3

SUPERVISOR'S USE ONLY

91524

915240

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 3 Physics, 2015

91524 Demonstrate understanding of mechanical systems

9.30 a.m. Friday 20 November 2015
Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Demonstrate understanding of mechanical systems.	Demonstrate in-depth understanding of mechanical systems.	Demonstrate comprehensive understanding of mechanical systems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Resource Booklet L3–PHYSR.

In your answers use clear numerical working, words and/or diagrams as required.

Numerical answers should be given with an SI unit, to an appropriate number of significant figures.

If you need more room for any answer, use the extra space provided at the back of this booklet.

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Merit

TOTAL 15

ASSESSOR'S USE ONLY

QUESTION ONE: SATELLITES

ASSESSOR USE ONLY

Mass of Earth = 5.97×10^{24} kg

Universal gravitational constant = 6.67×10^{-11} N m^2 kg^{-2}

Digital television in New Zealand can be accessed by using a satellite dish pointed at a satellite in space. The satellite used to transmit the signals appears to stay still above the equator.

The satellite, with a mass of 300 kg, is actually travelling around the Earth in a geostationary orbit at a radius of 4.22×10^7 m from the centre of the Earth.

(a) Name the force that is keeping the satellite in this circular orbit, and state the direction in which this force is acting.

Gravitational force is keeping the satellite in orbit. The ~~dir~~ direction is towards the centre off the earth.

The candidate correctly names the force and direction in which this force acts.

(b) Calculate the force acting on the satellite.

$$F_g = \frac{6.67 \times 10^{-11} \times 300 \times 5.97 \times 10^{24}}{(4.22 \times 10^7)^2}$$

$$= 67.08 = 67.1\text{N}$$

Correct working and answer.

(c) Show that the speed of the satellite is 3.07×10^3 m s^{-1}.

$$F_g = F_c$$

$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$\frac{GM}{r} = v^2$$

$$\sqrt{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{4.22 \times 10^7}} = v$$

Correct equation and evidence for showing the speed of satellite.

$$= 3071.039$$

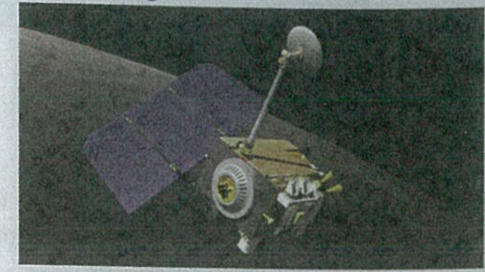
$$= 3.07 \times 10^3 \text{ms}^{-1}$$

ASSESSOR'S USE ONLY

$v = \frac{d}{t}$ $t = \frac{d}{v}$

(d) Kepler's law states that, for any orbiting object, $T^2 \propto r^3$, where r is the radius of the orbit, and T is the time period for the orbit.

NASA uses a robotic spacecraft to map the Moon. The Lunar Reconnaissance Orbiter orbits the Moon at an average height of 50.0×10^3 m with a period of 6.78×10^3 s. The Moon has a radius of 1.74×10^6 m.



https://upload.wikimedia.org/wikipedia/commons/9/95/Lunar_Reconnaissance_Orbiter_001.jpg

Use Kepler's law to estimate the mass of the Moon.

In your answer you should:

- use the relevant formulae to derive Kepler's law
- use Kepler's law to determine the mass of the Moon.

$F_c = F_g$

$\frac{mv^2}{\cancel{r}} = \frac{GM\cancel{m}}{r\cancel{^2}}$

$\frac{v^2}{\cancel{m}} = \frac{GM}{r}$

$\frac{F_c \times r}{m} = \frac{GM}{r}$

$\frac{mv^2}{r} \times r = \frac{GM^2}{r^2}$

$= \cancel{\frac{mv^2}{}} = \cancel{\frac{GM^2}{r^2}}$

$T^2 = \cancel{4\pi} \ \frac{1}{r^3}$

$T = \frac{2\pi \times 4.22 \times 10^7 \times 3.07 \times}{}$

$= 8.14 \times 10^{11}$ s

$\cancel{T^2 = r^3}$

$\cancel{V^2 = 4\pi^2}$

Incomplete derivation of Kepler's Law. For Achieved, attempt using Fg, Fc and V is made. For Merit, correct derivation of Kepler's law is required. For Excellence, correct derivation of Kepler's Law and the correct calculation for the mass of moon is required.

M5

ASSESSOR'S USE ONLY

QUESTION TWO: GRAVITY ELEVATORS

Earth's average radius = 6.38×10^6 m.

