**2**

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SUPERVISOR'S USE ONLY

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# Level 2 Mathematics and Statistics, 2017

## 91261 Apply algebraic methods in solving problems

2.00 p.m. Friday 24 November 2017
Credits: Four

| Achievement | Achievement with Merit | Achievement with Excellence |
|---|---|---|
| Apply algebraic methods in solving problems. | Apply algebraic methods, using relational thinking, in solving problems. | Apply algebraic methods, using extended abstract thinking, in solving problems. |

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

**You should attempt ALL the questions in this booklet.**

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

**You are required to show algebraic working in this paper. Guess-and-check methods, and correct answer(s) only, will generally limit grades to Achievement.**

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

**YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.**

Achievement

**TOTAL** 13

ASSESSOR'S USE ONLY

**QUESTION ONE**

ASSESSOR'S USE ONLY

(a) A function $f$ is given by $f(x) = x^5 + 3x^2 - 7x + 2$.

Find the gradient of the graph of the function at the point where $x = 1$.

$f'(x) = 5x^4 - 6x - 7$

$f'(1) = 5 + 6 - 7$

$f'(1) = 4$

$m = 4$

(b) Find the equation of the tangent to the graph of the function

$f(x) = 6 + 14x - 2x^3$

at the point (2,18) on the graph.

$f'(x) = -6x^2 + 14$

$f'(2) = -24 + 14$

$f'(2) = -10$

(c) The movement of an object is recorded from the time it passes a fixed point. After $t$ seconds it has a speed $v$ m $s^{-1}$, which can be modelled by the function

$v(t) = 0.5t^2 - 2t + 1$

IAUS

Use calculus to find how long it takes to reach an acceleration of 2.8 m $s^{-2}$.

$a(t) = t - 2$

$= 2.8 - 2$

$= 0.8$

(d) A tangent to the graph of the function $f(x) = 3x^2 - 4x$ has a gradient of 2, and passes through the point $(5,a)$, where $a$ is a constant.

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Find the value of $a$.

$f'(x) = 6x - 4$

$f'(x) = 6x - 4$ $\quad$ $2 = 6x - 4$

$a = 3x^2 - 4x$ $\quad$ $x = 1$.

$a = 3\times1^2 - 4\times1$

$a = -1$

$(5, -1)$

(e) The function $f(x) = x^3 + ax^2 + bx + 2$ has turning points when $x = -1$ and $x = 3$.

Find the values of $a$ and $b$.

