See back cover for an English translation of this cover

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91577M

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

SUPERVISOR'S USE ONLY

Tuanaki, Kaupae 3, 2014

91577M Te whakahāngai i te taurangi o ngā tau matatini hei whakaoti rapanga

9.30 i te ata Rātū 18 Whiringa-ā-rangi 2014
Whiwhinga: Rima

Paetae	Kaiaka	Kairangi
Te whakahāngai i te taurangi o ngā tau matatini hei whakaoti rapanga.	Te whakahāngai i te taurangi o ngā tau matatini mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai i te taurangi o ngā tau matatini mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mehemea e ōrite ana te Tau Ākonga ā-Motu (NSN) kei tō pepa whakauru ki te tau kei runga ake nei.

Whakautua e koe ngā pātai KATOA kei roto i te pukapuka nei.

Whakaaturia ngā mahinga KATOA.

Me mātua riro mai i a koe te pukaiti o ngā Tikanga Tātai me ngā Papatau L3–CALCMF.

Ki te hiahia koe ki ētahi atu wāhi hei tuhituhi whakautu, whakamahia te (ngā) whārangi kei muri i te pukapuka nei, ka āta tohu ai i ngā tau pātai.

Tirohia mehemea kei roto nei ngā whārangi 2–10 e raupapa tika ana, ā, kāore hoki he whārangi wātea.

HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

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PĀTAI TUATAHI

(a) I te mea he tauwehe a $x - 2$ nō $g(x) = x^3 - 2px^2 + px - 5$, kimihia te uara o p ina he tūturu a p.

(b) Mēnā ko $u = 3 - 3\mathrm{i}$, kimihia u^4 ki te āhua $r\,\mathrm{cis}\,\theta$.

(c) Whakaotihia te whārite $x = \sqrt{33 - 4x} + 3$.

ASSESSOR'S USE ONLY

QUESTION ONE

(a) Given that $x - 2$ is a factor of $g(x) = x^3 - 2px^2 + px - 5$, find the value of p where p is real.

(b) If $u = 3 - 3\mathrm{i}$, find u^4 in the form $r\,\mathrm{cis}\,\theta$.

(c) Solve the equation $x = \sqrt{33 - 4x} + 3$.

(d) Whakaotihia te whārite $z^4 = -4k^2\text{i}$, ina he tūturu a k.

Tuhia ō ōtinga ki te āhua pākoki e pā ana ki a k.

(e) Kimihia te whārite o te huanui e whakaahuahia ana e $|z - 1 + 2\text{i}| = |z + 1|$.

MĀ TE KAIMĀKA ANAKE

(d) Solve the equation $z^4 = -4k^2\text{i}$, where k is real.

Write your solutions in polar form in terms of k.

(e) Find the equation of the locus described by $|z - 1 + 2\text{i}| = |z + 1|$.

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PĀTAI TUARUA

MĀ TE KAIMĀKA ANAKE

(a) Whakaotihia te whārite $x^2 - 4x + 16 = 0$.

Tuhia ō otinga ki te āhua o $a \pm \sqrt{b}\,\mathrm{i}$, ina ko a me b he tau whakahau.

(b) Mēnā ko $w = 2\operatorname{cis}\frac{\pi}{3}$, tātaihia w^4.

Whakautua te pātaiki te āhua $a + b\mathrm{i}$, ina he tūturu a a me b.

(c) He otinga a $w = 2 - 3\mathrm{i}$ nō te whārite $3w^3 - 14w^2 + Aw - 26 = 0$, ina he tūturu a A.

Kimihia te uara o te A me ērā atu otinga e rua o te whārite.

ASSESSOR'S USE ONLY

QUESTION TWO

(a) Solve the equation $x^2 - 4x + 16 = 0$.

Give your solutions in the form $a \pm \sqrt{b}\,\mathrm{i}$, where a and b are rational numbers.

(b) Given that $w = 2\operatorname{cis}\frac{\pi}{3}$, find w^4.

