See back cover for an English translation of this cover

# Te Pāngarau me te Tauanga, Kaupae 2, 2021 91267M Te whakamahi tikanga tūponotanga hei whakaoti rapanga 

Ngā whiwhinga: Whā

| Paetae | Kaiaka | Kairangi |
| :--- | :--- | :--- |
| Te whakamahi tikanga tūponotanga hei <br> whakaoti rapanga. | Te whakamahi tikanga tūponotanga mā <br> te whakaaro tūhonohono hei whakaoti <br> rapanga. | Te whakamahi tikanga tūponotanga <br> mā te whakaaro waitara hei whakaoti <br> rapanga. |

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.
Tirohia mēnā kei a koe te Puka Tikanga Tātai L2-MATHMF.
Tuhia ō mahinga KATOA.
Ki te hiahia koe ki ētahi atu wāhi hei tuhituhi whakautu, whakamahia te wāhi wātea kei muri i te pukapuka nei.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2-25 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi ki roto i tētahi wāhi kauruku whakahāngai (\% \% ). Ka tapahia pea tēnei wāhi ina mākahia te pukapuka.

ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

## TŪMAHI TUATAHI

E tino mōhiotia ana a Aotearoa mō tōna ahurea kawhe, ā, kua nui kē atu te kawhe e wātea ana i roto i te 20 tau. Engari, kei te tipu te māharahara kua nui rawa te inu kawhe a ngā taiohi, ā, ko ngā pānga kino pea ko ngā whanonga moe me ngā whakatutukitanga i te kura.
(a) I te māharahara tētahi kura mō tēnei take ka whakahaerehia he rangahau tuihono, kirimuna o ana ākonga pakeke 300. I uia mēnā he rite tonu te inu kawhe a ngā ākonga (i te 3 wā i te wiki i te iti rawa), $\bar{a}$, mēnā kei te pā mai ngā raru moe. Kua whakarāpopototia ngā otinga ki te ripanga i raro nei.

Ripanga 1: Te inu kawhe me ngā raru moe mō ngā ākonga o te kura

|  | Ngā raru moe | Kāore he raru <br> moe | Tapeke |
| :--- | :---: | :---: | :---: |
| He rite tonu te inu kawhe | 68 | 54 | 122 |
| Kāore i te rite tonu te inu <br> kawhe | 75 | 103 | 178 |
| Tapeke | 143 | 157 | 300 |

(i) E ai ki ngā otinga o tēnei rangahau, he aha te ōwehenga o ngā ākonga pakeke he rite tonu te inu kawhe?
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(ii) He aha te tūponotanga mō tētahi ākonga ka tīpako matapōkeretia mai itēnei rangahau he raru moe ōna, $\bar{a}$, he rite tonu te inu kawhe?
(iii) Ka tirotiro a Jack ki ngā raraunga mō ngā ākonga he raru te moe me tana kite he iti iho rātau he rite tonu te inu kawhe. E whakapae ana ia he iti iho te tūponotanga ka raru te moe a te hunga inu kawhe i te hunga kāore e inu kawhe.

Whakamāramahia te whakaaro o Jack, ā, ME parahau he aha e hē ai ia.
Me uru ki tēnei ko ngā tātaitanga tōtika hei tautoko i tō tuhinga.
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## QUESTION ONE

New Zealand is well known for its café culture, and the availability of coffee has increased significantly over the last 20 years. However, there is growing concern that teenagers are consuming more caffeine, with potential negative effects on sleep behaviours and performance at school.
(a) A school was concerned about this issue and conducted an online, anonymous survey of 300 of its senior students. It asked if students drink coffee regularly (at least 3 times a week) and if they have any sleep-related issues. The results are summarised in the table below.

