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2

91262M

912625

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

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KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

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Tohua tēnei pouaka mēnā KĀORE koe i tuhituhi i roto i tēnei pukapuka

Te Pāngarau me te Tauanga, Kaupae 2, 2021

91262M Te whakamahi tikanga tuanaki hei whakaoti rapanga

Ngā whiwhinga: Rima

Paetae	Kaiaka	Kairangi
Te whakamahi tikanga tuanaki hei whakaoti rapanga.	Te whakamahi tikanga tuanaki mā te whakaaro tūhonohono hei whakaoti rapanga.	Te whakamahi tikanga tuanaki mā te whakaaro waitara hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tirohia mēnā kei a koe te Puka Tikanga Tātai L2–MATHMF.

Tuhia ō mahinga KATOA.

Ki te hiahia koe ki ētahi atu wāhi hei tuhituhi whakautu, whakamahia te wāhi wātea kei muri i te pukapuka nei.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2-27 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi ki roto i tētahi wāhi kauruku whakahāngai ( ). Ka tapahia pea tēnei wāhi ina mākahia te pukapuka.

ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

TŪMAHI TUATAHI

(a) Ka tohua he pānga f mā: $f(x) = 4x^3 - 2x^2 - 7x + 4$

Whakamahia te tuanaki hei whiriwhiri i te rōnaki o te kauwhata o te pānga kei te pūwāhi $x = 3$

(b) Mō te pānga pūtoru $f(x) = \frac{1}{2}x^3 + \frac{1}{2}x$ whiriwhiria te whārite o te pātapa ki te kōpiko i $x = 2$

QUESTION ONE

(a) A function f is given by: $f(x) = 4x^3 - 2x^2 - 7x + 4$

Use calculus to find the gradient of the graph of the function at the point where $x = 3$

(b) For the cubic function $f(x) = \frac{1}{2}x^3 + \frac{1}{2}x$ find the equation of the tangent to the curve at $x = 2$

(c) Ka taea te tokomaha o ngā kaimātaki o ia rā mō tētahi kaipānui hou kei tētahi ratonga roma kēmu ataata te whakatauira mā te whārite e whai ake:

$$V = -11t^2 + 528t \ \{0 \le t \le 48\}$$

E whakaatu ana a V i te tokomaha o ngā kaimātaki o ia rā, ā, e whakaatu ana ko t te wā ā-marama.

(i) Kimihia te (ngā) tere o te huri o ngā kaimātaki o ia rā e ai ki t, ina ko te tokomaha o ngā kaimātaki o ia rā he 3520.

(ii) He aha te tokomaha mōrahi o ngā kaimātaki o ia rā ka whiwhi i tēnei kaipānui hou i ngā marama 48 tuatahi?

(c) The number of daily viewers for a new presenter on a video gaming streaming service can be modelled by the following equation:

$$V = -11t^2 + 528t \; \{0 \leq t \leq 48\}$$

Where V represents the number of daily viewers and t represents time in months.

(i) Find the rate(s) of change of daily viewers, with respect to t, when the number of daily viewers is 3520.

(ii) What is the maximum number of daily viewers that this new presenter gets during the first 48 months?

(iii) Ka taea te tokomaha o ngā kaimātaki o ia rā mō tētahi kaipānui matatau kei tētahi ratonga roma kēmu ataata te whakatauira mā te whārite e whai ake:

$$V = 1.6t^3 - 130t^2 + 2900t \ \{0 \leq t \leq 48\}$$

E whakaatu ana a V i te tokomaha o ngā kaimātaki o ia rā, ā, e whakaatu ana ko t te wā ā-marama.

Ina eke te roma ki te 10 000 o ngā kaimātaki o ia rā, ka whiwhi moni te kaipānui. Mēnā ka heke ngā kaimātaki o ia rā ki raro i te 10 000, kua kore atu ēnei moni whiwhi, ā, kua kore te kaipānui e whiwhi moni.

Whakamahia ngā tikanga tuanaki hei whakatau mēnā ka mutu te whiwhi moni a te roma mā te kaipānui, i muri i te tīmata ki te whiwhi moni.

(iii) The number of daily viewers for an experienced presenter on a video gaming streaming service can be modelled by the following equation:

$$V = 1.6t^3 - 130t^2 + 2900t \ \{0 \leq t \leq 48\}$$

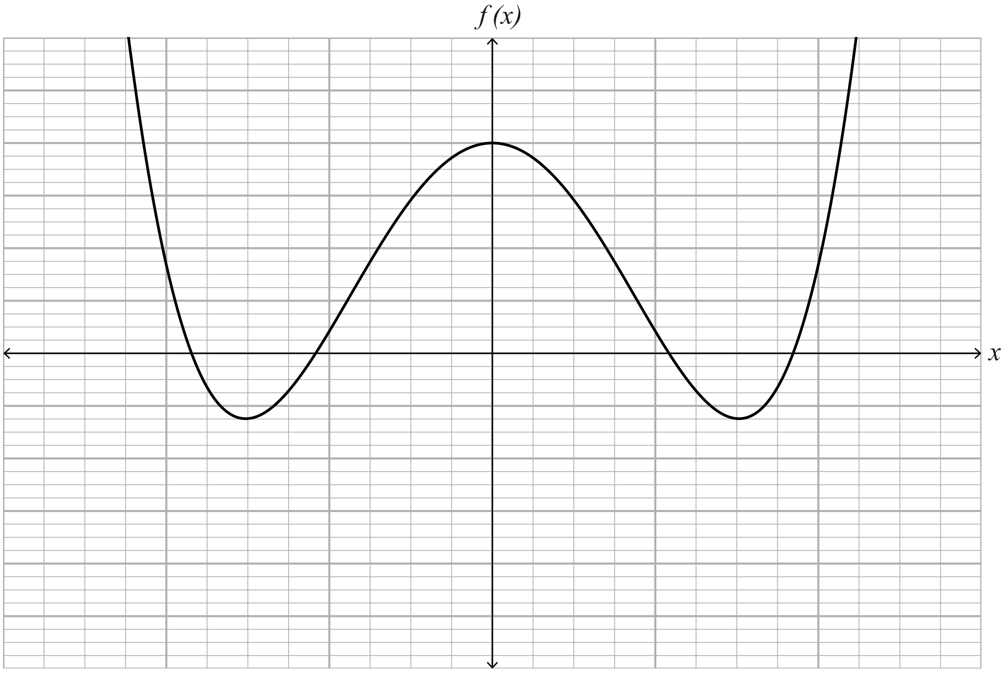
Where V represents the number of daily viewers and t represents time in months.

