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91267M



NEW ZEALAND QUALIFICATIONS AUTHORITY MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD KIA NOHO TAKATŪ KI TŌ ĀMUA AO! Tohua tēnei pouaka mēnā KĀORE koe i tuhi kōrero ki tēnei pukapuka



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Te Pāngarau me te Tauanga, Kaupae 2, 2022

91267M Te whakamahi tikanga tūponotanga i te wā e whakaoti rapanga ana

Ngā whiwhinga: E whā

Paetae	Kaiaka	Kairangi
Te whakamahi tikanga tūponotanga i te wā e whakaoti rapanga ana.	Te whakamahi tikanga tūponotanga i te wā e whakaoti rapanga ana, mā roto i te whakaaro whai pānga.	Te whakamahi tikanga tūponotanga i te wā e whakaoti rapanga ana, mā roto i te whakaaro waitara e whānui ana.

Tirohia kia kitea ai e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Whakaaturia ngā whiriwhiringa KATOA.

Tirohia kia kitea ai kei a koe te Pepa Ture Tātai L2-MATHMF.

Ki te hiahia wāhi atu anō koe mō ō tuhinga, whakamahia ngā whārangi kei muri o tēnei pukapuka.

Tirohia kia kitea ai e tika ana te raupapa o ngā whārangi 2–27, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi ki tētahi wāhi e kitea ai te kauruku whakahāngai (<//>
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). Ka poroa pea taua wāhanga ka mākahia ana te pukapuka.

HOATU TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

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HE WAI, HE WAI, I NGĀ WĀHI KATOA ...

ΤΕ ΤŪΜΑΗΙ ΤυΑΤΑΗΙ

I Aotearoa, ka whakapau te tangata i ngā rita toharite nei e 227 o te wai i ia rā, ā, ko te 16 rita te ine mahora. Whakapaetia ka whakatauiratia te whakapaunga wai e tētahi tuari māori.

(a) Whiriwhiria te tūponotanga ka 227 ki te 247 rita te nui o te wai ka whakapaua i te rā e tētahi tangata o Aotearoa ka tīpako matapōkeretia.

(b) (i) Mehemea ka tīpako matapōkeretia ngā tāngata e rua o Aotearoa, he aha te tūponotanga ka iti iho i te 210 rita te nui o te wai ka whakapaua i te rā e rāua tahi?

 (ii) Mehemea ka tīpakohia te tokorua o Aotearoa i tētahi wāhi i Aotearoa e rite tonu ana te pānga o te kopaka o te wai, ka pēhea tā tērā panoni i to whakautu i te wāhanga (i) i runga nei? Āta whakamāramahia o whakaaro.

WATER, WATER, EVERYWHERE ...

QUESTION ONE

In New Zealand, the average person uses a mean of 227 litres of water per day, with a standard deviation of 16 litres. Assume that water consumption can be modelled by a normal distribution.

(a) Find the probability that a randomly chosen New Zealander uses between 227 and 247 litres in a day.

(b) (i) If two New Zealanders are randomly chosen, what is the probability that they both use less than 210 litres per day?

(ii) If the two New Zealanders were both chosen from an area that often experiences water shortages, how would you expect this to change your answer to part (i) above?
 Explain your reasoning clearly.

(c) Kei te noho tahi a Suzanne me tōna whānau, e 3 ngā tamariki, me tana makau. Kei te tihi o ana whakaaro ngā rautaki penapena wai kua tūtohua e te kaunihera, nā konā, ka akiaki ia i tōna whānau kia poto iho ngā uwhiuwhi. E ai ki te kōrero kua pānuitia e Suzanne, e 8 meneti te wā toharite o ngā uwhiuwhi a te tangata o Aotearoa, ā, ko te 2 meneti te ine mahora. Ka ine ia i te roa o ngā uwhiuwhi a tōna whānau i ngā wiki e 2, ā, ka rekoatatia ngā hua i te kauwhata i raro nei.