Table 1: Coffee drinking and sleep issues for school students

|  | Sleep issues | No sleep issues | Total |
| :--- | :---: | :---: | :---: |
| Drink coffee regularly | 68 | 54 | 122 |
| Don't drink coffee regularly | 75 | 103 | 178 |
| Total | 143 | 157 | 300 |

(i) According to the results of this survey, what proportion of the senior students drink coffee regularly?
(ii) What is the probability that a randomly selected student from this survey who has sleep issues also drinks coffee regularly?
(iii) Jack looked at the data for students with sleep issues and noticed that fewer of them drink coffee regularly. He claims that this means that it is less likely for coffee drinkers to have sleep issues than those who don't drink coffee.

Explain Jack's reasoning AND justify why he is wrong.
Include appropriate calculations to support your answer.
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Ripanga 1: Te inu kawhe me ngā raru moe mō ngā ākonga o te kura (he mea tuarua mai i te whārangi 2)

|  | Ngā raru moe | Kāore he raru <br> moe | Tapeke |
| :--- | :---: | :---: | :---: |
| He rite tonu te inu kawhe | 68 | 54 | 122 |
| Kāore i te rite tonu te inu <br> kawhe | 75 | 103 | 178 |
| Tapeke | 143 | 157 | 300 |

(b) E tino mōhiotia ana he maha ngā inu ngoi rorotu e inumia ana e ngā taiohi i Aotearoa he nui ake te matū whakakori (caffeine) i roto tēnā i te kawhe.
I pātai taua rangahau anō mō te inu i ngā inu ngoi. I kitea:

- 36 ngā ākonga kāore i te inu kawhe engari he inu i ngā inu ngoi.
- E 7 anake ēnei ākonga (he inu i ngā inu ngoi engari kaua te kawhe) kāore he raru moe.
(i) Whakamahia ēnei meka ME ngā mōhiohio kei te Ripanga 1 hei whakaoti i te ripanga i raro.

Ripanga 2: Pānga o te matū whakakori ki ngā ākonga o te kura

|  | He raru moe | Kāore he <br> raru moe | Tapeke |
| :--- | :---: | :---: | :---: |
| Inu kawhe | 68 | 54 | 122 |
| He inu i ngā inu ngoi (engari kaua te <br> kawhe) |  |  |  |
| Kāore he inu kawhe, ngā inu ngoi rānei |  |  |  |
| Tapeke | 143 | 157 | 300 |

(ii) Mēnā 850 ngā ākonga pakeke i te kura kāore e inu kawhe, ngā inu ngoi rānei, e hia ō rātau e tūmanakohia ana he raru te moe?
(iii) E whakapae ana tētahi tuhipānui nūpepa ko ngā taiohi i Aotearoa he rite tonu te inu ingā inu ngoi he matū whakakori kei roto he rearua ake pea te pā mai o ngā raru moe tēnā i te hunga kāore e inu ana i ngā inu whai matū whakakori.

Matapaki mēnā he rawaka te tuku taunakitanga a te rangahau o tēnei kura hei tautoko i te whaimana o tēnei kerēme.
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Table 1: Coffee drinking and sleep issues for school students (repeated from page 2)

|  | Sleep issues | No sleep issues | Total |
| :--- | :---: | :---: | :---: |
| Drink coffee regularly | 68 | 54 | 122 |
| Don't drink coffee regularly | 75 | 103 | 178 |
| Total | 143 | 157 | 300 |

(b) It is well known that many of the popular energy drinks consumed by teenagers in New Zealand contain caffeine in even higher doses than coffee.
The same survey also asked about the consumption of energy drinks. It found that:

- 36 students didn’t drink coffee but did drink energy drinks.
- Only 7 of these students (who drank energy drinks but not coffee) had no sleep issues.
(i) Use these facts AND the information given in Table 1 to complete the table below.

Table 2: Effect of caffeine on sleep for school students

|  | Have sleep <br> issues | No sleep <br> issues | Total |
| :--- | :---: | :---: | :---: |
| Drink coffee | 68 | 54 | 122 |
| Drink energy drinks (but not coffee) |  |  |  |
| Don't drink either coffee or energy drinks |  |  |  |
| Total | 143 | 157 | 300 |

(ii) If there were 850 senior students in the school who don't drink either coffee or energy drinks, how many would be expected to have sleep issues?
(iii) A newspaper article claimed that teenagers in New Zealand who consume drinks containing caffeine regularly are nearly twice as likely to have sleep-related issues as those who don't consume drinks containing caffeine.