Once the stream reaches 10 000 daily viewers, it becomes monetised (the presenter earns money). If the daily viewership falls below 10 000, the presenter will lose this income and will no longer make any money.

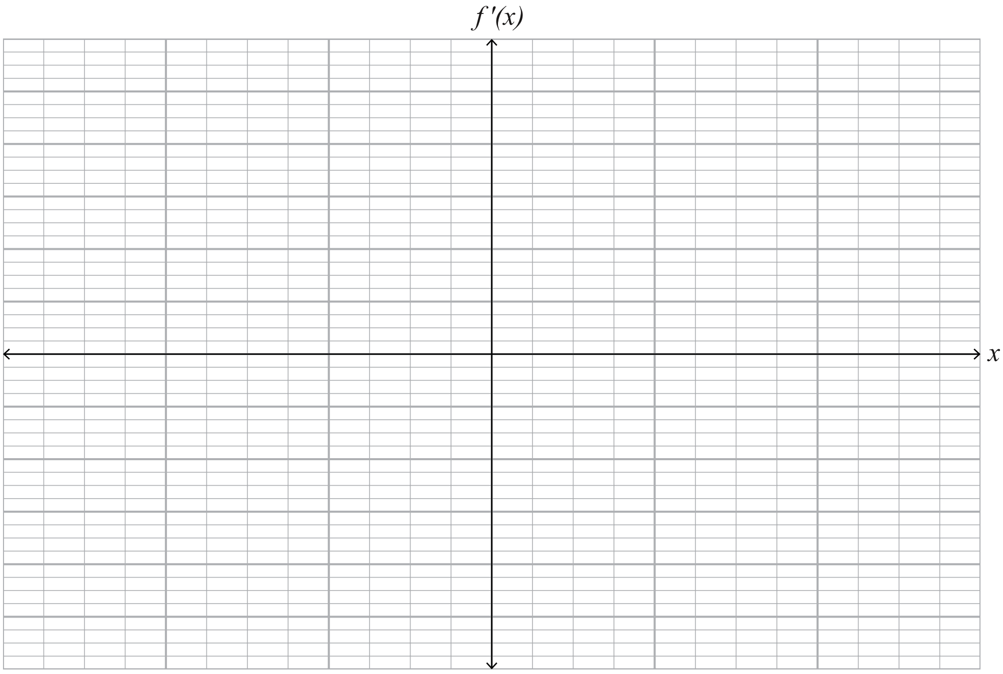
Use calculus methods to determine if the stream, after it becomes monetised, ever stops earning money for the presenter.

TŪMAHI TUARUA

(a) E whakaatuhia ana te kauwhata o te pānga $y = f(x)$ ki ngā tuaka i raro nei.



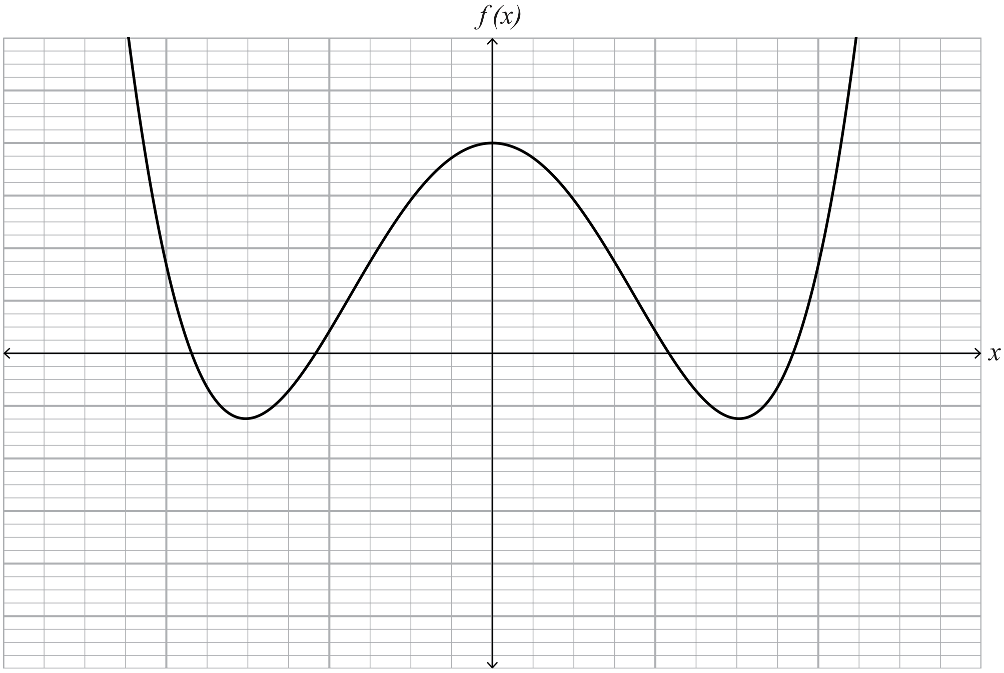
Tuhia te kauwhata o te pānga rōnaki $y = f'(x)$ ki ngā tuaka o raro.
He ōrite te āwhata huapae o ngā huinga tuaka e rua.