In the 2012 science fiction movie *Total Recall*, a gravity-powered elevator called "The Fall" is used to transport passengers between the Northern and Southern hemispheres, straight through the Earth. If a straight tunnel could be dug through the Earth from the North Pole to the South Pole, protected from the heat inside the Earth and the journey unaffected by friction, an elevator could be used, harnessing the gravity of the planet.

Once dropped, the elevator would accelerate downwards and then decelerate once it had passed through the midpoint and – in the absence of friction – would just arrive at the far side of the Earth.

Adapted from: http://www.killerasteroids.org/impact.php

An equation can be used to summarise acceleration of the elevator.

$a = -1.54 \times 10^{-6}\, y$, where y = distance from the midpoint

(a) One of the passengers on the elevator stands on bathroom scales at the start of the journey.

Describe why the bathroom scales read zero.

The passenger has no support force acting on him as he is in freefall because he is not in contact with the ground. This mems he is not exerting a force on the ground hence the scales read zero.

The answer clearly describes why the bathroom scales read zero.

(b) Calculate:

(i) The maximum acceleration of the elevator.

$a = -w^2 y$

$= -1.54 \times 10^{-6} \times 6.38 \times 10^6$

$= 9.83\ ms^{-2}$

Correct equation and evidence for calculating the maximum acceleration.

(ii) The maximum linear velocity of the elevator.

$v = rw$

$= \cancel{6.38} \sqrt{1.54 \times 10^{-6}} \times 6.38 \times 10^6$

$= 7917.37$

$= 7.92 \times 10^3\ ms^{-1}$

Correct equation and evidence for calculating the maximum linear velocity.

(c) Explain how the equation given shows that the elevator is undergoing simple harmonic motion.

The ~~acc~~ acceleration is always proportional to the displacement and since $a = -1.54 \times 10^{-6} y$, the acceleration is proportional to the displacement. It also shows that acceleration is 0 at the midpoint which is a characteristic of simple harmonic motion.

The candidate stated the proportionality relationship between acceleration and displacement with the reference to the given equation as SHM. For Merit, complete explanation that acceleration or restoring force is proportional to the displacement and acts in the opposite direction to each other with reference to SHM equation is required.

ASSESSOR'S USE ONLY

(d) Calculate the time the journey from the North Pole to the South Pole would take.

$w = \sqrt{1.54 \times 10^{-6}}$

$w = 1.24 \times 10^{-3}\ \text{rads}^{-1}$

$w = \frac{2\pi}{T}$

$T = \frac{2\pi}{1.24 \times 10^{-3}}$

$= 5063$

$= \cancel{5060}\ 5.06 \times 10^{3}\ \text{s}$

Correct equation,substitution and answer for a period. To gain Excellence, the candidate needs to half the period to find the time for the journey from North to South Pole.

M6

QUESTION THREE: CATS AND GRAVITY

ASSESSO USE ONL

Cats have the ability to orient themselves in a fall, allowing them to avoid many injuries even when dropped upside down. Cats can do this even without tails to help them and they do not need to be rotating first.

The sequence of events for a typical 3.00 kg cat:

- The cat determines which way is up (by rotating its head).
- The cat exerts internal forces to twist the front half of its body to face down (by twisting its spine around its centre of mass and aligning its rear legs).
- Then the cat exerts internal forces to twist the back half of its body to face down (by arching its back).
- The cat lands safely.

The cat can be modelled as a pair of equal mass cylinders (the front and back halves of the cat) linked at the centre of mass of the cat. The moment of inertia, $I \propto mr^2$.

https://catsnco.files.wordpress.com/2013/02/falling_cat03.jpg

(a) Describe the motion of the centre of mass of the cat during its fall, and explain why the linear momentum of the cat is increasing.

The centre of mass is ~~travelling at a constant velocity~~ accelerating at $9.81\,ms^{-2}$. It is in a constant position as the cat rotates around its centre of mass. The velocity of the cat is increasing as it falls due to gravity and since $P = mv$, and mass is constant, as velocity increases the momentum also increases.

Correct explanation

Considering only the first half of the fall:

With the cat's legs tucked in, the front half of the cat can be modelled as a cylinder of radius 0.060 m.

During the first part of the fall the cat uses its muscles to twist its front legs around quickly to reach an angular velocity of 1.20 rad s^{-1}.

(b) If the angular momentum of the front half of the cat is 3.24×10^{-3} kg m^2 s^{-1}, calculate the rotational inertia of the front half of the cat.