$f'(x) = 3x^2 + 2ax + b = 0$

$= 3(-1)^2 + 2(-1)a + b$

$= 3 - 2a + b$

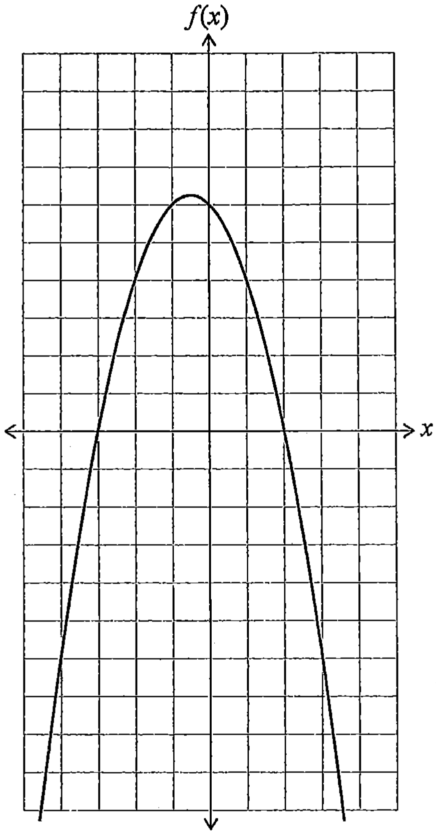
$f''(x) = 6x + 2a.$

A4

ASSESSOR'S USE ONLY

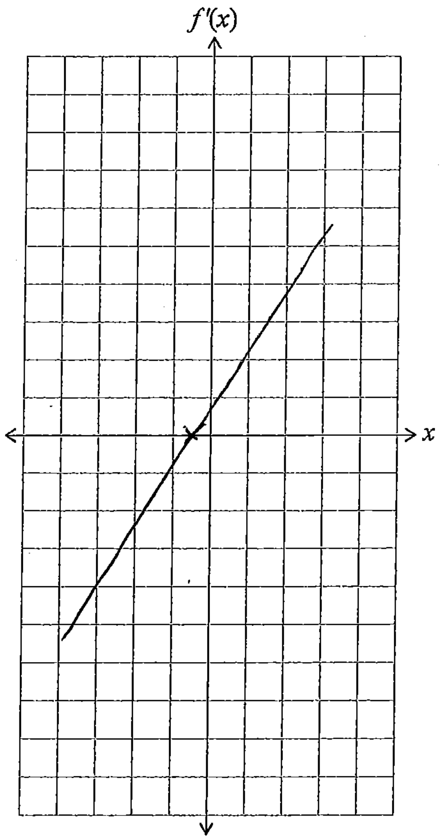
## QUESTION TWO

(a) The diagram below shows the graph of the function $y = f(x)$




Sketch the graph of the gradient function $y = f'(x)$ on the axes below.

Both sets of axes have the same scale.




*If you need to redraw this graph, use the grid on page 11.*

ASSESSOR'S USE ONLY

(b) The graph of a function $f(x) = 2x^3 + bx^2 - 2$ has a turning point when $x = -1$.

Find the value of $b$.

$f(x) = 2x^3 + bx^2 - 2.$

$f'(x) = 6x^2 + 2bx - 2.$

$f'(-1) = 6 \times {}^{-}1^2 + 2b \times {}^{-}1 - 2.$

$\wedge \quad = -6 - 2b - 2$

$= -8 - 2b.$

(c) Use calculus to show that the line $y = 15x - 12$
is a tangent to the graph of the function $f(x) = 4x^2 - x + 4$.

$y = 15x - 12$

$f(x) = 4x^2 - x + 4$

$f'(x) = 8x - 1$

$15 = 8x - 1$

$8x = 16$

$x = 2$

$y = 15(2) - 12$

$= 18$

(d) Use calculus to find the value of $k$ if the line $y = 6x + k$ is a tangent to the graph of the function $f(x) = x^2 + 2x - 1$.

$f'(x) = 2x + 2$

$= 2x + 2 = 0$

$x = {}^{-}1 \qquad (-1, -4).$

$f(-1) = -4$

**There is more space for your answer on the following page.**

ASSESSOR'S USE ONLY

$y - {}^{-}4 = 6(x - {}^{-}1)$

$y + 4 = 6x + 1$

$y = 6x - 3$

$k = 3$

(e) Use calculus to prove that the graph of the function

$y = x^3(3 - x)$

has a local maximum when $x = \frac{9}{4}$.

Justify that the turning point is a local maximum.

$y = x^3(3 - x)$

$y = 3x^3 - x^4$

$y' = 9x^2 - 4x^3 \quad = x^2(9 - 4x)$

when $y' = 0 \qquad x = 0$ or $x = 9/4$

$9/4 = 2.25$

$y'' = 18x - 12x^2$

$y''(0) = 0$

$y''(2.25) = -20.25$

M5

## QUESTION THREE

ASSESSOR'S USE ONLY

(a) The gradient graph of a function $f(x)$ is given by

$$f'(x) = 6x^2 - 2x + 4$$

The point (1,3) lies on the graph.

Find the equation of the function $f(x)$.