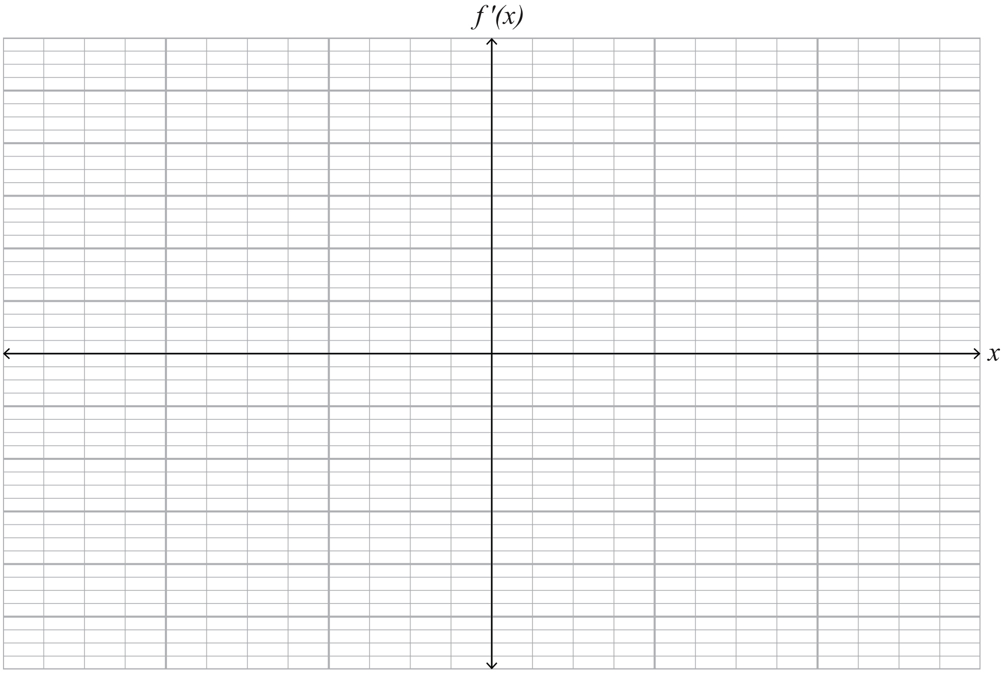
Ki te hiahia koe ki te tuhi anō i tēnei kauwhata, whakamahia te tukutuku i te whārangi 22.

QUESTION TWO

(a) The graph of a function $y = f(x)$ is shown on the axes below.



Sketch the graph of the gradient function $y = f'(x)$ on the axes below.
Both sets of axes have the same horizontal scale.



If you need to redraw this graph, use the grid on page 23.

(b) Ka tohua he pānga f mā: $f(x) = 5 + 3x + cx^2 - 2x^3$

I te pūwāhi kei te kauwhata o te pānga ina ko $x = 2$, ko te rōnaki he -5.

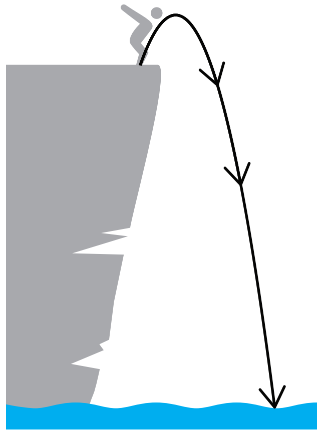
He aha te uara o c ?

(c) (i) Ka peke tētahi kairuku pari ki te takiwā i runga ake o tētahi pari kātahi ka taka atu ki te wai i raro.

He pūmau tana whakahohoro i te -9.8 m s^{-2}.

Ko te tere poutū tīmata o te peke a te kairuku he 2.8 m s^{-1}.

Mā te whakamahi i ngā tikanga tuanaki, whiriwhiria te tere o te kairuku i te kotahi hēkona i muri i tana peketanga.



(ii) Ka tau te kairuku ki te wai i te tere o te –22.68 m s^{-1} (e tohu ana ngā tere tōraro kei te heke whakararo te kairuku).

Tātaihia te teitei mōrahi (i runga ake o te wai) i taea e te kairuku.

Ka hiahia pea koe ki te tīmata mā te whiriwhiri i te teitei o te pari i runga ake o te wai.

(b) The function f is given by: $f(x) = 5 + 3x + cx^2 - 2x^3$

At the point on the graph of the function where $x = 2$, the gradient is -5.

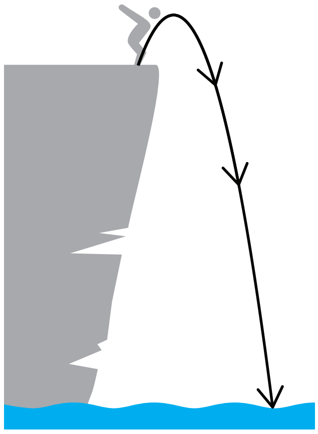
Find the value of c.

(c) (i) A cliff diver jumps up into the air above a cliff and then falls down into the water below.

Their acceleration is constant at -9.8 m s^{-2}.

The diver jumps up with an initial vertical velocity of 2.8 m s^{-1}.

Using calculus methods, find the velocity of the diver one second after they jumped.



(ii) The diver hits the water at a velocity of -22.68 m s^{-1} (negative velocities indicate the diver is moving down).

Find the maximum height (above the water) that the diver reached.

You may wish to begin by finding the height of the cliff above the water.

TŪMAHI TUATORU

(a) Ka pātahi tētahi kōpiko mā te pūwāhi (1 , 2.5), ā, ko tana pānga pāronaki: $f'(x) = 6x^2 + 5x - 1$

Tātaihia ngā taunga o te pūwāhi i te kōpiko $y = f(x)$ ina ko $x = 2$.

(b) Kei te haere tētahi kutarere hiko i tētahi arahīkoi torotika. Ka tohua tana tere t hēkona i muri i te hipa i tētahi tohu mā te $v(t) = 0.3t^2 + 1$ m s^{-1}.

E hia te tawhiti o te kutarere mai i te tohu ina ko $t = 3$ hēkona?

