 \times 7.25$

$= 7830$

So the max possible revenue is \$7830

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u.

(d) A tangent to the graph of the function $y = -\frac{1}{3}x^3 + kx + 4$ at a certain point P, has gradient of -7 and intersects the graph again at $(-6,64)$.

Use calculus to find the co-ordinates of the point P.

ASSESSOR'S USE ONLY

$\frac{dy}{dx} = -x^2 + k$

$= -x^2 + k = -7$

when $x = -6$

Λ $36 + k = -7$

$k = -43$

$\frac{dy}{dx} = -x^2 - 43$

$y - 64 = -7(x+6)$

$y - 64 = -7x - 42$

$y = -7x + 22$ //

u.

A4

Achievement Exemplar 2018

Subject	Level 2 Mathematics and Statistics	**Standard**	91262	**Total score**	14

Q	**Grade score**	**Annotation**
1	M5	This is M5 because the x interval boundaries had been determined in (d) but only one interval was correctly stated. Further evidence to gain M6 could have been obtained in (c) had the derivative been found correctly then consistently applied to find the local maximum.
2	M5	This is M5 because in (c) the parameters given were correctly applied to determine the distance equation. To have gained a higher grade (E7) a correct answer in context was required. If the candidate had applied calculus correctly in (b), or drawn the graph in (d) through the given y intercept, or found both x ordinates in (e), then a grade of M6 could have been given.
3	A4	If the candidate had correctly calculated the revenue in (c) or correctly calculated k in (d) a grade of M5 or M6 could have been given.