Discuss whether this school's survey provides sufficient evidence to support the validity of this claim.

## TŪMAHI TUARUA

Kua nui ake te hira o "ngā miraka" mai i ngā tipu i Aotearoa i ēnei tau. Whakaratoa anō ai e ngā toa kawhe ngā kōwhiringa maha ki ā rātau kiritaki pērā i te miraka hoi, te miraka aramona, te miraka kokonati rānei.
(a) He toa kawhe tā Jungwoo, ā, kua whakaemihia e ia ngā raraunga o ana hokohoko kawhe i roto i ēnei wiki kua hipa kia mōhio ai ia kia hia te nui o ngā hua miraka kē me whakaputu ia.

- $40 \%$ o ana hokohoko kawhe i hokona ki ngā kiritaki tamariki ake.
- He hauwhā o ngā kiritaki tamariki ake i tono i tētahi momo miraka kē.
- $80 \%$ o ngā kiritaki pakeke ake i ota i tētahi kawhe me te miraka kau.
- O ngā kiritaki tamariki ake i ota i tētahi kawhe me te miraka kē, $15 \%$ o rātau i ota i te miraka hoi.
- He haurua o ngā kiritaki pakeke ake i ota i tētahi kawhe me te miraka kē, i ota i te miraka hoi.
(i) Tātaihia te tūponotanga he tamariki ake tētahi kiritaki i roto i tēnei toa kawhe, ā, ka ota i tētahi kawhe me te miraka kau.
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(ii) Tātaihia te tūponotanga ka otahia e tētahi kiritaki i tīpako matapōkeretia tētahi kawhe me te miraka hoi.
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(iii) Mai i ngā kiritaki i tono miraka kē, he aha te ōrau ka ota i tētahi kawhe me te miraka hoi?
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(iv) Ko te tūponotanga i otahia e tētahi kiritaki i tīpako matapōkeretia tētahi kawhe me te miraka aramona he 0.066 . O rātau i tono i ngā miraka kē, he ōrite anō te ōwehenga o rātau i ota i te miraka aramona i ia awhe taipakeke (tamariki ake, pakeke ake rānei), à, ko ērā atu o ngā kiritaki i ota i tētahi kawhe me te miraka kokonati.

E hia ngā wā he nui ake te tūponotanga ka otahia he kawhe me te miraka kau tēnā i te miraka kokonati?

## QUESTION TWO

The use of plant-based "milks" in New Zealand has become much more popular in recent years. Cafés also now offer their customers a number of alternative milk options such as soy, almond, or coconut milk.
(a) Jungwoo owns a café and has collected data of his coffee sales over the last few weeks to find out how much he should stock alternative milk products.

- $40 \%$ of his coffee sales were to younger customers.
- One quarter of younger customers requested an alternative milk type.
- $80 \%$ of older customers ordered a coffee with cow's milk.
- Of the younger customers who ordered a coffee with alternative milk, $15 \%$ ordered soy milk.
- Half of the older customers who ordered a coffee with alternative milk, ordered soy milk.
(i) Find the probability that a customer in this café is younger and orders a coffee with cow's milk.
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(ii) Find the probability that a randomly chosen customer orders coffee with soy milk.
(iii) Out of the customers who request an alternative milk, what percentage order a coffee with soy milk?
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(iv) The probability that a randomly selected customer ordered an almond milk coffee is 0.066 . Of those who requested alternative milks, there were equal proportions of those ordering almond milk in each age range (younger or older), while the rest of the customers ordered a coffee with coconut milk.

