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QUALIFY FOR THE FUTURE WORLD
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Level 3 Physics, 2015

91524 Demonstrate understanding of mechanical systems

9.30 a.m. Friday 20 November 2015
Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Demonstrate understanding of mechanical systems.	Demonstrate in-depth understanding of mechanical systems.	Demonstrate comprehensive understanding of mechanical systems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Resource Booklet L3–PHYSR.

In your answers use clear numerical working, words and/or diagrams as required.

Numerical answers should be given with an SI unit, to an appropriate number of significant figures.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–8 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Low Excellence

TOTAL **20**

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QUESTION ONE: SATELLITES

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Mass of Earth = 5.97×10^{24} kg

Universal gravitational constant = 6.67×10^{-11} N m^2 kg^{-2}

Digital television in New Zealand can be accessed by using a satellite dish pointed at a satellite in space. The satellite used to transmit the signals appears to stay still above the equator.

The satellite, with a mass of 300 kg, is actually travelling around the Earth in a geostationary orbit at a radius of 4.22×10^7 m from the centre of the Earth.

(a) Name the force that is keeping the satellite in this circular orbit, and state the direction in which this force is acting.

$F_c = F_g$ centripetal force acting towards the center of earth supplied by earth's gravity

The candidate correctly names the force and direction in which this force acts.

(b) Calculate the force acting on the satellite.

$F_g = \frac{GMm}{r^2}$ $\quad F_g = \frac{6.67\times10^{-11} \times 300 \times 5.97\times10^{24}}{(4.22\times10^7)^2} = 67.08\text{ N}$

$F_g = 67.1\text{ N}$ (3sf)

Correct working and answer.

(c) Show that the speed of the satellite is 3.07×10^3 m s^{-1}.

$F_g = F_c$ since gravity is only force suplying centripetal force

$\therefore F_c = 67.08\text{ N}$ $\quad F_c = \frac{mv^2}{r}$

$67.08 = \frac{300 v^2}{4.22\times10^7}$ $\quad \frac{4.22\times10^7 \times 67.08}{300} = v^2 = 9435995.261$

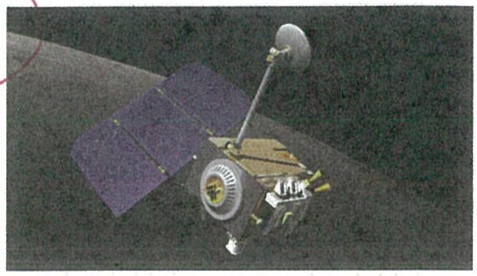
$v = \sqrt{9435995.261} = 3071.8\text{ ms}^{-1}$

$v = 3.07 \times 10^3\text{ ms}^{-1}$ (3sf)

Correct equation and evidence for calculating the speed of satellite.

Working On Back Page ~~need m~~ 3 have r, T, ω, f

(d) Kepler's law states that, for any orbiting object, $T^2 \propto r^3$, where r is the radius of the orbit, and T is the time period for the orbit.



https://upload.wikimedia.org/wikipedia/commons/9/95/Lunar_Reconnaissance_Orbiter_001.jpg

NASA uses a robotic spacecraft to map the Moon. The Lunar Reconnaissance Orbiter orbits the Moon at an average height of 50.0×10^3 m with a period of 6.78×10^3 s. The Moon has a radius of 1.74×10^6 m.

Use Kepler's law to estimate the mass of the Moon.

In your answer you should:

- use the relevant formulae to derive Kepler's law
- use Kepler's law to determine the mass of the Moon.

$r =$

radius (orbital) $= 50 \times 10^3 + 1.74 \times 10^6 = 1790000$ m $= 1.79 \times 10^6$ m