$$f(x) = \frac{6x^3}{3} - \frac{2x^2}{2} + 4x + C$$

$$= 2x^3 - x^2 + 4x + C$$

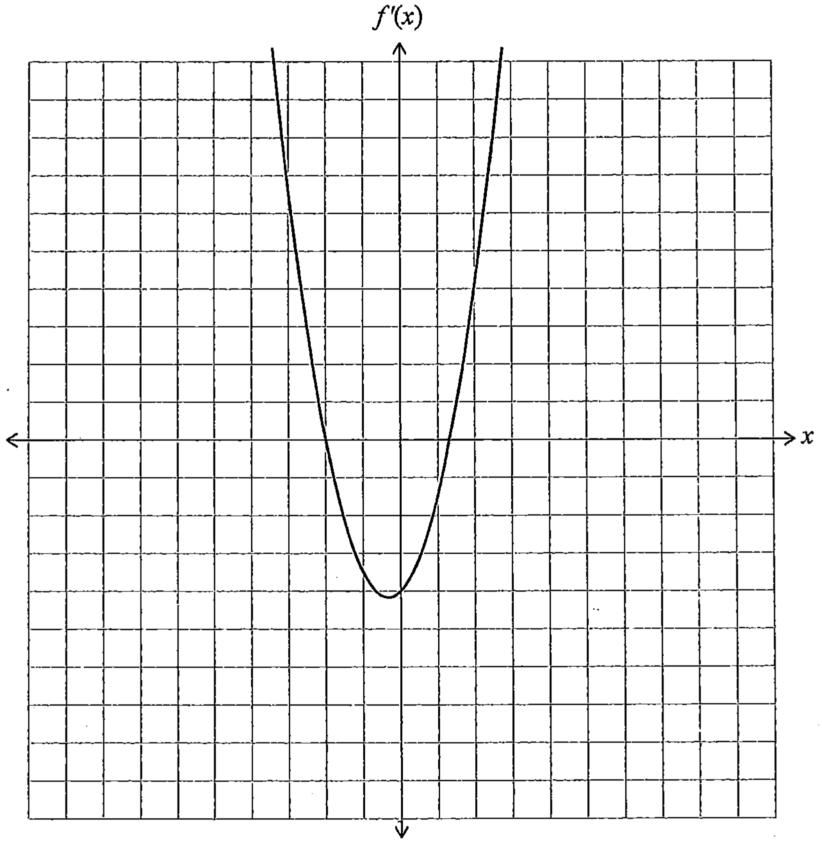
$$= 2 - 1 + 4 + c.$$

$$C = -5$$

$$f(x) = 2x^3 - x^2 + 4x - 5.$$

ASSESSOR'S USE ONLY

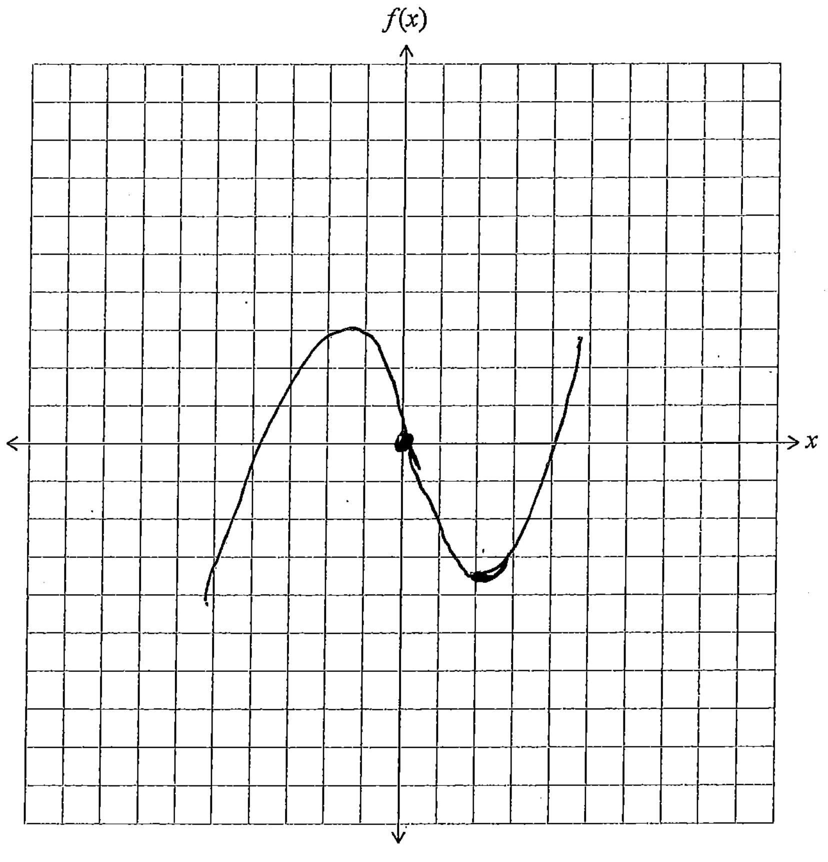
(b) The diagram below shows the graph of a gradient function $y = f'(x)$.




The point (0,0) is on the graph of the function $y = f(x)$.

On the axes below sketch the function $f(x)$.

Both sets of axes have the same scale.




If you need to redraw this graph, use the grid on page 11.

ASSESSOR'S USE ONLY

(c) An object can move in either direction on a straight track and has a constant acceleration of $-4$ cm s$^{-2}$.

