



Mana Tohu Mātauranga o Aotearoa New Zealand Qualifications Authority

Level 2 Mathematics and Statistics 2023

91262 Apply calculus methods in solving problems

Credits: Five

| Achievement | Achievement with Merit | Achievement with Excellence |
|---|---|--|
| Apply calculus methods in solving problems. | Apply calculus methods, using relational thinking, in solving problems. | Apply calculus methods, using extended abstract thinking, in solving problems. |

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

91262

Make sure that you have the Formulae Sheet L2–MATHF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (or write in i). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION ONE

(a) A function f is given by $f(x) = 4x^2 - 12x$.

Find the point on the curve where the gradient of f(x) is equal to 4.

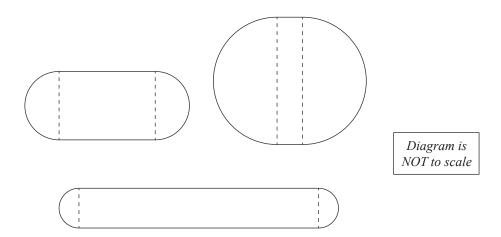
(b) Find the function whose gradient function is $f'(x) = 4x^3 - 6x^2 - 4x$, and which passes through the point (2,-5).

(c) Use calculus to find the values of x for which the graph of the function $\int_{1}^{1} dx$

$$f(x) = \frac{x^4}{4} - \frac{2x^3}{3} - 12x^2 + 10$$
 is increasing.

(d) A school is marking out a 400 m running track on its field. They want to be able to use the infield area (the rectangle between the semi-circles) to run PE games and activities. Due to this, they wish to maximise the rectangular area enclosed by the track. The track must consist of two straight sections and two semi-circular sections, and must be 400 m.

Three examples of possible field configurations are shown below:



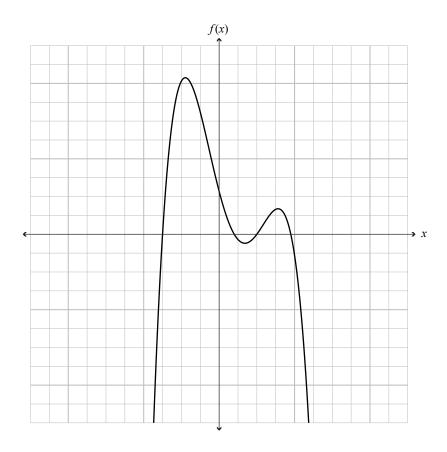
Find the dimensions of the rectangle that maximise its area.

Use calculus methods to show that this is a maximum.

Circumference of a circle: $C = 2\pi r$

QUESTION TWO

The diagram below shows the graph of a function y = f(x). (a)