QUESTION THREE

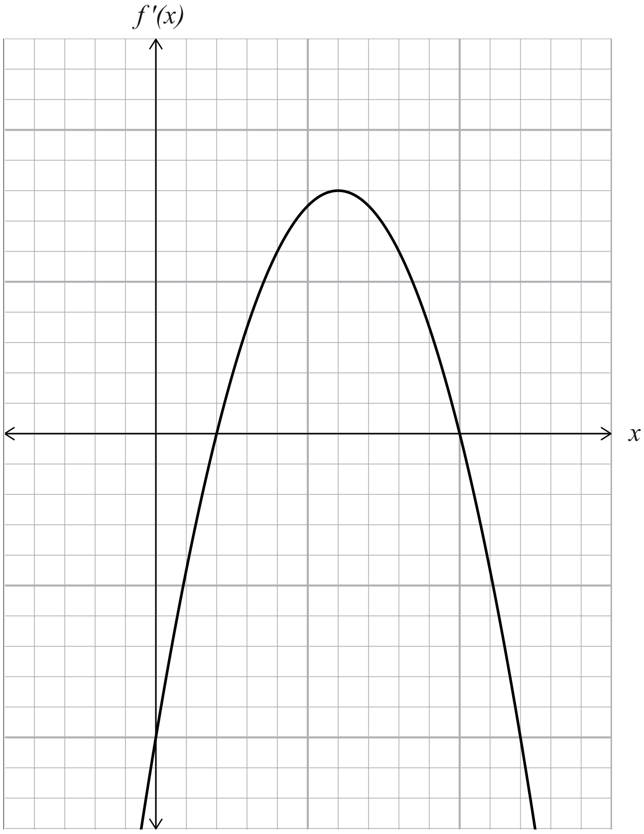
(a) A curve passes through the point (1 , 2.5) and has the derivative function: $f'(x) = 6x^2 + 5x - 1$

Find the co-ordinates of the point on the curve $y = f(x)$ where $x = 2$.

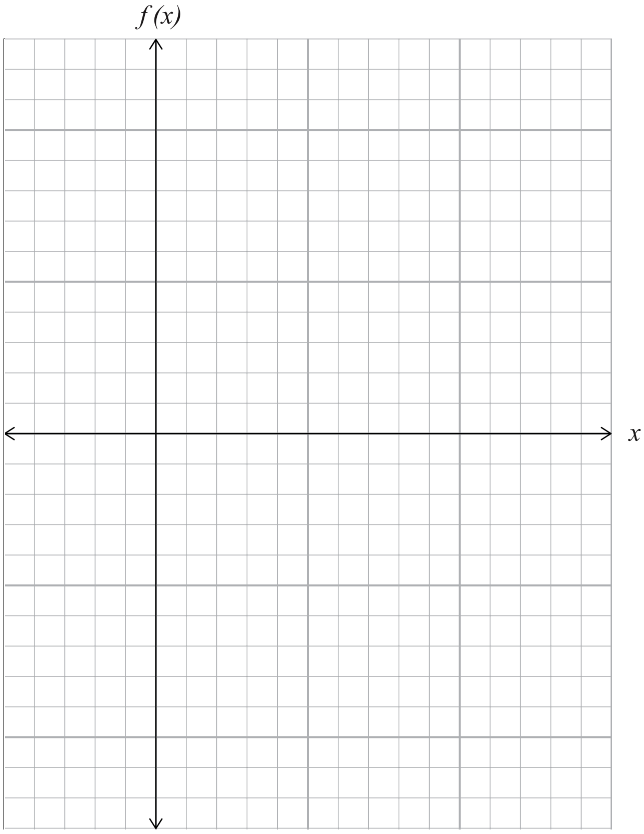
(b) An electric scooter is travelling down a straight footpath. Its velocity t seconds after passing a sign is given by $v(t) = 0.3t^2 + 1$ m s^{-1}.

How far is the scooter from the sign when $t = 3$ seconds?

(c) E whakaatu ana te hoahoa i raro nei i te kauwhata o tētahi pānga rōnaki $f'(x)$.

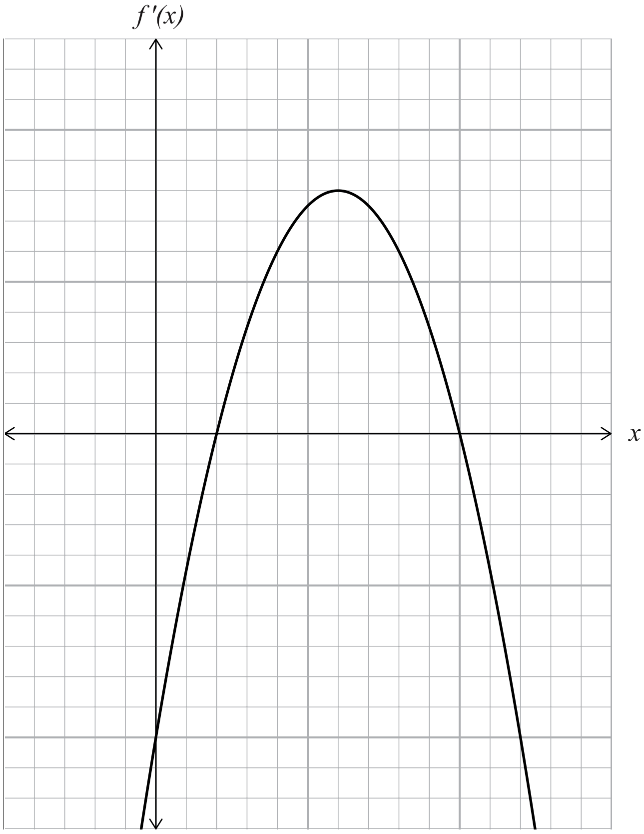


Ki ngā tuaka o raro, tātuhia te kauwhata o te pānga $f(x)$.