A fixed point P is marked on the track.

When a recording of the object's motion begins, the object:

- is 12 cm from P
- is moving away from P, and
- has a velocity of 6 cm s$^{-1}$.

(i) Using calculus, find the speed of the object 5 seconds after its motion began being recorded.

$a(t) = -4$

$v = -4t + c$

$= -4t + c$

$6 = -4t + c$

$6 = -4 \times 0 + c$

$c = 6$

(ii) What is the maximum distance of the object from the point P?

Justify that this is the maximum distance.

$v = -4t + 6$

$0 = -4t + 6$

$t = 6/4 = 3/2 = 1{\cdot}5$ seconds.

$s(t) = -2t^2 + 6t$

when $t = 1{\cdot}5$ $\quad s(1{\cdot}5) = -2(1{\cdot}5)^2 + 6(1{\cdot}5)$

$= 4{\cdot}5$

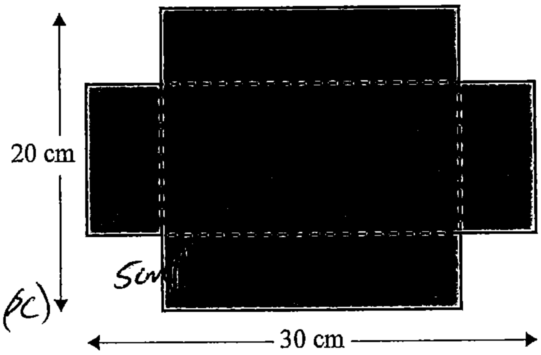
max distance when $t = 1{\cdot}5$s is $4{\cdot}5$cm.

Question Three continues on the following page.

(d) Find the maximum volume of an open box (i.e. a box with a base and sides, but no lid) that can be made from a rectangular piece of cardboard measuring 20 cm by 30 cm, by removing the corner squares and folding along the dotted lines.

ASSESSOR'S USE ONLY

Justify that this is the maximum volume.




$V = L \times w \times h$

$V = (30-2x)(20-2x)(x).$

$=(600 - 60x - 40x + 4x^2)(x)$

$V = 600x - 60x^2 - 40x^2 + 4x^3$

$= 4x^3 - 100x^2 + 600x.$

$\frac{dV}{dx} = 12x^2 - 200x + 600$

$0 = 12x^2 - 200x + 600$

$= \cancel{(6x \quad )(2x \quad )}$

$= 4(3x^2 - 50x + 150)$

$= 4(3x - \quad)(x - \quad)$

$\cancel{x = 3.92}$

$\overset{30}{}\quad\overset{5}{}$

$(3x-15)(x-10)$

$\cancel{3x^2 - 30x - 15x}$

A4

| Subject: | Mathematics | Standard: | 91262 | Total score: | 13 |
|---|---|---|---|---|---|

| Q | Grade score | Annotation |
|---|---|---|
| 1 | A4 | 1(a) Correct derivative and gradient.<br>1(b) Correct gradient only, no equation for tangent.<br>1(c) Correct a(t) equation, incorrect substitution to find t at a = 2.8.<br>1(d) Correct derivative and solved to get x = 1. Incorrect equation for a.<br>1(e) Correct derivative and f'(x) = 0 shown. Correct substitution of x = -1 but no x = 3 substitution. |
| 2 | M5 | 2(a) Correct x intercept but wrong slope.<br>2(b) Incorrect derivative and f'(x) = 0 not shown, incorrect substitution.<br>2(c) Correct f'(x) equated to slope of y = 15x -12, no evidence of f(2) = 18.<br>2(d) f'(x) = 0 was incorrect, inconsistent application of f(-1) and wrong identification that k = 3.<br>2(e) Correct derivative and both solutions stated for y' = 0. No concluding statement to justify y''(2.25) = -20.25 was a maximum. |
| 3 | A4 | 3(a) Correct integration with constant c but f(x) = 1 not applied.<br>3(b) Correct shape and (0,0) intersect, max and min points not aligned with x intercepts.<br>3(c)(i) Correct equation for v, speed at t=5s not found.<br>3(c)(ii) Correct t= 1.5s, incorrect s(t) equation and no justification that a max.<br>3(d) Correct equation for volume and derivative, unable to find correct solutions for max/min situation. |