$L = 3.24 \times 10^{-3}\ Kgm^2s^{-1}$ ~~$\times$ 1.2~~

~~$= 3.89 \times 10^{-3}$~~

$I = \frac{3.24 \times 10^{-3}}{1.2} = 2.7 \times 10^{-3}\ Kg^2m^2 rads^{-1}$

Correct working and answer.

(c) The cat is able to twist the front half of its body, even though the total angular momentum of the cat must remain zero.

Explain why the total angular momentum of the cat must remain zero, and explain what must happen to the rear of the cat's body.

Due to the conservation of angular momentum, the angular momentum must remain zero. The rear of the cat's body must ~~would~~ ~~have~~ rotate in the opposite direction in order to keep the angular momentum equal to zero.

Correctly explained that rear half of cat must rotates but in the opposite direction to keep the angular momentm equal to zero. For Merit grade, candidate needs to explain that no external torques act so total angular momentum must be conserved so rear half of cat must rotates but in the opposite direction.

(d) During the first half of its fall, the cat stretches out its rear legs. The rear half of the cat can be modelled as a cylinder of radius 0.120 m.

Explain how the cat can rotate the front and rear of its body at different speeds.

In your answer you should:

- calculate the angular momentum of the rear half of the cat
- explain why there is a difference in rotational speed between the front half of the cat and the rear half of the cat
- calculate the angular velocity of the rear of the cat.

$L = mvr$

$= 1.5 \times 0.12 \times$

$F = mr^2 = 1.5 \times 0.12^2$

$= 0.0216$

The front half of the cat has a different rotational inertia and the back half of the cat has a different inertia ~~hence~~ ~~the~~ and since $E_{K\,ROT} = \frac{1}{2} I w^2$, ~~so~~. the ~~angular~~ rotational speed is different for both.

Incorrect working and explanation.

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A4

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Merit

TOTAL 16

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QUESTION ONE: SATELLITES

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The satellite, with a mass of 300 kg, is actually travelling around the Earth in a geostationary orbit at a radius of 4.22×10^7 m from the centre of the Earth.

(a) Name the force that is keeping the satellite in this circular orbit, and state the direction in which this force is acting.

Centrepetal Force Acting forwards the centre of the Earth F_c ~~Fone~~ is the gravitational attraction Force

The candidate correctly names the force and direction in which this force acts.

(b) Calculate the force acting on the satellite.

$F_g = \frac{GMm}{r^2}$ $\quad F_g = \frac{(6.67\times10^{-11} \times 5.97\times10^{24} \times 300)}{(4.22\times10^7)^2}$

$F_g =$ ~~[illegible]~~ 67.1N (3sf)

Correct working and answer.

(c) Show that the speed of the satellite is 3.07×10^3 m s^{-1}.

$F_c = \frac{mv^2}{r}$ $\quad F_c = F_g \therefore 67.1 = \frac{mv^2}{r}$

$67.1 = \frac{300v^2}{4.22\times10^7}$ $\quad \frac{(67.1 \times 4.22\times10^7)}{300} = v^2$

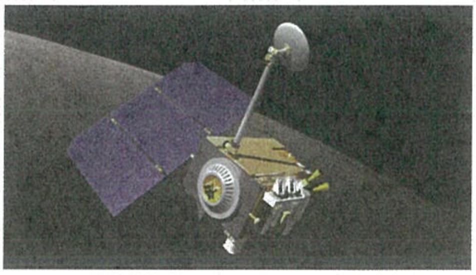
$v^2 = 9438733.33$ $\quad v = 3072.25$

$v = 3070$ (3sf) m/s $= 3.07\times10^3$ ms^{-1}

Correct equation and evidence for showing the speed of satellite.

(d) Kepler's law states that, for any orbiting object, $T^2 \propto r^3$, where r is the radius of the orbit, and T is the time period for the orbit.

NASA uses a robotic spacecraft to map the Moon. The Lunar Reconnaissance Orbiter orbits the Moon at an average height of 50.0×10^3 m with a period of 6.78×10^3 s. The Moon has a radius of 1.74×10^6 m.



https://upload.wikimedia.org/wikipedia/commons/9/95/Lunar_Reconnaissance_Orbiter_001.jpg

Use Kepler's law to estimate the mass of the Moon.

In your answer you should:

- use the relevant formulae to derive Kepler's law
- use Kepler's law to determine the mass of the Moon.