How many times more likely is it for a customer to order a coffee with cow's milk than coconut milk?
(b) He pai ake ki a Kathy te inu i te kawhe mōwai (me te miraka) engari he mate pāwera ia ki te miraka kau, ā, kāore e pai ki a ia te miraka hoi. Ina haere ia ki tētahi toa kawhe mō te wā tuatahi, mēnā kāore he miraka kē, ka ota ia i tētahi kawhe pango roa (me te kore miraka). Ka kite ia he $30 \%$ te wā e pā mai ana tēnei.

I kite ia, o ngā toa kawhe e whakarato ana i ngā miraka kē, tata ki te $40 \%$ he miraka aramona me te miraka kokonati ā rātau. Mēnā he miraka aramona me te kokonati ā te toa kawhe, ka whiua e ia he kapa ka kōwhiri matapōkeretia e ia tētahi.
Ka kite anō a Kathy mēnā kotahi anake te momo miraka kē kore-hoi a tētahi toa kawhe, he rearua ake te tūponotanga he miraka aramona tēnā i te kokonati.


Nō reira, he nui ake te tūponotanga ka whiwhi kawhe pango roa a Kathy, he kawhe mōwai miraka aramona, he kawhe mōwai miraka kokonati rānei?

Tautokona tō tuhinga ki ngā tātaitanga hāngai.
(b) Kathy prefers to drink a flat white coffee (with milk) but is allergic to cow's milk and dislikes soy milk. When she goes to a café for the first time, if it does not have an alternative milk available, she orders a long black coffee (with no milk). She finds this happens about $30 \%$ of the time.
She has found that, out of cafés that offer alternative milks, about $40 \%$ have both almond milk and coconut milk available. If the café has both almond and coconut milk available, she flips a coin and chooses either one randomly.
Kathy has also noticed that if a café has only one non-soy alternative milk available, it is twice as likely to be almond as coconut milk.


Overall, is it more likely for Kathy to have a long black, an almond-milk flat white, or a coconutmilk flat white?

Support your answer with relevant calculations.

## TŪMAHI TUATORU

Ka kite tētahi kaiwhakahaere toa kawhe he rerekē te wā tatari a ngā kiritaki mō ā rātau kawhe hei hari atu. Ka whakaemihia e ia he raraunga i roto i te wiki ka kite ia ko te wā toharite e tatari ana he 7 meneti, me te ine mahora whakatau tata o te 1.5 meneti.

Me kī ka taea te whakamahi he tuari māori hei whakatauira i te wā tatari mō tētahi kawhe hari atu i tēnei toa kawhe.

Me whakaatu rawa ngā mahinga, hoahoa hoki/ranei.
(a) Tātaihia te tūponotanga ka whiwhi tētahi kiritaki i tana kawhe i roto ite 5 meneti.
(b) I tētahi rā, he 150 ngā kiritaki a tētahi toa kawhe i ota kawhe hei hari atu i tētahi wā ahiahi.

E hia o ēnei ki tāu e tūmanako ana ka tatari mō te 8 ki te 10 meneti mō tā rātau kawhe hei hari atu?

## QUESTION THREE

A café manager notices that the amount of time a customer waits to receive their takeaway coffee varies. He collects some data over a week and finds the mean time waited is 7 minutes, with an estimated standard deviation of 1.5 minutes.

Assume that a normal distribution can be used to model the waiting time for a takeaway coffee at this café.

Working and/or diagrams must be shown.
(a) Find the probability that a customer receives their coffee within 5 minutes.
(b) One day, the café has 150 customers who order takeaway coffee over the afternoon period.

How many of these would you expect to have to wait for between 8 and 10 minutes for their takeaway coffee?
(c) I kite te kaiwhakahaere ina rārangitia ana tētahi kaimahi ake, he iti iho ite 5 meneti te wā e tatari ana te $30 \%$ o ngā kiritaki mō ā rātau kawhe. Ko te wā toharite mō te mahi me te whakarato a tēnei kaimahi i te kawhe a te kiritaki he tata ki te 7 meneti tonu.