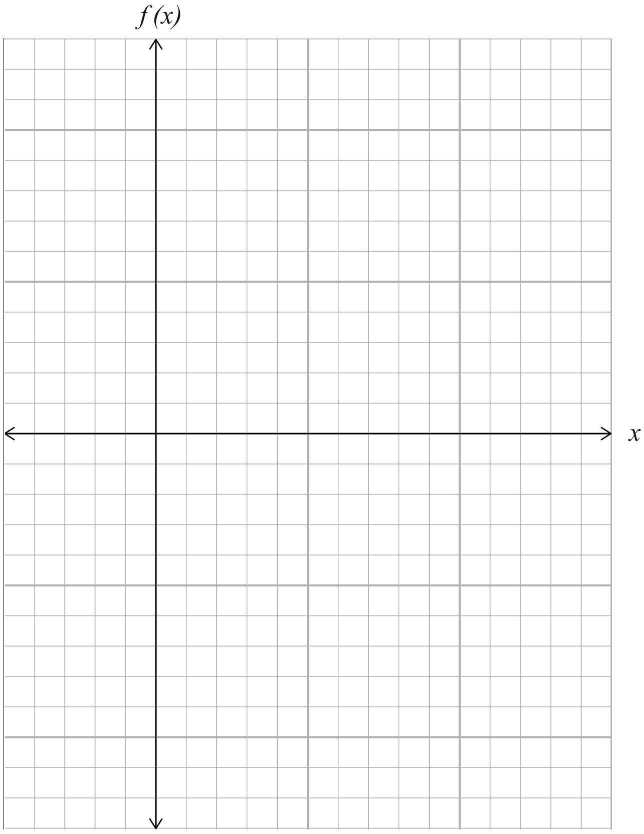


Ki te hiahia koe ki te tuhi anō i tēnei kauwhata, whakamahia te tukutuku i te whārangi 24.

(c) The diagram below shows the graph of a gradient function $f'(x)$.



On the axes below sketch the graph of the function $f(x)$.

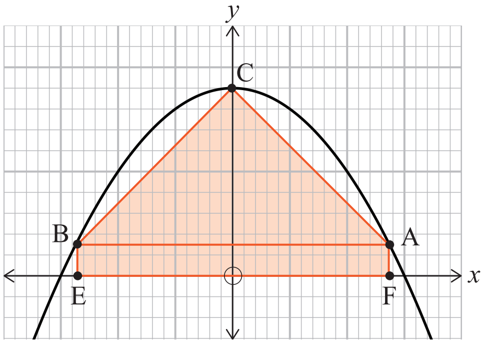


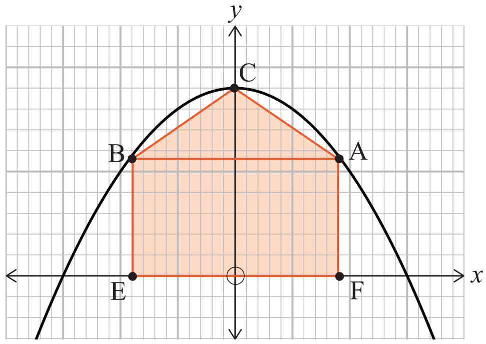
If you need to redraw this graph, use the grid on page 25.

(d) E rua ngā whirihora ka taea o tētahi āhua o te whare kua tātuhia i raro i tētahi unahi.

Ka tātuhia he āhua o te whare i raro ina ko:

- C kei (0,9).
- A me B ngā pūwāhi kei te unahi $y = 9 - x^2$.
- Ki te tuaka-x a E me F.





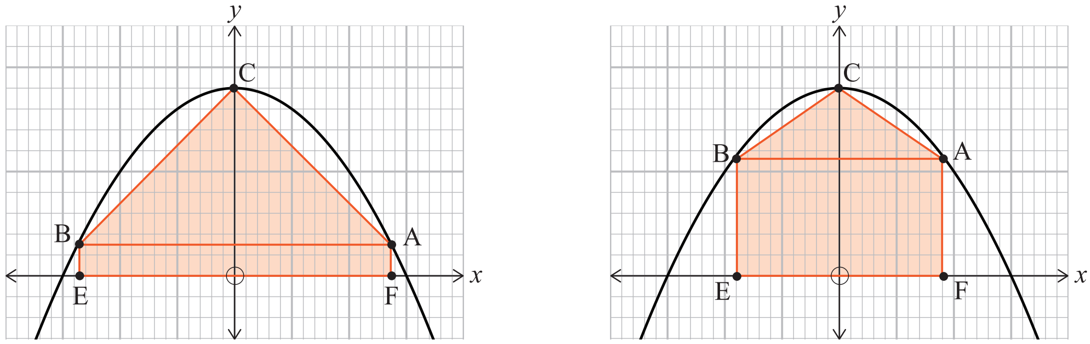
(i) Tātaihia te teitei pātū (AF, BE rānei) ina he mōrahi te horahanga o te āhua o te whare.

Parahautia koinei te horahanga mōrahi.

(d) Two possible configurations of a house shape drawn below a parabola are shown below.

A house shape is drawn as shown where:

- C is at (0,9).
- A and B are points on the parabola $y = 9 - x^2$.
- E and F are on the x-axis.

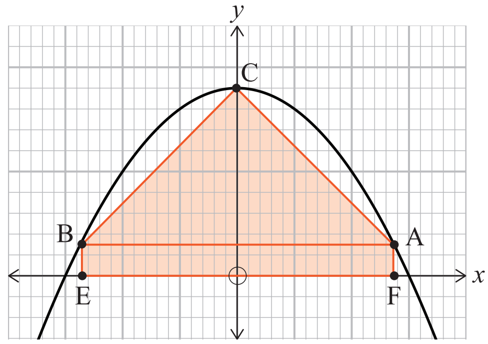


(i) Find the height of the wall (AF or BE) when the area of the house shape is a maximum.

Justify that this is the maximum area.

(ii) Mō te rapanga arowhānui e herea ana te āhua o te whare e tētahi unahi ko:

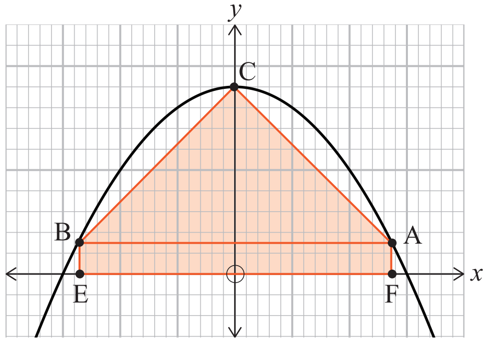
- C kei $(0,d)$.
- A me B kei te unahi $y = d - kx^2$.
- Ki te tuaka-x a E me F.