~~$F_g = \frac{GMm}{r^2}$~~ ~~$F_g = 6.67 \times 10^{-11} \times$~~

$v = r\omega \quad \therefore \quad r = \frac{v}{\omega}$

$T^2 \propto r^3$

$F_c = \frac{mv^2}{r} \quad \therefore \quad r = \frac{mv^2}{F_c}$

thus $r = \frac{v}{\omega} = \frac{mv^2}{F_c}$

$\therefore T^2 \propto r^3 = \left(\frac{mv^2}{F_c}\right)^3$

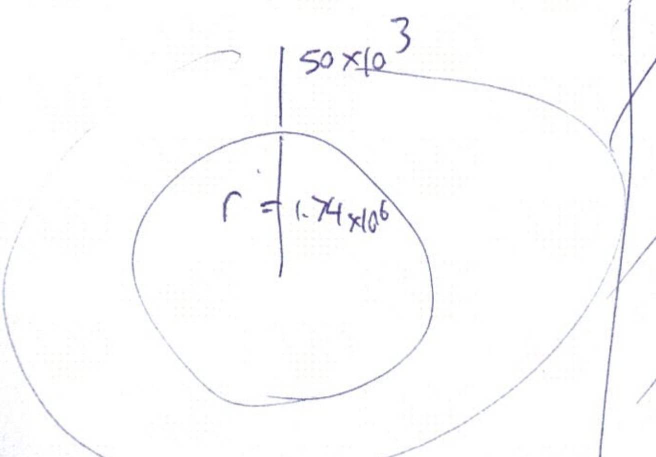
$T^2 \propto \left(\frac{mv^2}{F_c}\right)^3$

Incorrect and incomplete derivation of Kepler's Law. For Achieved, attempt using Fg, Fc and V is made. For Merit, correct derivation for Kepler's Law is required. For Excellence, correct derivation and calculation for the mass of moon is required.

$F_g = F_c \quad F_g = \frac{GMm}{r^2}$

$T^2 \propto \left(\frac{mv^2}{\frac{GMm}{r^2}}\right)^3 \quad T^2 \propto \left(\frac{v^2 r^2}{GM}\right)^3$

On back



$v = \frac{d}{t}$ d(orbit) $= 2\pi r$

d(orbit) $= 2\pi \times 1.74 \times 10^6$

V(linear orbital) $= \frac{2\pi \times 1.79 \times 10^6}{6.78 \times 10^3}$ s

$v = 1658.835$ ms^{-1}

$\omega = 2\pi f$

$\omega = 1.4749 \times 10^{-4}$

M5

QUESTION TWO: GRAVITY ELEVATORS

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Earth's average radius = 6.38×10^6 m.

In the 2012 science fiction movie *Total Recall*, a gravity-powered elevator called "The Fall" is used to transport passengers between the Northern and Southern hemispheres, straight through the Earth. If a straight tunnel could be dug through the Earth from the North Pole to the South Pole, protected from the heat inside the Earth and the journey unaffected by friction, an elevator could be used, harnessing the gravity of the planet.

Once dropped, the elevator would accelerate downwards and then decelerate once it had passed through the midpoint and – in the absence of friction – would just arrive at the far side of the Earth.

Adapted from: http://www.killerasteroids.org/impact.php

An equation can be used to summarise acceleration of the elevator.

$a = -1.54 \times 10^{-6}\, y$, where y = distance from the midpoint

(a) One of the passengers on the elevator stands on bathroom scales at the start of the journey.

Describe why the bathroom scales read zero.

The scales read zero because there is no reaction force of the scales on the passenger since both of them only experience downwards Fg (Gravitational force of earth) because the lifts without friction, The passenger experiences apparent weightlessness as there is no reaction force ~~of the scales~~ on him.

The answer clearly describes why the bathroom scales read zero.

(b) Calculate:

(i) The maximum acceleration of the elevator.

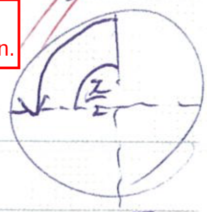
max accel at surface ∴ $r = 6.38 \times 10^6 = y$

$a = -1.54 \times 10^{-6} \times 6.38 \times 10^6 = -9.8252\ ms^{-2}$

$a_{max} = 9.83\ ms^{-2}$ towards center (3sf)

Correct equation and evidence for calculating the maximum acceleration.

(ii) The maximum linear velocity of the elevator.



$a = -\omega^2 y$

$\omega^2 = 1.54 \times 10^{-6}$ $\quad \omega = 1.241 \times 10^{-3}$

$v = r\omega$

$v = 1.2141 \times 10^{-3} \times 6.38 \times 10^6 = 7917.37\ ms^{-1}$

$V_{max} = 7920\ ms^{-1}$ (3sf)

Correct equation and evidence for calculating the maximum linear velocity.

$a = -\omega^2 y$

(c) Explain how the equation given shows that the elevator is undergoing simple harmonic motion.