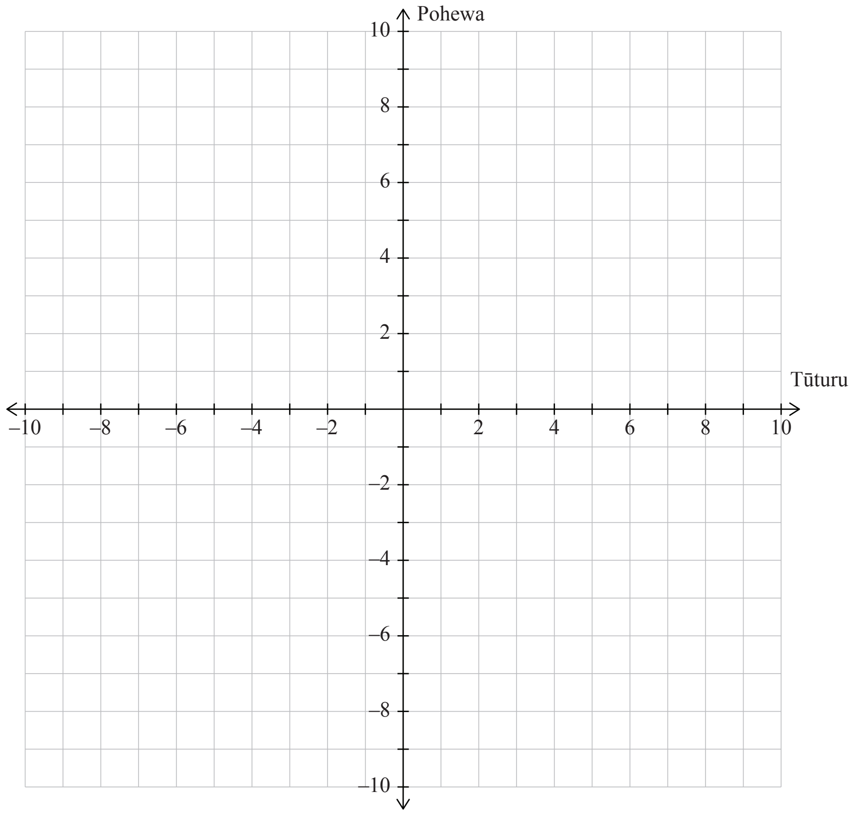
Give your answer in the form $a + b\mathrm{i}$, where a and b are real.

(c) $w = 2 - 3\mathrm{i}$ is a solution of the equation $3w^3 - 14w^2 + Aw - 26 = 0$, where A is real.

Find the value of A and the other two solutions of the equation.

(d) He pai te tau matatini z mō $|z - 3 - 4\mathrm{i}| = 2$.

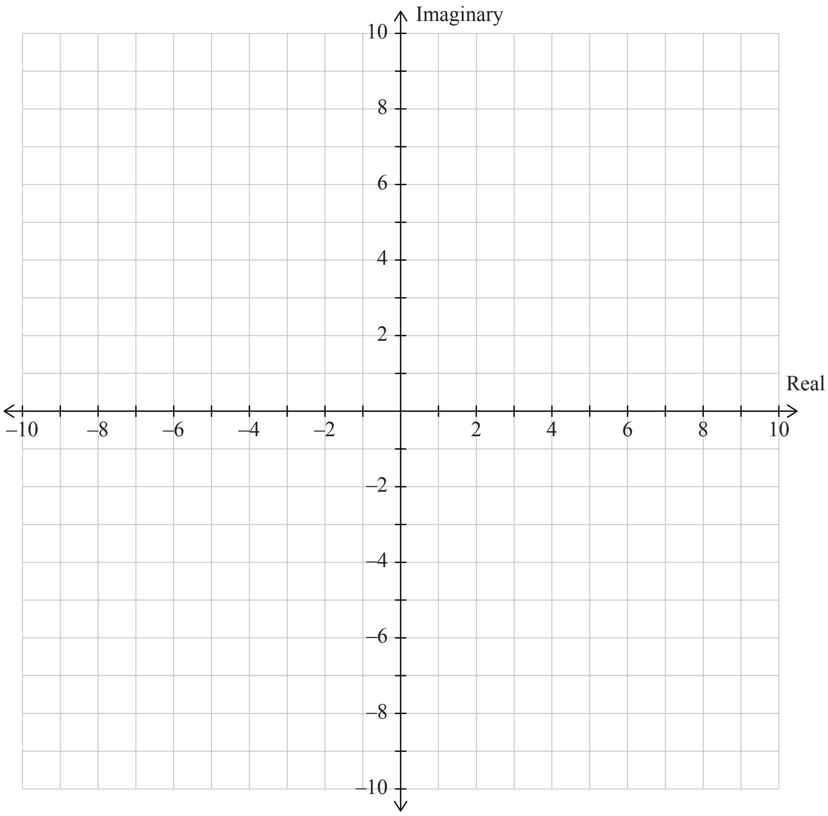
(i) Tāngia te huanui o ngā pūwāhi e tohu ana ko z i te hoahoa Argand i raro



(ii) He aha te uara mōrahi o $\mathrm{Re}(z)$?