Whakaaturia mai ko te horahanga mōrahi e haupunitia ana e te āhua o te whare ka pā mai ina ōrite ana te horahanga o te wāhanga tapawhā hāngai ABEF me te horahanga o te wāhanga tapatoru ABC.

(ii) For the generalised problem where the house shape is bounded by a parabola where:

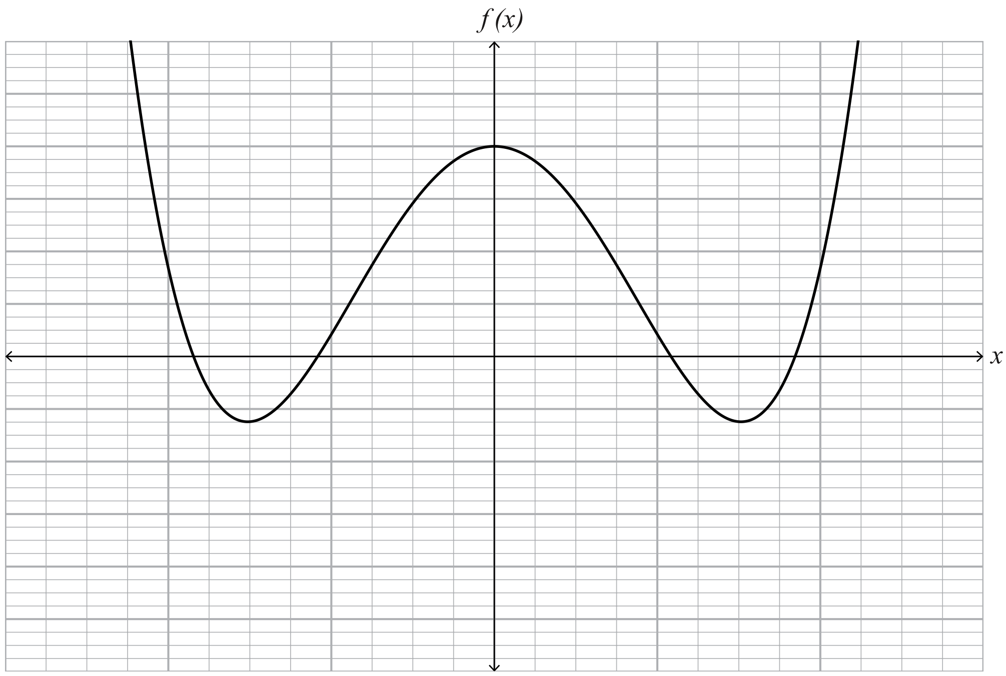
- C is at $(0,d)$.
- A and B are on the parabola $y = d - kx^2$.
- E and F are on the x-axis.



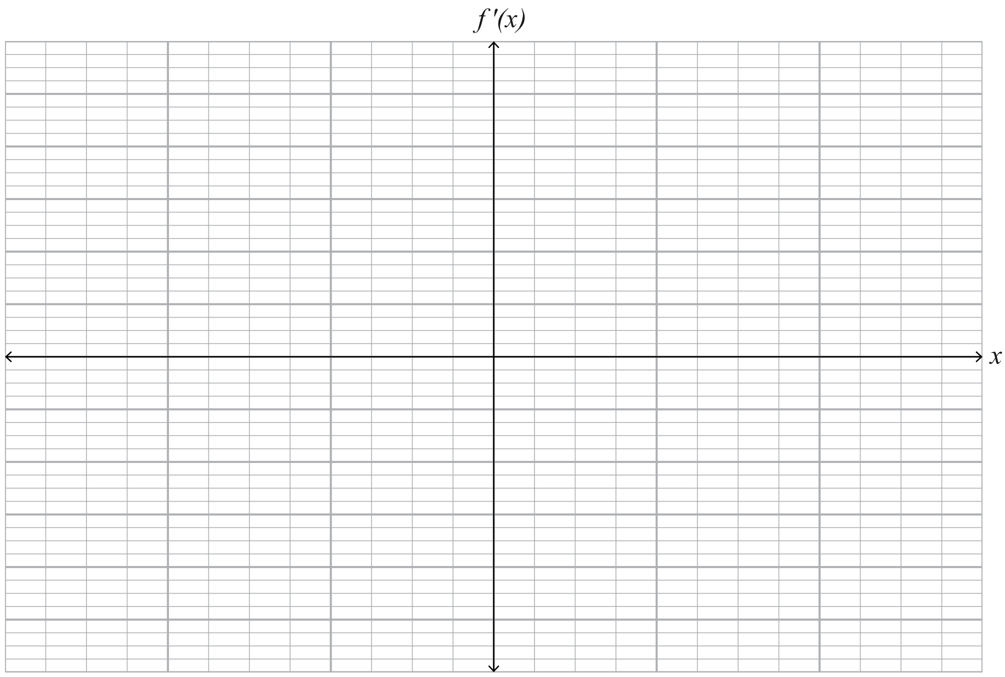
Show that the maximum area enclosed by the house shape occurs when the area of the rectangular section ABEF and the area of the triangular section ABC are equal.

NGĀ TUKUTUKU TĀPIRI

Ki te hiahia koe ki te tātuhi anō i tō urupare ki te Tūmahi Tuarua (a), whakamahia te tukutuku i raro nei. Kia mārama tonu tō tohu ko tēhea te tuhinga ka hiahia koe kia mākahia.