$a = -1.54\times10^{-6} y$

The given equation shows the negative proportionality between a (acceleration of the elevator) and y (the displacement from the equilibrium, the center of earth) thus $a \propto -y$. This means that when a, hence Force, is at maximum, displacement is at minimum (at equilibrium) and when displacement is maximum at the amplitude (the poles), Force and acceleration are minimum, these are the conditions for oscillating simple harmonic motion, thus the elevator is experiencing undamped SHM (no friction).

Correctly explains that acceleration or restoring force is proportional to the displacement but act in the opposite direction to each other using a SHM equation as a reference.

(d) Calculate the time the journey from the North Pole to the South Pole would take.

~~$\omega = \frac{\Delta\theta}{\Delta t}$ $\Delta t = \omega \times \Delta\theta$ $\Delta\theta = \pi$~~

~~$a = -\omega^2 y$ $a = -1.54\times10^{-6} y$ $\therefore \omega^2 = 1.54\times10^{-6}$~~

~~$\omega = \sqrt{1.54\times10^{-6}} = 1.240967 \times10^{-3}$ rad s^{-1}~~

~~$v = 7917.58$ m s^{-1}~~

~~$\Delta t = \sqrt{1.54\times10^{-6}} \times \pi =$~~ ~~$v = A\omega \sin\omega t$~~

$\theta = \pi$ from pole to pole $(\frac{2\pi}{2} = \pi)$

$\omega = \frac{\Delta\theta}{\Delta t}$ $\therefore t = \frac{\theta}{\omega}$

$a = \omega^2 y$ and $a = -1.54\times10^{-6} y$

thus $\omega^2 = 1.54\times10^{-6}$

$\therefore \omega = 1.241\times10^{-3}$ rads^{-1}

$\theta = \pi$

$t = \frac{\theta}{\omega}$ $\therefore$

$t = \frac{\pi}{1.241\times10^{-3}} = 2531.5675$ s

$t = 2530$ s (3sf)

Correct equation,substitution and answer for the time for the journey from North Pole to South Pole.

other working

$v = -A\omega \sin \omega t$ $\omega = \frac{\Delta\theta}{\Delta t}$ $\therefore \omega t = \theta$ $\Rightarrow v = -A\omega \sin\theta$

$\theta = \frac{\pi}{2}$ $\omega^2 = 1.54\times10^{-6}$ $\omega = 1.241\times10^{-3}$

$v = 6.38\times10^6 \times 1.241\times10^{-3} \times \sin(\frac{\pi}{2}) = 7917.58$

$v = 7920$ m s^{-1} (3sf)

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QUESTION THREE: CATS AND GRAVITY

Cats have the ability to orient themselves in a fall, allowing them to avoid many injuries even when dropped upside down. Cats can do this even without tails to help them and they do not need to be rotating first.

The sequence of events for a typical 3.00 kg cat:

- The cat determines which way is up (by rotating its head).
- The cat exerts internal forces to twist the front half of its body to face down (by twisting its spine around its centre of mass and aligning its rear legs).
- Then the cat exerts internal forces to twist the back half of its body to face down (by arching its back).
- The cat lands safely.

The cat can be modelled as a pair of equal mass cylinders (the front and back halves of the cat) linked at the centre of mass of the cat. The moment of inertia, $I \propto mr^2$.

(a) Describe the motion of the centre of mass of the cat during its fall, and explain why the linear momentum of the cat is increasing.

The center of mass of the cat ~~experiences no external forces therefor it will~~ only experiences Fg the force of gravity downwards* which causes a linear acceleration due to gravity since $a = \frac{\Delta v}{\Delta t}$ the cat's velocity downwards is increasing which means it's linear momentum $P = mv$ increases as ~~$\propto$~~ $P \propto v$, when the cats mass is a constant 3kgs $\uparrow v = \uparrow P$

*assumed friction force is negligeble

https://catsnco.files.wordpress.com/2013/02/falling_cat03.jpg

Correct explanation

Considering only the first half of the fall:

With the cat's legs tucked in, the front half of the cat can be modelled as a cylinder of radius 0.060 m. r 2sf