(d) A complex number z satisfies $|z - 3 - 4\mathrm{i}| = 2$.

(i) Sketch the locus of points that represents z on the Argand diagram below



(ii) What is the maximum value of $\mathrm{Re}(z)$?

(e) Ko te tau matatini z ko $z = \dfrac{1+3\mathrm{i}}{p+q\mathrm{i}}$, ina ko p me q he tūturu, ā, ko $p > q > 0$.

Mēnā ko $\operatorname{Arg}(z) = \dfrac{\pi}{4}$, whakaaturia ko $p - 2q = 0$.

MĀ TE KAIMĀKA ANAKE

(e) The complex number z is given by $z = \dfrac{1+3\text{i}}{p+q\text{i}}$, where p and q are real and $p > q > 0$.

Given that $\operatorname{Arg}(z) = \dfrac{\pi}{4}$, show that $p - 2q = 0$.

ASSESSOR'S USE ONLY

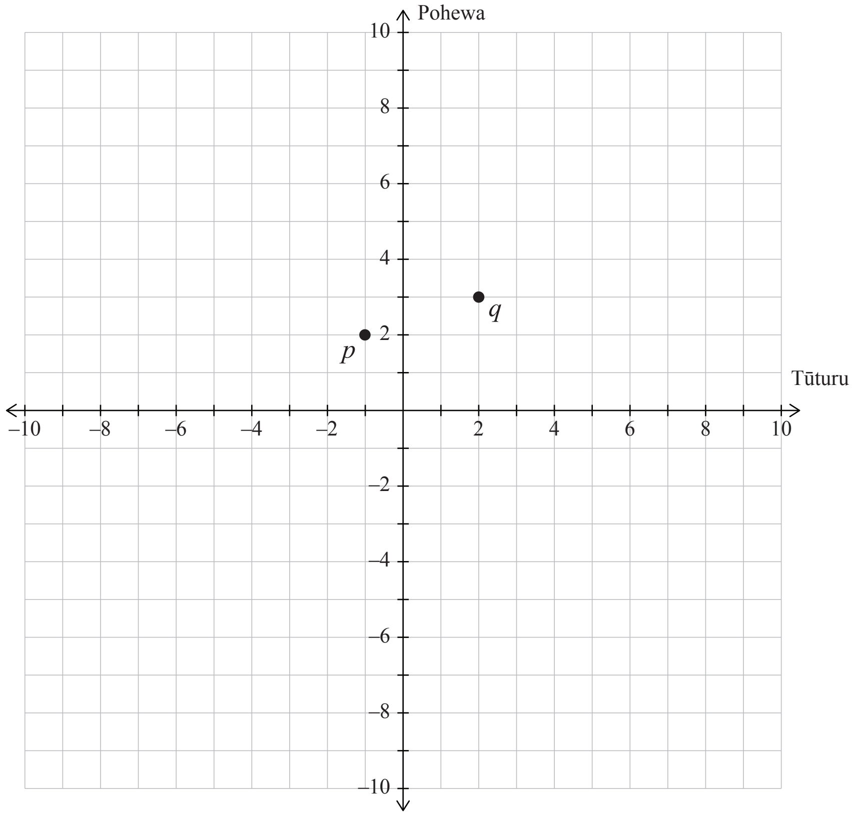
PĀTAI TUATORU

(a) Rohaina, kātahi ka whakamāmā ake ki tōna whānuitanga ka taea, te kīanga e whai ake:

$$\left(2-\sqrt{3}\right)\left(5+2\sqrt{3}\right)\left(4-3\sqrt{3}\right)$$

Whakautua te pātai ki te āhua $a+b\sqrt{3}$, ina ko a me b he tau tūturu.

(b) E tohua ana ngā tau matatini p me q ki te hoahoa Argand i raro.



Mēnā $r = 2p - 3q$, kimihia r ka māka ki te hoahoa Argand i runga ake.

