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SUPERVISOR'S USE ONLY

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Level 2 Mathematics and Statistics, 2018

91262 Apply calculus methods in solving problems

9.30 a.m. Wednesday 14 November 2018
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You must show the use of calculus in answering all questions in this paper.

Check that this booklet has pages 2–15 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Merit

TOTAL 18

ASSESSOR'S USE ONLY

QUESTION ONE

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(a) A function f is given by $f(x) = x^3 - 6x + 2$.

Find the gradient of the graph of the function at the point where $x = 4$.

$3x^2 - 6 = f'(x)$

$3(4)^2 - 6 = f'(x)$

$42 = f'(x)$ //

u.

(b) A rectangle is expanding in area so that at all times its length is three times its width.

Find the rate of change of the area of the rectangle with respect to its width when the area of the rectangle is 75 cm^2.

$A = Lw$ $\quad$ $L = 3w$

$A = 3w \times w$

$= 3w^2$ $\quad$ $75 = 3w^2$

$\frac{dA}{dw} = 6w$ $\quad$ $w = 5$.

$= 6(5)$

$= 30$ cm^3 per cm //

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ASSESSOR'S USE ONLY

(c) The derivative of a function f is given by $f'(x) = -3x^2 + 12x$.
The graph of the function has a local minimum at the point (0,5).

Use calculus to find the value of the local maximum of the function.

$0 = -3x^2 + 12x$

$0 = -3x(x - 4)$ $\quad x = 4, 0$

$-x^3 + 6x + c = f(x)$

$-(0)^3 + 6(0) + c = 5$

$c = 5.$

$-(4)^3 + 6(4)^2 + 5 = f(x)$

$-64 + 96 + 5 = f(x)$

$f(x) = 37$ $\quad (4, 37)$ is max //

r

(d) Use calculus to find the values of x for which the graph of the function $f(x) = \frac{2}{3}x^3 + \frac{9}{2}x^2 - 5x - 18$ is increasing.

$f'(x) = 2x^2 + 9x - 5$

$0 < 2x^2 + 9x - 5$

when $x > -5$ and $x < 0.5$ the function is increasing //

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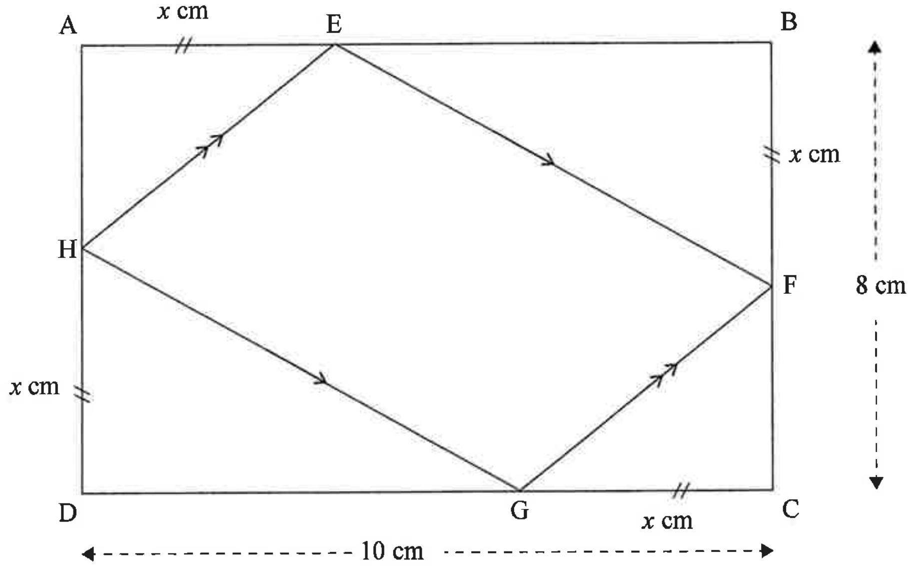
ASSESSOR'S USE ONLY

(e) A rectangle ABCD measures 10 cm by 8 cm. A parallelogram EFGH can be drawn inside the rectangle, as shown in the diagram below.

Suppose that the distance from each corner of the rectangle to the next vertex of the parallelogram, in a clockwise direction, is x cm.

That is, AE = BF = CG = DH = x.



Use calculus to find the smallest possible area that the parallelogram can have.