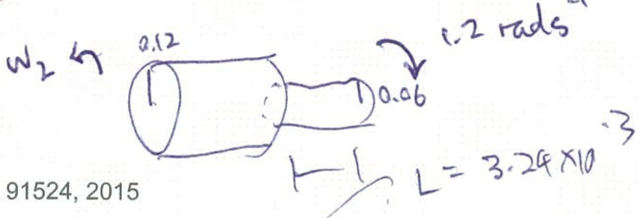
During the first part of the fall the cat uses its muscles to twist its front legs around quickly to reach an angular velocity of 1.20 rad s^{-1}. w

(b) If the angular momentum of the front half of the cat is 3.24×10^{-3} kg m^2 s^{-1}, calculate the rotational inertia of the front half of the cat. L

$L = I\omega \quad \therefore I = \frac{L}{\omega} \quad I = \frac{3.24 \times 10^{-3}}{1.2} = 2.7 \times 10^{-3}$

$I = 2.7 \times 10^{-3}$ kgm² (2sf)

Correct working and answer for the rotational inertia.



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(c) The cat is able to twist the front half of its body, even though the total angular momentum of the cat must remain zero.

Explain why the total angular momentum of the cat must remain zero, and explain what must happen to the rear of the cat's body.

The ~~total~~ angular momentum must remain zero because no external forces, unbalanced torques, act on the cat during the fall. only internal forces turn parts of the cat thus angular momentum is conserved ($L_{rear} - L_{front} = 0$) since ~~I~~ $L = I\omega$, and L is conserved, the cat must increase the radius of ~~it's~~ rear by extending it's legs, this increase in radius increases the ~~mass~~ rotational inertia (I) of the rear half allowing the front half ~~of~~ the cat to increase ω, ~~[illegible]~~ since $I \propto \frac{1}{\omega}$ in $L = I\omega$ when L is conserved

Correctly explains that no external torques act so total angular momentum must be conserved so rear half of cat must rotate but in the opposite direction.

(d) During the first half of its fall, the cat stretches out its rear legs. The rear half of the cat can be modelled as a cylinder of radius 0.120 m. r

Explain how the cat can rotate the front and rear of its body at different speeds.

In your answer you should:

- calculate the angular momentum of the rear half of the cat
- explain why there is a difference in rotational speed between the front half of the cat and the rear half of the cat
- calculate the angular velocity of the rear of the cat.

~~[illegible]~~ $L_{rear} - L_{front} = 0$ when L in conserved

$\therefore L_{rear} = L_{front}$ $\quad L_{rear} = 3.24 \times 10^{-3}$ $\quad mass_{rear} = \frac{3}{2} = 1.5\ kg$ (3sf)

$L = mvr$ ~~[illegible]~~ $\quad L_{rear} = 3.24 \times 10^{-3}\ kgm^2s^{-1}$ (turning opposite front end)

$v = \frac{L}{mr}$ $\quad v = \frac{3.24 \times 10^{-3}}{1.5 \times 0.12}$

$v = 0.018\ ms^{-1}$

$v = r\omega$ $\quad \omega = \frac{v}{r}$ $\quad \therefore \omega = 0.150\ rads^{-1}$ (3sf) (turning opposite front end of cat)

The difference in the rotational speed of the two ~~halfs~~ parts of the cat is due to the difference ~~change~~ in rotational inertia of the rear and front parts whilst conserving ~~[illegible]~~ angular momentum. since their are no external ~~[illegible]~~ angular forces, the net Angular momentum $= 0$ $\Sigma L = 0$ thus the front and rear parts have equall Angular momentum however, the mass of the rear of the cat is distributed further away by extending the legs to increase radius. This mean it has a larger rotational inertia, resistance to $\Delta\omega$, whilst by tucking in it's front legs the cat's front greatly decreases rotational inertia, greatly increa[ses] ω whilst conserving angular momentum since $I \propto \frac{1}{\omega}$ when L is constant ($L = I\omega$)

Correct explanation for Low Excellence. For High Excellence, correct calculation for the angular velocity of the rear of the cat is required with the explanation.

E7

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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Q1 d) $T^2 \propto R^3$

Seen

$r = 5\times10^4 + 1.74\times10^6 = 1790000\text{m}$

distance(orbit) $= 1790000\text{m}$

$v = \frac{d}{t}$ $\quad$ v linear orbit $= \frac{1790000 \times 2\pi}{6.78\times10^3} = 1658.835 \text{ ms}^{-1}$

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