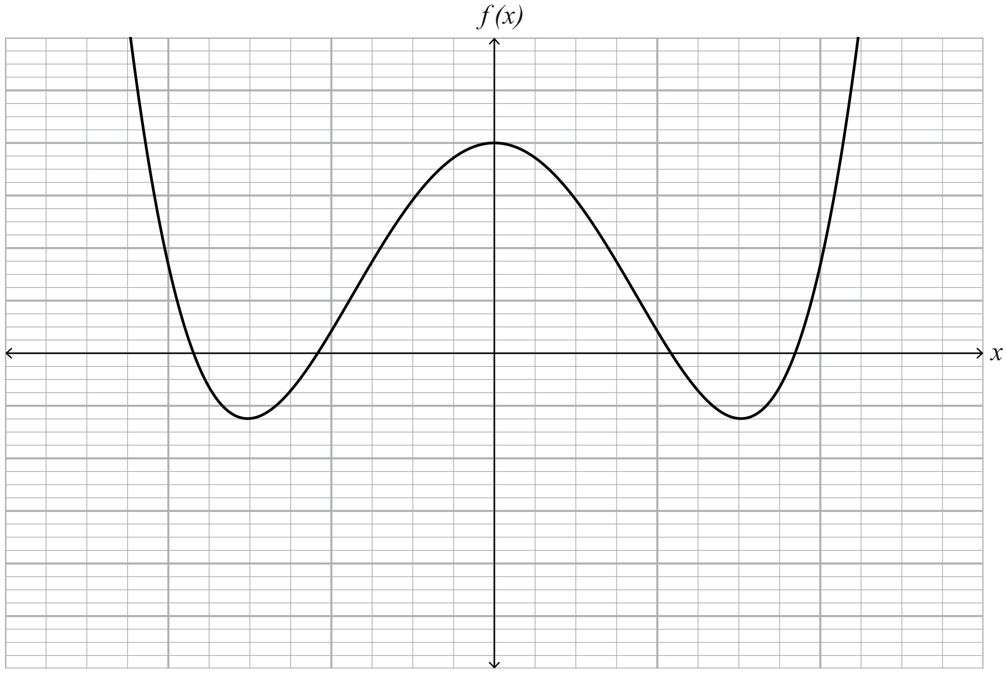


Tuhia te kauwhata o te pānga rōnaki $y = f'(x)$ ki ngā tuaka o raro.
He ōrite te āwhata huapae o ngā huinga tuaka e rua.

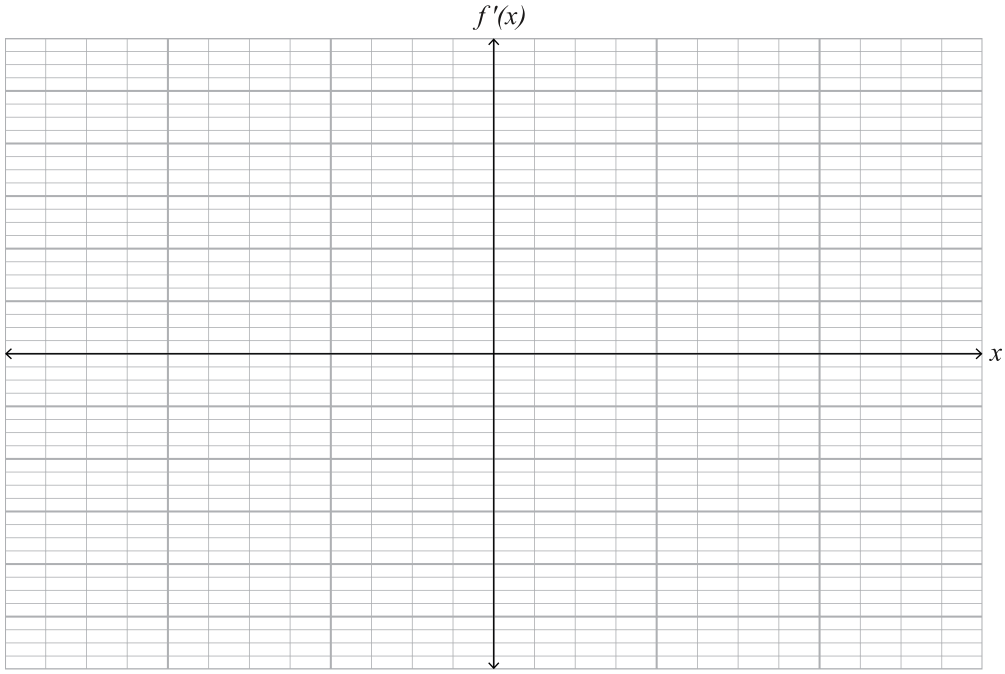


SPARE GRIDS

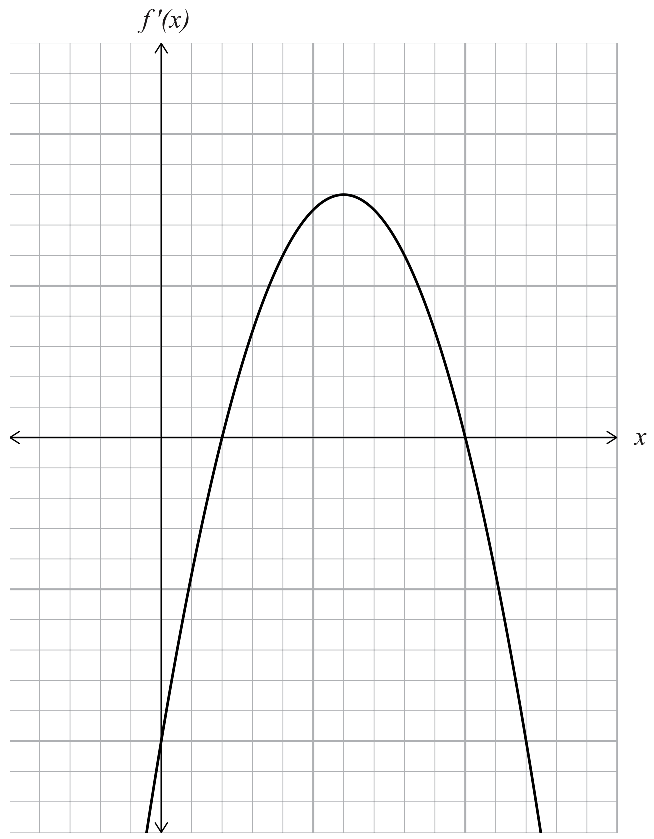
If you need to redo Question Two (a), use the grid below. You should make it clear which answer you want marked.