Justify that your answer is a minimum.

$A = (10 \times 8) - 2\left(\frac{1}{2}x(10-x)\right) - 2\left(\frac{1}{2}x(8-x)\right)$

$= 80 - 2(5x - \frac{1}{2}x^2) - 2(4x - \frac{1}{2}x^2)$

$= 80 - 10x - x^2 - 8x - x^2$

$= 80 - 18x - 2x^2$

$\frac{dA}{dx} = -18 - 4x$ con

$0 = -18 - 4x$

$x = -4.5$ //

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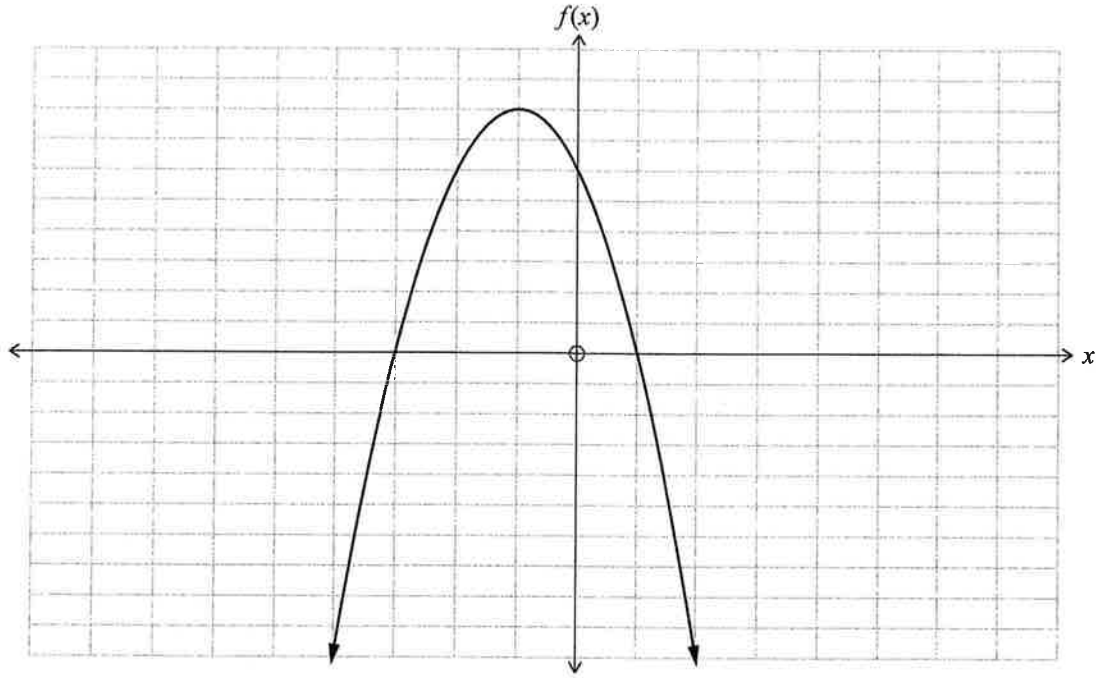
M6