ASSESSOR'S USE ONLY

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$r = 1.74\times10^6 + 50\times10^3 = 1790000\text{ m}$

$M = \frac{mv^2r^2}{rmG}$

$\cancel{T = \frac{d}{v}}$ $\cancel{V = \frac{d}{T}}$ $\cancel{F_c = \frac{m\frac{d^2}{T^2}}{r}}$ $\boxed{M = \frac{v^2 r}{G}}$

$\cancel{F_c = \frac{mT^2}{rd^2}}$ $\cancel{\frac{GMm}{r^2} = \frac{mT^2}{rd^2}}$

~~d = 2π × 1.74×10^6~~

$\cancel{M = \frac{T^2r^2}{rd^2}}$ $\cancel{M = \frac{T^2 r}{d^2}}$

~~10932742.43~~

$\cancel{M = \frac{(6.78\times10^3)^2 \times 1.74\times10^6}{10932742.43^2}}$

~~M =~~ On Back

seen

M5

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

1D) $M = \frac{v^2 r}{G}$ $v = \frac{d}{t}$

$v = \frac{(2\pi \times 1790000)}{6.78 \times 10^3}$

gem

$v = 1658.835 m/s$

$M = \frac{(1658.835^2 \times 1.74 \times 10^6)}{6.67 \times 10^{-11}}$

$M = 7.18 \times 10^{22}$ kg (3sf)

Correct calculation for the mass of moon. For higher grades, derivation of Kepler's Law is required.

91524

QUESTION TWO: GRAVITY ELEVATORS

ASSESSOR'S USE ONLY

Earth's average radius = 6.38×10^6 m.

In the 2012 science fiction movie *Total Recall*, a gravity-powered elevator called "The Fall" is used to transport passengers between the Northern and Southern hemispheres, straight through the Earth. If a straight tunnel could be dug through the Earth from the North Pole to the South Pole, protected from the heat inside the Earth and the journey unaffected by friction, an elevator could be used, harnessing the gravity of the planet.

Once dropped, the elevator would accelerate downwards and then decelerate once it had passed through the midpoint and – in the absence of friction – would just arrive at the far side of the Earth.

Adapted from: http://www.killerasteroids.org/impact.php

An equation can be used to summarise acceleration of the elevator.

$a = -1.54 \times 10^{-6}\, y$, where y = distance from the midpoint

(a) One of the passengers on the elevator stands on bathroom scales at the start of the journey.

Describe why the bathroom scales read zero.

The elevator is falling so there is no reaction force pushing up on the scales. The floor of the elevator is falling downwards at the same rate as the scales and the person therfore the person is not putting any force on the scales as there is no reaction force opposing the falling objects so a sense of weightlessness is felt. //

The answer clearly describes why the bathroom scales read zero.

(b) Calculate:

(i) The maximum acceleration of the elevator.

$a_{max} = A\omega^2$ $\quad a_{max} = 6.38 \times 10^6 \times 1.54 \times 10^{-6}$

$a_{max} = 9.83\ ms^{-2}$ (3sf) //

Correct equation and evidence for calculating the maximum acceleration.

(ii) The maximum linear velocity of the elevator.

$V_{max} = A\omega$ $\quad V_{max} = \left(6.38 \times 10^6 \times \left(\sqrt{1.54 \times 10^{-6}}\right)\right)$

$V_{max} = 7917.37$ m/s

$V_{max} = 7917$ m/s (4sf) //

Correct equation and evidence for calculating the maximum linear velocity.

(c) Explain how the equation given shows that the elevator is undergoing simple harmonic motion.

The acceleration of the elevator is inversly proportional to the displacment of the elevator. ~~the so~~ A restoring force is always acting towards the equilibrium position. The -1.54×10^{-6} shows that the acceleration is in the opposite direction to the displacment. A condition of simple harmonic motion.

ASSESSOR'S USE ONLY

The candidate stated that acceleration acts in the opposite direction to the displacement with reference to the negative from the constant given for the equation. For Merit, complete explanation that acceleration or restoring force is proportional to the displacement and act in the opposite direction to each other with reference to SHM equation.

(d) Calculate the time the journey from the North Pole to the South Pole would take.

$\frac{1}{2}T$

~~T=2π~~

$V_F = V_S + At$

$7917 = 0 + 9.83\,t$

$\frac{2\pi\sqrt{\frac{l}{g}}}{2}$

$\frac{7917}{9.83} = t$ 805.39 s

$t \times 2 = \frac{1}{2}T$

$= 1610.78$ seconds $= 26.85$ minutes (4sf)

Irrelevant equation

M5

QUESTION THREE: CATS AND GRAVITY

ASSESSOR'S USE ONLY

Cats have the ability to orient themselves in a fall, allowing them to avoid many injuries even when dropped upside down. Cats can do this even without tails to help them and they do not need to be rotating first.