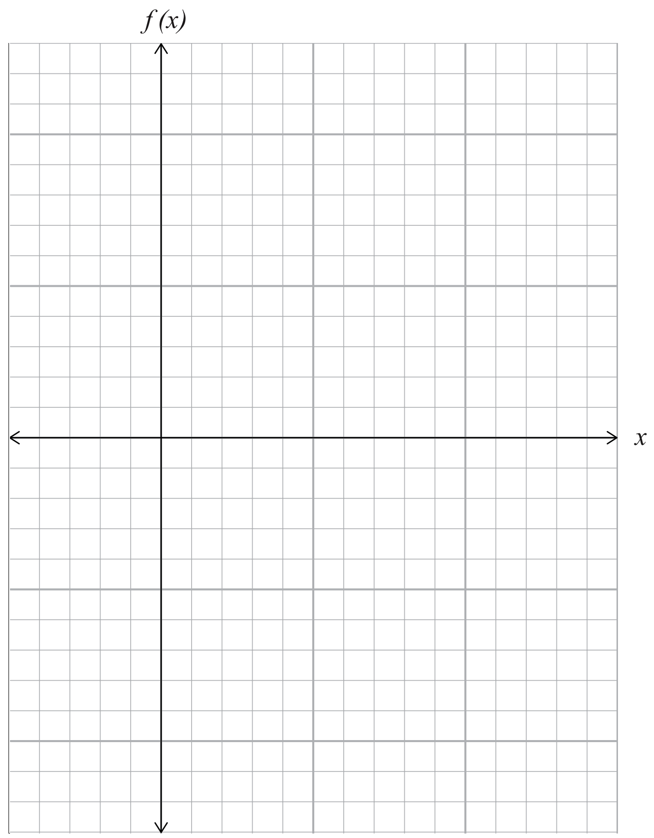
Sketch the graph of the gradient function $y = f'(x)$ on the axes below.
Both sets of axes have the same horizontal scale.



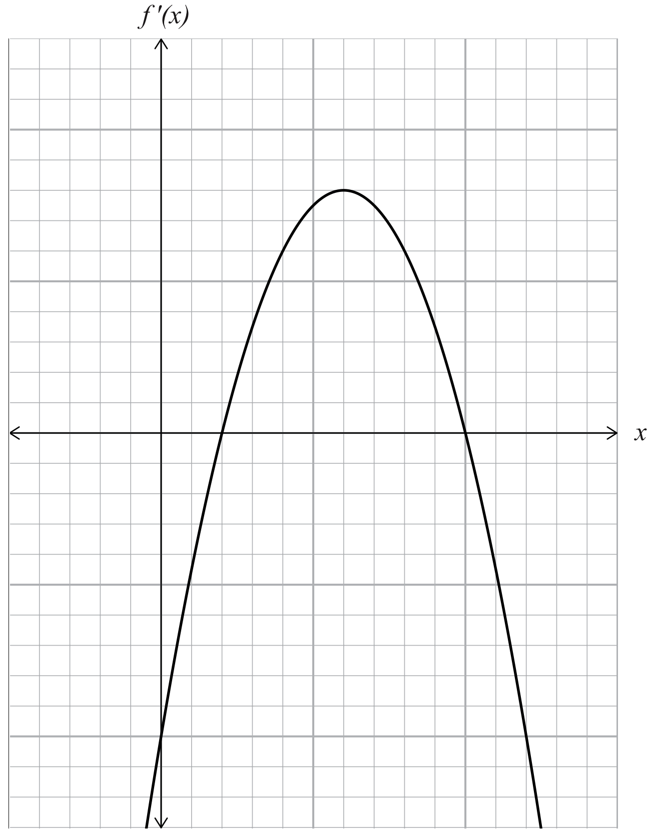
Ki te hiahia koe ki te tātuhi anō i tō urupare ki te Tūmahi Tuatoru (c), whakamahia te tukutuku i raro nei. Kia mārama tonu tō tohu ko tēhea te tuhinga ka hiahia koe kia mākahia.



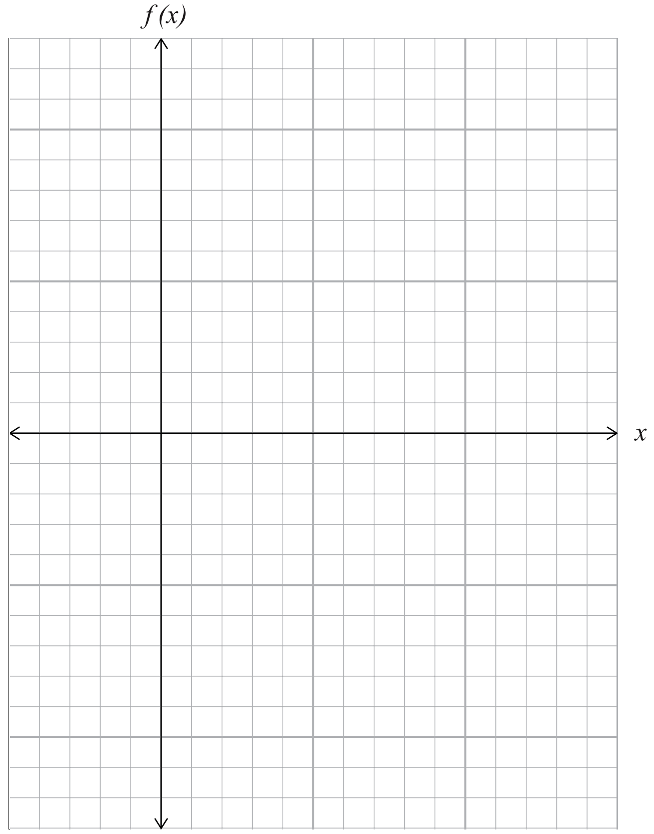
Ki ngā tuaka o raro, tātuhia te kauwhata o te pānga $f(x)$.



If you need to redo Question Three (c), use the grid below. You should make it clear which answer you want marked.



On the axes below sketch the graph of the function $f(x)$.



He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

English translation of the wording on the front cover

Level 2 Mathematics and Statistics 2021

91262M Apply calculus methods in solving problems

Credits: Five

91262M

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHMF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.