



Mana Tohu Mātauranga o Aotearoa New Zealand Qualifications Authority

Level 3 Calculus 2023

91577 Apply the algebra of complex numbers in solving problems

Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply the algebra of complex numbers in solving problems.	Apply the algebra of complex numbers, using relational thinking, in solving problems.	Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

Show ALL working.

91577

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (^{or (Warke Mr)}_{solver (Warke Mr)}). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

AUFATION AND

(a) Write
$$(5-2\sqrt{p})^2$$
 in the form $a + bp + c\sqrt{p}$ where $a, b, and c$ are integers.
(b) Find the value(s) of r so that the quadratic equation $4x^2 - 4x + 3r - 2 = 0$ has no real roots.
(c) If $z = p + qi$ and $w = a + bi$ and $\operatorname{Re}\left(\frac{z}{w}\right) = 0$, then show that $ap = -bq$.

NRITE IN TE IN THIS AR

DONOT WRIT

OT WRITE IN THIS AREA DO NO DO NOT WRITE I O NOT WRITE I

UTWENTE IN THISARD NTHISAREA DONOT WAS AREA DONOT W AREA DONOT W AREA DONOT W NOT WARE IN THI NOT WARE IN THIS WARE IN THISAREA DONOT WARE IN THIS WARE IN THISARD ISARDA DONOT A DONOT WAR A DONOT WAR

IN THIS AREA D WITTENTHIS AR E WITTENTHIS AR E WITTENTHIS AR E WITTENTHIS AR E WITTENTHIS AREA IN AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN ALLEA DO NOT WRITE AREA DO NO

DO NOT WRITE I

TE IN THIS AREA TE IN THIS AREA HIS AREA, DD NC

IN THIS AREA D IN THIS AREA I WRITE IN THIS! I TE IN THIS AREA HISAREA DO NO

S AREA DO NOT DO NOT WRITE IN

WRITE IN THIS A TE IN THIS AREA HIG AREA DO NO

AREA DO NOT SARA DO NOT SARA DO NOT SARA DO NOT WITE IN THIS WITE I

REA DO NOT WA REAL LO NOT WHERE WRITE IN THIS P DO NOT WRITE NOT WRITE IN TH

T WRITE IN THIS I TE IN THIS AREA THIS AREA, DO N T WRITE IN THIS

IN THIS AREA E

(d) One solution of the equation $z^3 - 8z^2 + 6z + d = 0$ is z = 5 - i.

If d is real, find the value of d and the other two solutions of the equation.

The complex numbers *u* and *v* are u = 3 + i and v = 1 + 2i. (e)

Determine the possible value(s) of the real constant k if $\left| \frac{u}{v} + k \right| = \sqrt{k+2}$.

QUESTION TWO

(a) If
$$u = q^6 \operatorname{cis} \frac{5\pi}{8}$$
 and $v = q^2 \operatorname{cis} \frac{2\pi}{5}$, write $\frac{u}{v}$ in the form $r \operatorname{cis} \theta$.

(b) If z = 1 + ki and w = 7 - ki, then find |z - w|, giving your answer in terms of k.

(c) Find
$$\operatorname{Arg}(z)$$
 if $\frac{13z}{z+1} = 11 - 3i$.

6	d 17
0	н 4 С
live the equation $z^3 + 64m^{12} = 0$, where <i>m</i> is a real constant.	U
	17
rite your solution(s) in polar form, in terms of <i>m</i> .); 2 2
	۳ ۲ ۲ ۲ ۲ ۲
	N VE
	T
	N.
	V V 1 1 1 0 0 0
	E
	/ E
	R
	π
	ת ד
	C N
	Τι
	77 H T N T
	IN 1
	N
	d N
	The second se
	i. Pe
	S N
	н
	т Т
	T) jai
	r Her K Her Her Her Her Her Her Her Her Her Her

WRITE IN

O NOT EA DONOT WE WRITE IN THIS P NOT WRITE ITE IN THIS

(d)

(e) The straight line with equation y = mx - 1, where *m* is a real constant and m > 0, is a tangent to the locus described by $|z - 2 + i| = \sqrt{3}$.

Find the Cartesian equation of the locus AND the value of m.

QUESTION THREE

(a) When the polynomial $2x^3 + px^2 + 7x - 3$ is divided by x + 3, the remainder is 30.

Find the value of *p*.

(b) The complex numbers u and v are u = n - i and v = 2 - 3i.

Given that $\frac{u}{v} = 3 + 4i$, find the value of *n*.

(c) Solve the following equation for x, in terms of w. $4\sqrt{(4x-w)} = 5 - 8\sqrt{x}$ (d) Find the values of x and y, given that x and y are real, and $\frac{1}{x + yi} = 1 - \frac{1}{1 + pi}$ giving your answers in terms of p, where p is a positive, real constant.

Question Three continues on the next page. Calculus 91577, 2023

If z + 2i = iz + k, where k is a real number, and $\frac{w}{z} = 2 + 2i$, where Im(w) = 8, find the value of k. (e)

	11
STION	Extra space if required. Write the question number(s) if applicable.
+	

		DO NOT WRI
		NOT WRITE IN OT WRITE IN TE IN THIS PR
	12	TE IN THIS AR
		HIS AREA DO
		DO NOT WRIT
	Extra space if required.	OT WRITE IN TE IN TEIS AF
		PD, Albert MI 1970 PD, ABBA AD PD, ABBA AD PD, ABBA AD PD, ABBA AD PD, ABBA AD PD, ABBA AD N, First AD N, First ABBA N, ABBA AD N, ABBA AD
QUESTION	Write the question number(s) if applicable.	DO'NOT WR
NUMBER		NOTWRITEIN
		VRITE IN THIS
		N THIS ARE
		THIS AREA
		NOT WRITE IN WRITE IN THIS
		N'THIS AREA
		NTHISARD REA DONO DO NOT WRIT OT WRITE IN WRITE IN THIS E IN THISARE SAREA DO I SAREA DO I
		OT WRITE IN
		WRITE IN THIS
		E IN THIS ARD
		IN THIS AREA WRITE IN THIS
		E IN THE NE
		EIN HIBRI ISAREA DO A DONOT OTWRITEIN WRITEIN HIS
		OT WRITE IN
		WRITE IN THIS
		TUIS AREA
		REA DO NOT
		TE IN THIS AR
		TEIN THIS AF
		THIS AREA REA DO NO
		REA DO NOT
		THIS FIN THIS
		DO NOT WRI NOT WRITE II
		TE IN THIS AR
		TE IN THIS AD TE IN THIS AD HIS AREA D WRITE IN TH NTHIS AREA WRITE IN THIS TE IN THIS A
		IN, THIS AREA
		HISAREA D SAREA DO NOTWRITE
		NOT WRITE
		DOMOTIVR
		NOT WRITE IN TH
		TE IN THIS A
		HIS AREA W
		N THIS AREA
		WRITE IN T
		SAREA DO
		DO-NOT WR
		NOT WRITE
		TE IN THIS N
		HIS AREA D
		IN THIS AREA
		HIS AREA D
		IS AREA DO
		HISAREA DO ISAREA DO NOT WRITE [NOT WRITE] WRITE IN TH
		WRITE'
		RITE IN THIS
		NTHISAREN ARËA DO M VITEINITHS UTEINITHS NTEINITHS NTHISAREA ARËA DO N REA DO NO
		RITEINITHS
		HIS AREA D
		N THIS
		AREA DO N
		WRITE IN TH DONOT WR NOT WRITE
		DO NOT WE
		WRITE IN

EIN THIS AND HISAREA DON T WRITE IN THIS IN THIS AREA D