I kī te kaiwhakahaere ko te tikanga me whiwhi hōnore te kaimahi mō āna mahi pai.
Whakamahia ngā meka i runga ake hei tātai i tētahi whakatau tata o te ine mahora o te wā e whiwhi kawhe ai mai i tēnei kaimahi, $\bar{a}$, ka matapaki mēnā e whakaae ana koe ki te whakaaro a te kaiwhakahaere.
(c) The manager noticed that when a particular staff member was rostered on, 30\% of customers waited less than 5 minutes to receive their coffee. The mean time this staff member takes to make and deliver a customer's coffee is still approximately 7 minutes.

The manager suggested this means the staff member should be rewarded for good service.
Use the facts above to calculate an estimate of the standard deviation of time taken to receive a coffee from this staff member, and discuss if you agree with the manager's suggestion.
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(d) E whakaaturia ana ētahi o ngā raraunga i whakaemihia e te kaiwhakahaere toa kawhe i roto ite wiki kua hipa e ai ki te Hoahoa 1 i raro. ( $n=50$ )

(i) Tātaihia te tau waenga, ngā hauwhātanga, me te inenga whānui i waenga hauwhātanga mō ngā raraunga kei te Hoahoa 1, ā, mō tētahi tauira ā-tuari māori he 7 meneti te toharite me te 1.5 meneti te ine mahora.

Tuhia $\bar{o}$ whakautu ki te ripanga i raro nei.

| Tauanga | Raraunga mai i te Hoahoa 1 | Tauira māori |
| :--- | :--- | :--- |
| Tau waenga |  |  |
| Hauwhā raro |  |  |
| Hauwhā runga |  |  |
| Inenga whānui i <br> waenga hauwhā |  |  |

(d) Some of the data that the café manager collected over the past week are shown in Figure 1 below. ( $n=50$ )

(i) Calculate the median, quartiles, and interquartile range for the data in Figure 1 and for a normal distribution model with mean of 7 minutes and standard deviation of 1.5 minutes. Enter your answers in the table below.

| Statistic | Data from Figure 1 | Normal model |
| :--- | :--- | :--- |
| Median |  |  |
| Lower quartile |  |  |
| Upper quartile |  |  |
| Interquartile range |  |  |

(ii) I te tīmatanga o tēnei tūmahi ka kīia atu koe ka taea te whakamahi te tauira tuari māori (me te 7 meneti te tau toharite me te 1.5 meneti te ine mahora) hei whakatauira i ngā wā tatari mō tētahi kawhe hei hari atu i tēnei toa kawhe.

Kei te tika tēnei whakapae?
Whakamahia ngā taunakitanga tau i tātaitia i te wāhanga (i) ka kōrero mō waenganui, mō te hora, me te hanga o ngā raraunga kei te Hoahoa 1 hei tautoko i tō tuhinga.
(ii) At the start of this question you are told to assume that a normal distribution model (with mean 7 minutes and standard deviation 1.5 minutes) can be used to model the waiting times for a takeaway coffee at this café.

Is this assumption appropriate?
Use the numerical evidence calculated in part (i) and refer to the centre, spread, and shape of the data in Figure 1 to support your answer.

He whārangi anō ki te hiahiatia. Tuhia te (ngā) tau tūmahi mēnā e tika ana.



## English translation of the wording on the front cover

## Level 2 Mathematics and Statistics 2021 91267M Apply probability methods in solving problems

Credits: Four

| Achievement | Achievement with Merit | Achievement with Excellence |
| :--- | :--- | :--- |
| Apply probability methods in solving <br> problems. | Apply probability methods, using <br> relational thinking, in solving problems. | Apply probability methods, using <br> extended abstract thinking, in solving <br> problems. |

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.
Make sure that you have Formulae Sheet L2-MATHMF.
Show ALL working.
If you need more room for any answer, use the extra space provided at the back of this booklet.
Check that this booklet has pages $2-25$ in the correct order and that none of these pages is blank.
Do not write in any cross-hatched area ( $\%$ ) . This area may be cut off when the booklet is marked.

## YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