Ngā wā o ngā uwhiuwhi i tō Suzanne whare

He pēhea ngā hua o te whare o Suzanne ina whakatauritea ki tētahi tauira tuari māori, ko te 8 meneti te toharite, ā, ko te 2 meneti te ine mahora?

Me matapaki koe i ngā āhuatanga e RUA o ēnei: o te pū, o te āhua, o te hora hoki, i tō urupare, ka mutu, me kaua e iti iho i te rua.



(c) Suzanne lives with her family of 3 children and her partner. She is conscious of water-saving efforts suggested by the council, and so encourages her family to have shorter showers. Suzanne reads that the average shower time of New Zealanders is 8 minutes, with a standard deviation of 2 minutes. She times the showers taken by her family over the next 2 weeks and records the results in the graph below.



Shower times in Suzanne's household

How do Suzanne's household results compare to a normal distribution model with a mean of 8 minutes and a standard deviation of 2 minutes?

You should discuss at least TWO of centre, shape, and spread in your response.

(d) I muri i ngā wiki e whā atu anō, ka rere anō te rangahau a Suzanne, ka kitea ai e āhua ōrite ana ngā wā uwhiuwhi i tōna whare ki tētahi tauira tuari māori, e 8 meneti te toharite, ā, e 2 meneti te ine mahora. Hei tā Suzanne, ko te 15% noa iho o ngā wā uwhiuwhi e pai ana.

He aha te wā uwhiuwhi roa katoa e pai ana ki ō Suzanne whakaaro?

(e) Ka whakamahi ngā pakihi i tona hauwhā o te wai i Tāmaki Makaurau. E ai ki te Kaunihera o Tāmaki Makaurau, "Kei raro iho i te 2000 rita te nui o te wai e whakamahia ana e tona 85 orau o ngā pakihi i ia rā."

(i) Mehemea ko te 450 rita te ine mahora o tā te pakihi whakamahi i te wai, tātaia te toharite o te whakamahinga wai a ngā pakihi, i runga i te whakapae e tika ana te tauira tuari māori.

(d) After four more weeks, Suzanne did another survey and found the shower times in her household approximated a normal distribution model, with a mean of 8 minutes and a standard deviation of 2 minutes. Suzanne considers only 15% of the shower times to be acceptable.

What is the longest shower time that Suzanne considers acceptable?

(e) Businesses use about a quarter of Auckland's water. Auckland Council states that, "About 85 per cent of businesses use less than 2000 litres of water per day."

(i) If the standard deviation for business water use is 450 litres, calculate the mean water usage for business customers, assuming that a normal distribution is an appropriate model.

(ii) I kī hoki te Kaunihera "Neke atu ana i te 15 000 rita te wai e whakamahia ana e tētahi wāhanga iti noa o ngā pakihi i Tāmaki Makaurau (1.5%)."

He aha ngā take ka ara ake i tēnei ki tō whakautu i te wāhanga (i), ā, he aha ai?

(ii) The Council also stated "A small number of businesses in Auckland (1.5%) use more than 15 000 litres per day."

What issues does this raise for your answer to part (i), and why?

TE TŪMAHI TUARUA

(a) I whakahaerehia e LAWA (Land (Whenua), Air (Hau Takiwā), Water (Wai) Aotearoa) tētahi rangahau, ā, i whakamātau hoki rātou i ētahi o ngā awa i Aotearoa i ngā momo whenua rerekē kia kitea ai āe rānei e haumaru ana aua wai rā hei wāhi kauhoe. Kua tāpaetia ngā hua o te rangahau i te Tūtohi 1.