QUESTION TWO

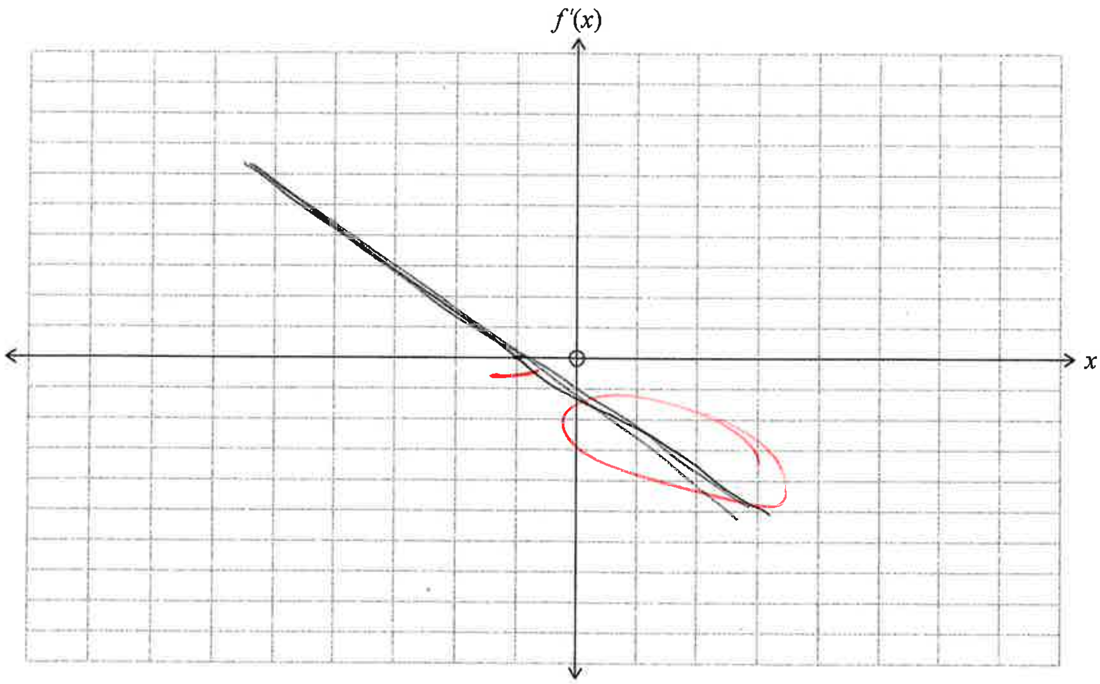
ASSESSOR'S USE ONLY

(a) The graph of a function $y = f(x)$ is shown on the axes below.



Sketch the graph of the gradient function $y = f'(x)$ on the axes below.

Both sets of axes have the same scale.



u

If you need to redo this question part, use the grids on page 12.

ASSESSOR'S USE ONLY

(b) A skyrocket is projected into the air so that t seconds after it is launched, its height, h metres, above the ground is given by

$$h(t) = 39.2t - 4.9t^2.$$

What is the maximum height that the skyrocket will reach?

$h'(t) = 39.2 - 9.8t.$ when $h'(t) = 0$

$39.2 - 9.8t = 0$

$-9.8t = -39.2$

$t = \frac{-39.2}{-9.8}$

$= 4$

$h(t) = 39.2(4) - 4.9(4)^2 = 78.4$ max height is 78.4 m //

(c) Adam is operating his drone. It is moving in a straight line and t seconds after passing a tree its acceleration, a m s^{-2}, is given by

$$a(t) = 6 - 12t.$$

Two seconds after the drone passed the tree, its velocity was 20 m s^{-1}.

How far was the drone from the tree when its velocity was 20 m s^{-1}?