QUESTION THREE

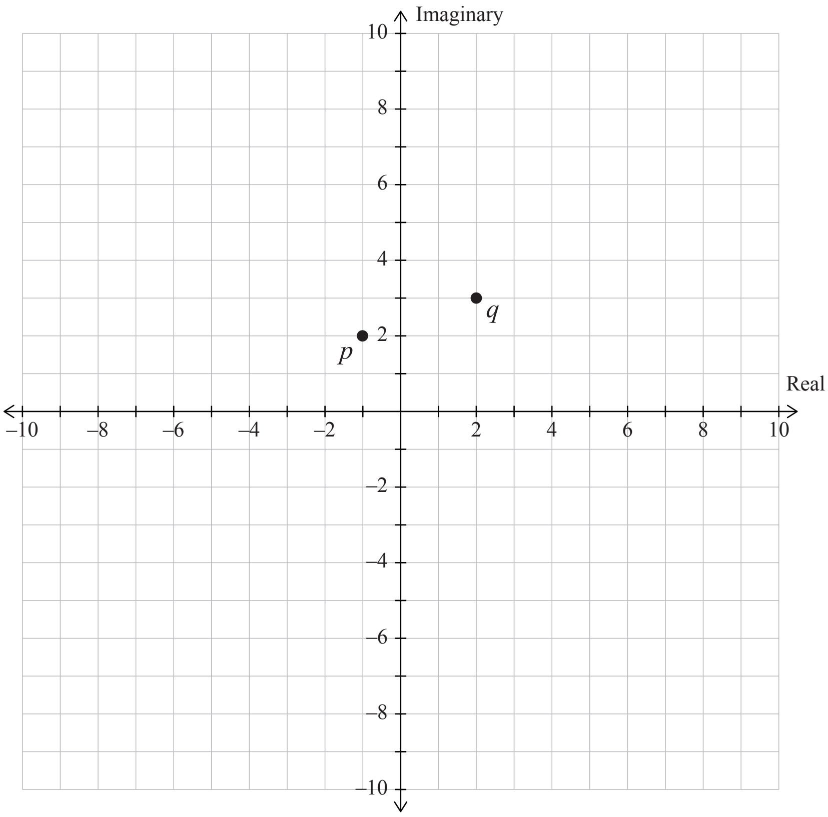
ASSESSOR'S USE ONLY

(a) Expand and simplify as far as possible the following expression:

$$\left(2-\sqrt{3}\right)\left(5+2\sqrt{3}\right)\left(4-3\sqrt{3}\right)$$

Give your answer in the form $a+b\sqrt{3}$, where a and b are real numbers.

(b) The complex numbers p and q are represented on the Argand diagram below.



If $r = 2p - 3q$, find r and mark it on the Argand diagram above.

(c) He aha te uara o p, ina he tūturu a p, **kāore** te kauwhata o $y = px^2 - 4px + 1$ e haukoti i te tuaka-x?

(d) Mēnā ko $z = 3 + 2\mathrm{i}$, kimihia te uara o $\overline{z}^2 + \frac{1}{z^2}$, whakautua te pātai ki te āhua $a + b\mathrm{i}$, ina he tūturu a a me b.

ASSESSOR'S USE ONLY

(c) For what values of p, where p is real, does the graph of $y = px^2 - 4px + 1$ **not** intersect the x-axis?

(d) Given that $z = 3 + 2\text{i}$, find the value of $\overline{z}^2 + \dfrac{1}{z^2}$, giving your answer in the form $a + b\text{i}$, where a and b are real.

(e) α, β, me γ ngā pūtake e toru o te whārite pūtoru $ax^3 + bx^2 + cx + d = 0$, ina he tau tūturu a a, b, c, me d.

(i) Hāponotia ko

$$\alpha + \beta + \gamma = \frac{-b}{a}, \quad \alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}, \quad \alpha\beta\gamma = \frac{-d}{a}$$

(ii) Nō reira hāponotia ko $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2 = \dfrac{bd}{a^2}$

MĀ TE KAIMĀKA ANAKE

(e) α, β, and γ are the three roots of the cubic equation $ax^3 + bx^2 + cx + d = 0$, where a, b, c, and d are real numbers.

(i) Prove that

$$\alpha + \beta + \gamma = \frac{-b}{a}, \quad \alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}, \quad \alpha\beta\gamma = \frac{-d}{a}$$

(ii) Hence prove that $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2 = \dfrac{bd}{a^2}$

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He puka anō mēnā ka hiahiatia.
Tuhia te (ngā) tāu pātai mēnā e hāngai ana.

TAU PĀTAI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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English translation of the wording on the front cover

91577M

Level 3 Calculus, 2014

91577 Apply the algebra of complex numbers in solving problems

9.30 am Tuesday 18 November 2014
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply the algebra of complex numbers in solving problems.	Apply the algebra of complex numbers, using relational thinking, in solving problems.	Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–19 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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