The sequence of events for a typical 3.00 kg cat:

- The cat determines which way is up (by rotating its head).
- The cat exerts internal forces to twist the front half of its body to face down (by twisting its spine around its centre of mass and aligning its rear legs).
- Then the cat exerts internal forces to twist the back half of its body to face down (by arching its back).
- The cat lands safely.

The cat can be modelled as a pair of equal mass cylinders (the front and back halves of the cat) linked at the centre of mass of the cat. The moment of inertia, $I \propto mr^2$.

(a) Describe the motion of the centre of mass of the cat during its fall, and explain why the linear momentum of the cat is increasing.

https://catsnco.files.wordpress.com/2013/02/falling_cat03.jpg

The centre of mass is falling straight down. ~~It~~ the path of the centre of mass is not altered while the cat is moving as no external forces are acting on it. The linear momentum of the cat is increasing because it is accelerating towards the ground due to the force of gravity $p = mv$

Candidate correctly states that linear momentum increases because it is accelerating towards the ground due to the force of gravity. To get Merit, candidate needs to explain that the centre of mass accelerates downwards and linear momentum increases due to the external force (weight) causing an increase in vertical velocity.

Considering only the first half of the fall:

With the cat's legs tucked in, the front half of the cat can be modelled as a cylinder of radius 0.060 m.

During the first part of the fall the cat uses its muscles to twist its front legs around quickly to reach an angular velocity of 1.20 rad s^{-1}.

(b) If the angular momentum of the front half of the cat is 3.24×10^{-3} kg m^2 s^{-1}, calculate the rotational inertia of the front half of the cat.

$L = I\omega$ $\quad 3.24 \times 10^{-3} = 1.20 I$ $\quad \dfrac{3.24 \times 10^{-3}}{1.20} = I$

$I = 2.7 \times 10^{-3}$ kgm^2

Correct working and answer.

(c) The cat is able to twist the front half of its body, even though the total angular momentum of the cat must remain zero.

ASSESSOR'S USE ONLY

Explain why the total angular momentum of the cat must remain zero, and explain what must happen to the rear of the cat's body.

The total angular momentum of the cat must remain zero because there are no external forces acting on the cat, therfore there are no ~~towrq~~ torques acting on the cats centre of mass. With no torques the ~~cat~~ ~~total~~ cat cannot angularly accelerate, therfore it must have zero angular momentum. ~~The rear of the cat must move on the other side of the CofM~~ to cancel out the ~~tot~~ overall angular momentium.

Correct explanation

(d) During the first half of its fall, the cat stretches out its rear legs. The rear half of the cat can be modelled as a cylinder of radius 0.120 m.

Explain how the cat can rotate the front and rear of its body at different speeds.

In your answer you should:

- calculate the angular momentum of the rear half of the cat
- explain why there is a difference in rotational speed between the front half of the cat and the rear half of the cat
- calculate the angular velocity of the rear of the cat.

~~ALGEBRUM~~ $I = \frac{1}{2}mr^2$ $I = \frac{1}{2} \times 3 \times 0.120^2$ $I = 0.0216$ ~~kg~~ kgm^2

There is a difference in speed because the front and rear of the cat have different mass distribution - Rotational Inertia's. with different rotational inertias the cat ~~can~~ will have different rotational velocities to create equal and oppisite angular momentium values to cancel each other out $L = I\omega$ if L increase ω must decrease etc.

L rear half must = and oppisite front half ∴ $L = -3.24 \times 10^{-3}\ \text{kgm}^2/\text{s}$

$I = 0.0216\ \text{kgm}^2$ $L = I\omega$ $\frac{-3.24 \times 10^{-3}}{0.0216} = \omega$

$\omega = -0.15\ \text{rads}^{-1}$ $0.15\ \text{rad}^{-1}$ in opposite direction.

M6

The candidate correctly explains the difference between the mass distribution and the rotational inertia, and then relates this to rotational velocity. To gain Excellence, they need to explain that the cat can change both the rotational inertia and the speed of the front and rear end of its body independently. This can be done by changing its mass distribution. By tucking in its legs, the cat's mass is concentrated closer to the axis of rotation so its angular velocity is greater since angular momentum is conserved or vise versa. Correct calculation of the angular velocity of the rear end of the cat is required.