$a = 6 - 12t$ $t = 2$ $v = 0$

$d = ?$ $v = 20$

$V = 6t - 6t^2 + c = 20$

$6(2) - 6(2)^2 + c = 20$

$12 - 24 + c = 20$

$c = 32$

$V = 6t - 6t^2 + 32$

$d = 3t^2 - 2t^3 + 32t + C$

$3(0)^2 - 2(0)^3 + 32(0) + C = 0$

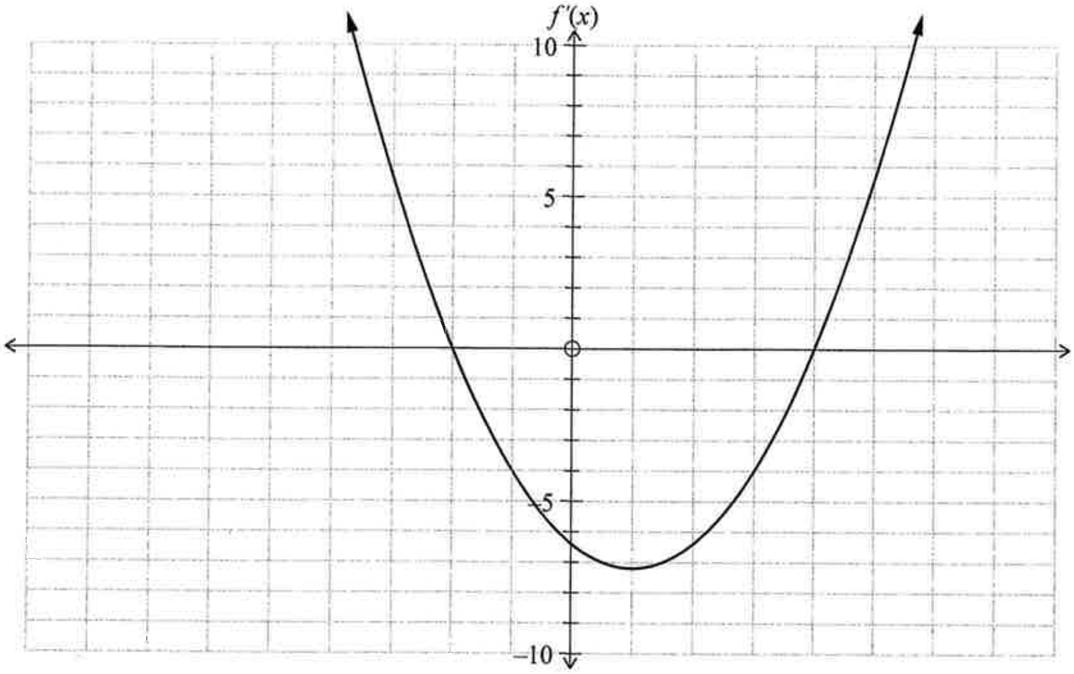
$C = 0$

$d = 3(2)^2 - 2(2)^2 + 32$

$d = 36$ m

(d) The diagram below shows the graph of the gradient function $y = f'(x)$ of a function $y = f(x)$.

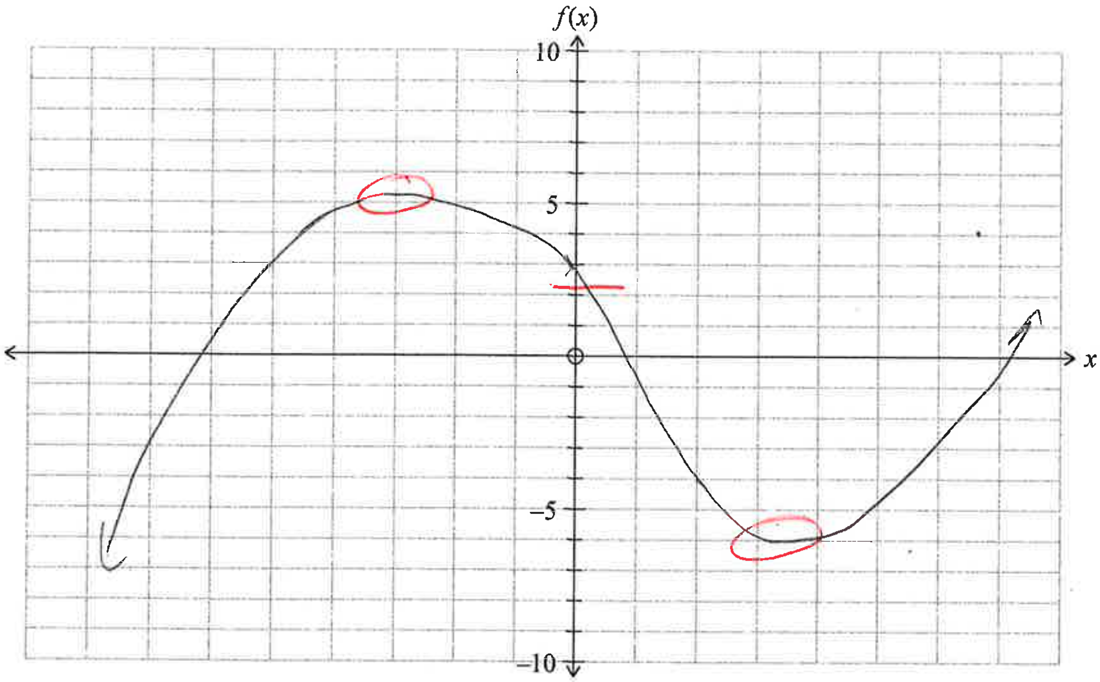
ASSESSOR'S USE ONLY



The graph of the function $y = f(x)$ passes through (0,3).

On the axes below sketch the graph of the function f.

Both sets of axes have the same scale.



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If you need to redo this question part, use the grids on page 13.

(e) The graph of the function $y = x^3 - 6x^2 + kx - 5$ has a turning point at $x = 3$.

Use calculus methods to find the coordinates of both turning points.

Determine the nature of each turning point, justifying your answer.