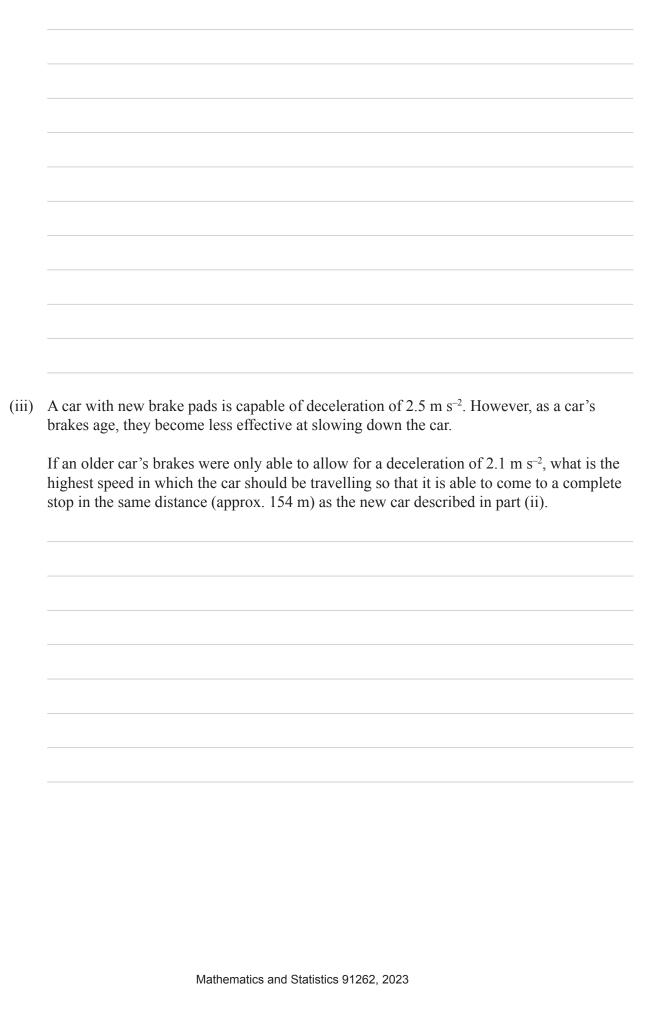


On the axes below, sketch the graph of the gradient function y = f'(x).

(b) Use calculus to show that the line y = 8x + 10 is a tangent to the graph of the function $f(x) = 3x^3 - x + 4$.

- (c) A new car is travelling at 27.78 m s⁻¹ (100 km/hr), when the driver sees an obstruction in the road ahead. The driver applies the brakes, and decelerates at a rate of 2.5 m s⁻².
 - (i) How long will it take the car to come to a complete stop?

(ii) Using calculus methods, show that the distance the car travels between the time the driver applies the brakes, and the car coming to a complete stop, is approximately 154 m.



QUESTION THREE

| (a) | (i) | Find the gradient of the curve $y = 2x^3 - 9x^2 + 12x + 3$ at the point (1,8). | | | | |
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| | (ii) | Find the coordinates of another point on the curve that has the same gradient as in part (i). | | | | |
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| (b) | The | The number of followers of a new social media account can be modelled by: | | | | |
| | | $F = 5t^3 - 840t^2 + 42180t \qquad (0 \le t \le 90)$ | | | | |
| | Whe | are t is days after the account is created, and F is the number of followers. | | | | |
| | (i) | What is the rate of increase of followers on day 5? | | | | |
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What is the maximum number of followers the social media account has during the first (ii) 90 days? (iii) How many days during the first 90 days does the social media account lose followers? Use calculus methods to justify your answer. Question Three continues on the following page.

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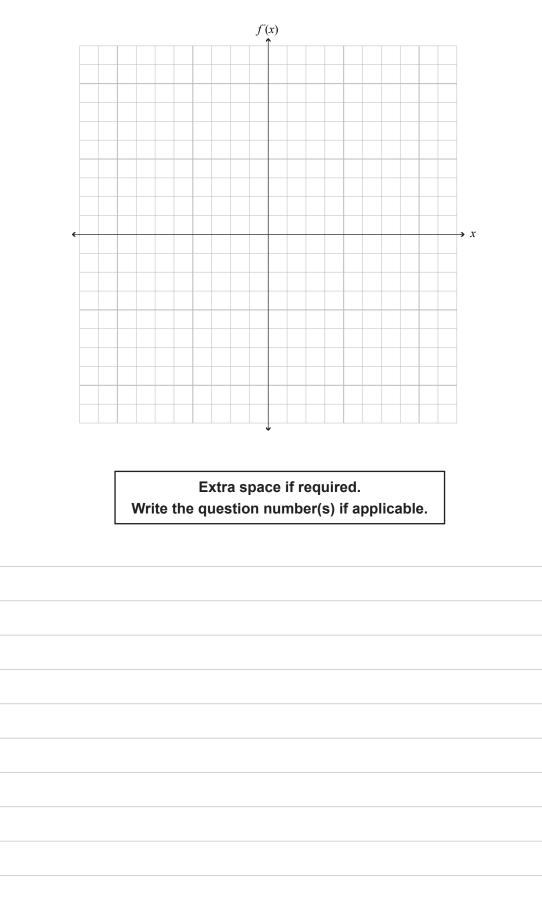
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| | x - 7 has a tangent at $x = 4$ that also crosses the curve at (0,-7). |
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| ind the value of <i>p</i> that makes | this true. |
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SPARE DIAGRAMS

QUESTION NUMBER

If you need to redraw your response to Question Two (a), use the grid below. Make sure it is clear which answer you want marked.



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