Te tūtohi 1: Te maha o ngā awa i ngā momo whenua e whā, e haumaru ana,
e mõrearea ana rānei hei wāhi kauhoe

	Ngā ururua māori	Ngā ururua rāwaho	Te pātītī	Ngā tāone	Te tapeke
Mōrearea ana hei wāhi kauhoe	48	11	424	57	540
Haumaru ana hei wāhi kauhoe	146	15	104	5	270
Te tapeke	194	26	528	62	810

Nō te LAWA River Water Quality 2020 Report ēnei raraunga

(i) Mehemea ka tīpako matapōkeretia tētahi awa, he aha te tūponotanga he awa i te tāone taua awa, ā, e mōrearea ana te awa rā hei wāhi kauhoe?

(ii) Ka nui ake rānei te tūponotanga ka haumaru tētahi awa i ngā ururua māori hei wāhi kauhoe, tēnā i tētahi awa i ngā ururua rāwaho?

E rerekē ana te hautanga tūturu o ngā awa i Aotearoa i ia momo whenua, tēnā i ngā hua o te rangahau kua whakarāpopotohia i te Tūtohi 1. E pēnei kē ana:

- e 48% o ngā awa kei ngā ururua māori
- e 5% kei ngā ururua rāwaho
- e 46% kei ngā wāhi pātītī
- 1% kei ngā tāone.
- (iii) Whakamahia te Tūtohi 1 me ēnei hautanga hei whakatau tata i te ōrau o ngā awa katoa i Aotearoa e mōrearea ana hei wāhi kauhoe.

(iv) E pēhea ana te nui o tō whakapono ka tika tēnei tūponotanga mō ngā awa katoa i Aotearoa?

QUESTION TWO

(a) LAWA (Land, Air, Water Aotearoa) conducted a survey and tested some New Zealand rivers in different land types to see if they were safe for swimming. The results from the survey are presented in Table 1.

	Native vegetation	Exotic forest	Pasture	Urban	Total
Unsafe for swimming	48	11	424	57	540
Safe for swimming	146	15	104	5	270
Total	194	26	528	62	810

Table 1: Numbers of rivers in four land categories which are safe or unsafe for swimming

Data sourced from LAWA River Water Quality 2020 Report

(i) What is the probability that a randomly chosen river out of those tested is an urban river and is unsafe for swimming?

(ii) Is a river in native vegetation more likely to be safe for swimming than a river in an exotic forest area?

The actual proportion of New Zealand rivers in each land type is different to those in the survey summarised in Table 1. In fact:

- 48% of rivers are in native vegetation
- 5% are in exotic forest
- 46% are in pasture areas
- 1% are in urban areas.
- (iii) Use Table 1 and these proportions to estimate the percentage of all New Zealand rivers that are unsafe for swimming.

(iv) How confident do you feel that this probability would be correct for all New Zealand rivers?

(b) Kei te hiahia a Mia rāua ko Joe ki te hararei ki te takiwā o tētahi awa i Aotearoa e haumaru ana hei wāhi kauhoe. Ka kite rāua i tēnei tūtohi e whakarāpopoto ana i ngā awa rorotu i Te Ika a Māui, i Te Wai Pounamu hoki, e rite tonu ana te aroturukihia kia kitea ai mehemea e haumaru ana hei wāhi kauhoe.

Te tūtohi 2: Ngā awa rorotu e haumaru ana, e mōrearea ana rānei hei wāhi kauhoe

	Te Ika a Māui	Te Wai Pounamu
Haumaru ana hei wāhi kauhoe	105	65
Mōrearea ana hei wāhi kauhoe	74	34

Nō lawa.org.nz ēnei raraunga, i a Hānuere, 2022

- (i) Mehemea ka tīpako matapōkeretia tētahi o aua awa e aroturukihia ana, he aha te tūponotanga ka haumaru taua awa hei wāhi kauhoe?
- (ii) Ka titiro a Mia ki ngā raraunga, ā, ka mea atu, "E nui ake ana i te taurua te tūponotanga ka mōrearea tētahi awa i Te Ika a Māui hei wāhi kauhoe, tēnā i tētahi awa i Te Wai Pounamu."

Ka mea atu a Joe, "Ko te 20% **noa iho** te nui ake o te tūponotanga ka mōrearea tētahi awa i Te Ika a Māui hei wāhi kauhoe, tēnā i tētahi awa i Te Wai Pounamu."

