91262

NEW ZEALAND QUALIFICATIONS AUTHORITY MANA TOHU MĀTAURANGA O AOTEAROA


# Level 2 Mathematics and Statistics 2022 <br> 91262 Apply calculus methods in solving problems 

Credits: Five

| Achievement | Achievement with Merit | Achievement with Excellence |
| :--- | :--- | :---: |
| Apply calculus methods in solving <br> problems. | Apply calculus methods, using relational <br> thinking, in solving problems. | Apply calculus methods, using extended <br> abstract thinking, in solving problems. |

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

## You should attempt ALL the questions in this booklet.

Show ALL working.
Make sure that you have the Formulae Sheet L2-MATHF.
If you need more room for any answer, use the extra space provided at the back of this booklet.
Check that this booklet has pages $2-16$ in the correct order and that none of these pages is blank.
Do not write in any cross-hatched area (\%). This area may be cut off when the booklet is marked.
YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

## QUESTION ONE

(a) A function $f$ is given by $f(x)=2 x^{4}+4 x^{3}-20 x^{2}-5$.

Use calculus to find the gradient of the graph of the function at the point where $x=3$.
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(b) For the function $f$ :

$$
f^{\prime}(x)=4-6 x+2 x^{2}
$$

The graph of $f(x)$ passes through the point $(3,4)$.
Find the equation of the function $f$.
(c) Use calculus to find the values of $x$ for which the graph of the function $f(x)=\frac{2 x^{3}}{3}+\frac{3 x^{2}}{2}-20 x-3$ is decreasing.
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(d) The tangent to the curve $f(x)=\mathrm{p} x-\mathrm{q} x^{2}$ at the point $(2,-10)$ has a gradient of -6 .

Find the values of the constants p and q .
(e) A new business is designing a logo based on a stylised ' H ' shape, as seen in the diagrams.

The logo is to be designed with two semi-circles and two straight lines. The owner of the new business wants to include space for the company's name inside the shape, so he has asked the designer to maximise the area inside the shape.

The owner plans to build a replica of the logo, and due to material constraints, the total perimeter of the shape must not be greater than 80 cm .

Three examples are shown here:

(i) Find the maximum area.
(ii) Use calculus methods to show that this is a maximum.

Circumference of a circle: $C=2 \pi r$
Area of a circle : $A=\pi r^{2}$

## QUESTION TWO

(a) The gradient graph of a function $y=f(x)$ is shown on the axes below.


Sketch a possible graph of the original function $y=f(x)$ on the axes below.

(b) A cricket match is being played. A cricketer hits the ball, and its height, $h$, above the grass cricket pitch in metres, can be modelled by the function:

$$
h=22.5 t-4.9 t^{2}+1
$$

where $t$ is the time in seconds since the ball was hit.
Using calculus methods, find the maximum height above the ground that the ball reaches.
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(c) (i) During the cricket match, spectators both come and leave the venue. The number of spectators can be modelled by the following equation:

$$
P=100 t^{2}-2 t^{4}+750 \quad(0 \leq t \leq 7.5)
$$

where $P$ represents the number of spectators in attendance, and $t$ represents the time in hours since the start of the match.

Show that the rate of change of spectators is -528 people per hour when $t=6$.
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(ii) What is the meaning of this value for the rate of change?
(iii) At another match, the number of spectators is modelled by the equation $P=k t^{2}-2 t^{4}+750$, where $k$ is a constant.
The number of spectators is growing fastest when $t=4$.
Find the value of $k$.
Explain your reasoning clearly, using correct mathematical statements.
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## QUESTION THREE

(a) Find the equation of the tangent of the curve $y=2 x(x-3)$, where $x=1$.
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(b) The diagrams below show the graphs for the functions of $y=f(x), y=g(x)$, and $y=h(x)$.

(i) On the axes below, sketch the graphs of their respective gradient functions.

(ii) The diagram below shows the graph of the function $y=p(x)$.


On the axes below sketch the graph of the gradient function $y=p^{\prime}(x)$.


If you need to redraw any of your graphs, use the grid on page 13.
(iii) Is it true to say that the gradient of the function $p(x)$ is zero when $x=0$ ?

Justify your response using the graphs on pages 8 and 9 , and/ or mathematical reasoning.
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(c) A calculus class is exploring 3-D geometrical art. The students decide that they want to create wireframe models of a tetrahedron and an octahedron, similar to those shown below.

| Tetrahedron | Octahedron |
| :---: | :---: |
| Surface area $=\sqrt{3} a^{2}$ <br> where $a$ represents the length <br> of each edge of the tetrahedron | Sure area $=2 \sqrt{3} b^{2}$ <br> of each edge of the octahedron |

They will make each edge with a piece of wire, and join each piece together, and then cover each surface with paper to create lanterns similar to those below.


Source: https://timesofindia.indiatimes.com/city/ahmedabad/20-unique-diy-mathematical-lamps-of-paper-by-ccl-iitgn/
They wish to create each model so that the total resulting surface area of the two shapes is a minimum.

They have a total of 180 cm of wire to cut and use for both models.

Determine the lengths of the edges, $a$ and $b$, required to minimise the total surface area of the two shapes when all of the wire is used.

Use calculus to justify that the surface area is a minimum.

## SPARE DIAGRAMS

If you need to redraw your response to Question Two (a), use the grid below. Make sure it is clear which answer you want marked.


If you need to redraw any of your responses to Question Three (b)(i), use the grids below. Make sure it is clear which answers you want marked.


If you need to redraw your response to Question Three (b)(ii), use the grid below. Make sure it is clear which answer you want marked.


Extra space if required. Write the question number(s) if applicable.

Mathematics and Statistics 91262, 2022

Extra space if required. Write the question number(s) if applicable.
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