$\frac{dy}{dx} = 3x^2 - 12x + k$

$0 = 3(3)^2 - 12(3) + k$

$k = 9$

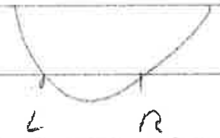
$\frac{dy}{dx} = 3x^2 - 12x + 9$

$0 = 3x^2 - 12x + 9$

$x = 3, 1$

(L) $x = 2$ $\quad f'(2) = 3(2)^2 - 12(2) + 9 = -3$

3 is the local minimum



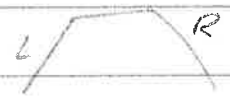
(R) $x = 4$ $\quad f'(4) = 3(4)^2 - 12(4) + 9 = 9$

because left is decreasing and right is increasing

(L) $f'(0) = 3(0)^2 - 12(0) + 9 = 9$

(R) $f'(2) = -3$ $\quad$ is the local max.

left is increasing and right is decreasing //



ASSESSOR'S USE ONLY

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M6

QUESTION THREE

ASSESSOR'S USE ONLY

(a) The gradient function of a curve is given by $\frac{dy}{dx} = -5x^4 + 6$.

The curve passes through (1,7).

Find the equation for y.

$y = -x^5 + 6x + c.$

$7 = -(1)^5 + 6(1) + c$

$c = 2$

$y = -x^5 + 6x + 2$ //

u.

(b) Suppose that, at the start of a particular day, 1000 people were trading in a market, and that t days after the start of that day, the number of traders, N, can be modelled by

$N(t) = 1000 + 400t + 100t^2$.

How many days will it take for the rate of change of the number of traders to be 14 400 per day?

$N'(t) = 400 + 200t$

$14400 = 400 + 200t$

$14000 = 200t$

$t = \frac{14000}{200}$

$= 7_{\wedge}$ days.

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u

It will take 7 days //

Question Three continues on the following page.

ASSESSOR'S USE ONLY

(c) A school is selling tickets for its drama production.

The revenue, $\$R$, from selling tickets for a price of $\$p$ each, can be modelled by the function

$$R(p) = 40p(29 - 2p)$$

Use calculus to find the maximum possible revenue (using this model).

$R(p) = 1160p - 80p^2$

$R'(p) = 1160 - 160p$

$0 = 1160 - 160p$

$p = 7.25 \Rightarrow R(p) = 1160p - 80p^2$

$= 1160(7.25) - 80(7.25)^2$

$= 4205$

Maximum possible revenue is \$4205 //

(d) A tangent to the graph of the function $y = -\frac{1}{3}x^3 + kx + 4$ at a certain point P, has gradient of –7 and intersects the graph again at (–6,64).

Use calculus to find the co-ordinates of the point P.

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$\frac{dy}{dx} = -x^2 + k$

$-7 = -x^2 + k$

$y - y_1 = m(x - x_1)$

$y - 64 = -7(x - {-6})$

$y - 64 = -7x - 42$

$y = -7x + 22$

$y = -\frac{1}{3}x^3 + kx + 4$

$64 = -\frac{1}{3}(-6)^3 + k(-6) + 4$

$64 = 72 - 6k + 4$

$6k = 72 + 4 - 64$

$6k = 12$

$k = 2$

r

$y = -\frac{1}{3}x^3 + 2x + 4$

$y = -\frac{1}{3}(-6)^3 + 2(-6) + 4$

$= 72 - 12 + 4$

$= 64$

P is $(-6, 64)$ //

M6

Merit Exemplar 2018

Subject	Level 2 Mathematics and Statistics	Standard	91262	Total score	18

Q	Grade score	Annotation
1	M6	A higher grade of either E7 or E8 could have been gained if the candidate had applied a sound technique in (d) to correctly determine the interval or realised that their negative solution in (e) was an indication of an earlier error. This mistake denied the opportunity to provide further evidence.
2	M6	This provides evidence for M6 because of a correct distance equation in (c) and the candidate answer in (e) giving x ordinates and justifying which were max and min. However, the answer in (e) has not given the coordinates as asked for in the question so the higher grade of E7 could not be given.
3	M6	If the candidate had solved the derivative equation in (d) to find the relevant values for x, they may have scored either E7 or E8 should they have gone on to find the coordinates and determine which was valid.