Whakamāramahia ngā whakaaro o **ia** tangata, Ā, whakatauria mehemea ka tautokohia e te pāngarau ēnei whakaaro e rua.

Whakamahia ngā tātaihanga e hāngai ana hei tautoko i tō tuhinga.

(b) Mia and Joe want to go on holiday near a swimmable river in New Zealand. They find this summary table of popular river sites in the North and South Island that are monitored regularly to check if they are swimmable.

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	North Island	South Island
Safe for swimming	105	65
Unsafe for swimming	74	34

- (i) What is the probability that a river chosen at random from these monitored sites is safe for swimming?
- (ii) Mia looks at the data and says, "It is more than twice as likely for a river in the North Island to be unsafe for swimming compared to a South Island river."

Joe says, "It's **only** 20% more likely for a North Island river to be unsafe for swimming compared to a South Island river."

Explain **each** person's reasoning AND decide if either reasoning is supported mathematically. Use relevant calculations to support your answer.

TE TŪMAHI TUATORU

(a) I whakahaerehia tētahi rangahau nui i te ipurangi, e uiui ana i ngā tāngata e 2500 o Aotearoa mō ō rātou whakaaro mō te whakamahinga o te wai.

E 87% te tokomaha e kūraruraru ana mō te kōpaka o te wai, ā:

- e 80% te hunga ka ngana ki te penapena i te wai i te nuinga o te wā (pērā i te whakapoto i ngā uwhiuwhi).
- 18% te hunga ka ngana noa iho ki te penapena wai i te wā e herea ana te whakamahinga.
- e 2% kāore e paku ngana ki te penapena i te wai.

O te hunga kāore i te kūraruraru mō te kōpaka o te wai:

- He $\frac{1}{2}$ ka rite tonu te ngana ki te penapena i te wai.
- He $\frac{1}{4}$ ka ngana noa iho ki te penapena wai i te wā e herea ana te whakamahinga.
- Kāore te toenga e paku ngana ki te penapena wai.

Whakamahia ēnei korero hei whakaoti i te hoahoa tūponotanga i raro nei.



Hoahoa 1

- (i) Mehemea ka tīpako matapōkeretia tētahi kaiurupare, he aha te tūponotanga e kūraruraru ana ia mō te kōpaka o te wai, ā, ka rite tonu tana penapena i te wai?
- (ii) O ngā kaiurupare rangahau e 2500, e hia i mea atu "ka penapena wai noa iho rātou i te wā e herea ana te whakamahinga"?

QUESTION THREE

- (a) A large online survey was carried out, asking 2500 New Zealanders for their views about water use.
 87% were concerned about water shortages, and:
 - 80% usually try to save water (such as taking shorter showers).
 - 18% only try to save water when restrictions are in place.
 - 2% never try to save water.

Of those not concerned about water shortages:

- $\frac{1}{2}$ usually try to save water.
- $\frac{1}{4}$ only try to save water when restrictions are in place.
- The rest never try to save water.

Use this information to complete the probability tree below.



- (i) What is the probability that a respondent chosen at random is concerned about water shortages and usually saves water?
- (ii) Of 2500 survey respondents, how many said they "only save water when restrictions are in place"?

(b) I whakaaro ake tētahi kaiako kura tuarua, mehemea rānei ka āhua orite ngā whakaaro o te hunga rangatahi ki ērā i kitea rā i te rangahau ā-ipurangi i whakaahuatia rā i te wāhanga (a). I uiuitia ngā tauira e 200 no ngā tau 9 ki te 13 i ngā kura tuarua e rua o te rohe pātata.

I kitea e ia:

- E 70% o ngā tauira e kūraruraru ana mō te āhuarangi hurihuri.
- O te hunga e kūraruraru ana mō te āhuarangi hurihuri, e 85% e kūraruraru ana hoki mō te kōpaka o te wai.
- E 55% o te hunga **kāore** e kūraruraru ana mō te āhuarangi hurihuri, ka kūraruraru mō te kōpaka o te wai.
- (i) Tāngia he hoahoa tūponotanga mo tēnei tūāhuatanga, ā, whakamahia te hoahoa hei whiriwhiri i te tūponotanga e kūraruraru ana tētahi tauira, ka tīpako matapokeretia, mo te kopaka o te wai.

(b) A secondary school teacher wondered if young people's views would be similar to those seen in the online survey described in part (a). She surveyed 200 students from years 9 to 13 at two local high schools.

She found:

- 70% of students were concerned about climate change.
- Of those who were concerned about climate change, 85% were also concerned about water shortages.
- 55% of those who were **not** concerned about climate change were concerned about water shortages.
- (i) Draw a probability tree of this situation and use it to find the probability that a student chosen at random is concerned about water shortages.

- (ii) I kitea hoki e te kaiako:
 - He taurua te nui ake o te tūponotanga ka penapena wai tētahi tauira, mehemea e kūraruraru ana ia mō te kōpaka o te wai, tēnā i tētahi kāore e kūraruraru ana.
 - He ōrite te tūponotanga ka penapena wai tētahi tauira, ahakoa e kūraruraru ana mō te āhuarangi hurihuri, ahakoa kāore rānei e kūraruraru ana.
 - Ko te 0.5632 te tapeke o te tūponotanga ka penapena wai tētahi tauira i tēnei rangahau.

Whiriwhiria te tūponotanga **kāore** tētahi tauira i te kūraruraru mō te āhuarangi hurihuri, ā, **kāore** ia i te kūraruraru mō te kōpaka o te wai, ā, **kāore** hoki ia i te ngana ki te penapena wai.

Ka rere tonu te Tūmahi Tuatoru i te whārangi e whai ake ana.

- (ii) The teacher also found that:
 - The probability that a student saved water was 2 times greater if they were concerned about water shortages than if they were not concerned.
 - The probability that a student saved water was the same whether they were concerned about climate change or not.
 - The overall probability that a student in this survey saved water was 0.5632.

Find the probability that a student is **not** concerned about climate change, is **not** concerned about water shortages, and does **not** try to save water.

Question Three continues on the next page. (iii) I whakamahi te kaiako i ngā hua o ngā rangahau e rua (i te wāhanga (a) me te wāhanga (b) i ngā whārangi i mua ake nei) ki te kī atu e 25% te iti iho o te tūponotanga ka penapena wai ngā tauira kura tuarua o Aotearoa i ō rātou kāinga, tēnā i te hapori whānui.

Matapakina te houtupu o tēnei korero.

Whakamahia ngā tātaihanga e hāngai ana, me ngā whakaaro tauanga hei tautoko i tō tuhinga.

(iii) The teacher used the findings of both surveys (from part (a) and part (b) on previous pages) to claim that New Zealand secondary school students are about 25% less likely to save water in their homes than the general population.

Discuss the validity of this claim.

Use relevant calculations and statistical considerations to support your answer.

Mathematics and Statistics 91267M, 2022

	He whārangi anō ki te hiahiatia. Tuhia te tau tūmahi mēnā e hāngai ana.	
TE TAU TŪMAHI	fund to taa tantan mond o hangar and.	

QUESTION	Extra space if required. Write the question number(s) if applicable.	

	He whārangi anō ki te hiahiatia. Tuhia te tau tūmahi mēnā e hāngai ana.	
TE TAU TŪMAHI	fund to taa tantan mond o hangar and.	

QUESTION	Extra space if required. Write the question number(s) if applicable.	
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Level 2 Mathematics and Statistics 2022

91267M Apply probability methods in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability methods in solving problems.	Apply probability methods, using relational thinking, in solving problems.	Apply probability methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae Sheet L2–MATHMF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2-27 in the correct order and that none of these pages is blank.

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). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.