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L3–CALCMF

993208

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Tuanaki, Kaupae 3, 2015

2.00 i te ahiahi Rāapa 25 Whiringa-ā-rangi 2015

TE PUKAPUKA O NGĀ TIKANGA TĀTAI ME NGĀ TŪTOHI mō 91577M, 91578M me 91579M

Tirohia tēnei pukapuka hei whakatutuki i ngā tūmahi o ō Pukapuka Tuhinga, Tūmahi hoki.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–7 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

KA TAEA TĒNEI PUKAPUKA TE PUPURI HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TE TUANAKI – ĒTAHI TURE WHAI HUA

TE TAURANGI

Ngā Whārite Pūrua

Mēnā $ax^2 + bx + c = 0$

kāti $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Ngā Taupū Kōaro

$y = \log_b x \Leftrightarrow x = b^y$

$\log_b(xy) = \log_b x + \log_b y$

$\log_b\left(\dfrac{x}{y}\right) = \log_b x - \log_b y$

$\log_b(x^n) = n \log_b x$

$\log_b x = \dfrac{\log_a x}{\log_a b}$

Ngā Tau Matatini

$z = x + \mathrm{i}y$

$\quad = r \operatorname{cis}\theta$

$\quad = r(\cos\theta + \mathrm{i}\sin\theta)$

$\bar{z} = x - \mathrm{i}y$

$\quad = r \operatorname{cis}(-\theta)$

$\quad = r(\cos\theta - \mathrm{i}\sin\theta)$

$r = |z| = \sqrt{z\bar{z}} = \sqrt{(x^2 + y^2)}$

$\theta = \arg z$

ina $\cos\theta = \dfrac{x}{r}$

ā, $\sin\theta = \dfrac{y}{r}$

Te Ture a De Moivre

Mēnā he tau tōpū a n, kāti,

$(r \operatorname{cis}\theta)^n = r^n \operatorname{cis}(n\theta)$

TE ĀHUAHANGA TAUNGA

Te Rārangi Torotika

Whārite $y - y_1 = m(x - x_1)$

TE TUANAKI

Kimi Pārōnaki

$y = f(x)$	$\dfrac{dy}{dx} = f'(x)$
$\ln x$	$\dfrac{1}{x}$
e^{ax}	$a\mathrm{e}^{ax}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$

Ngā Tikanga Pāwhaitua

$f(x)$	$\int f(x)\,\mathrm{d}x$
x^n	$\dfrac{x^{n+1}}{n+1} + c$ $(n \neq -1)$
$\dfrac{1}{x}$	$\ln\lvert x\rvert + c$
$\dfrac{f'(x)}{f(x)}$	$\ln\lvert f(x)\rvert + c$

Te Pānga Tawhā

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \cdot \frac{\mathrm{d}t}{\mathrm{d}x}$$

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) \cdot \frac{\mathrm{d}t}{\mathrm{d}x}$$

MATHEMATICS – USEFUL FORMULAE

ALGEBRA

Quadratics

If $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Logarithms

$y = \log_b x \Leftrightarrow x = b^y$

$\log_b(xy) = \log_b x + \log_b y$

$\log_b\left(\dfrac{x}{y}\right) = \log_b x - \log_b y$

$\log_b\left(x^n\right) = n\log_b x$

$\log_b x = \dfrac{\log_a x}{\log_a b}$

Complex numbers

$z = x + \mathrm{i}y$

$= r\ \mathrm{cis}\,\theta$

$= r(\cos\theta + \mathrm{i}\sin\theta)$

$\bar{z} = x - \mathrm{i}y$

$= r\ \mathrm{cis}\,(-\theta)$

$= r(\cos\theta - \mathrm{i}\sin\theta)$

$r = |z| = \sqrt{z\bar{z}} = \sqrt{(x^2 + y^2)}$

$\theta = \arg z$

where $\cos\theta = \dfrac{x}{r}$

and $\sin\theta = \dfrac{y}{r}$

De Moivre's Theorem

If n is any integer, then

$(r\ \mathrm{cis}\,\theta)^n = r^n\ \mathrm{cis}\,(n\theta)$

COORDINATE GEOMETRY

Straight Line

Equation $y - y_1 = m(x - x_1)$

CALCULUS

Differentiation

$y = f(x)$	$\dfrac{dy}{dx} = f'(x)$
$\ln x$	$\dfrac{1}{x}$
e^{ax}	ae^{ax}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$
$\cot x$	$-\mathrm{cosec}^2 x$

Integration

$f(x)$	$\int f(x)\,\mathrm{d}x$
x^n	$\dfrac{x^{n+1}}{n+1} + c$ $(n \neq -1)$
$\dfrac{1}{x}$	$\ln\lvert x\rvert + c$
$\dfrac{f'(x)}{f(x)}$	$\ln\lvert f(x)\rvert + c$

Parametric Function

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}t} \cdot \dfrac{\mathrm{d}t}{\mathrm{d}x}$

$\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right) \cdot \dfrac{\mathrm{d}t}{\mathrm{d}x}$

Te Ture mō te Otinga Whakarau[1]

$(f.g)' = f.g' + g.f'$ mēnā rānei $y = uv$ kāti $\dfrac{\mathrm{d}y}{\mathrm{d}x} = u\dfrac{\mathrm{d}v}{\mathrm{d}x} + v\dfrac{\mathrm{d}u}{\mathrm{d}x}$

Te Ture mō te Otinga Wehe

$\left(\dfrac{f}{g}\right)' = \dfrac{g.f' - f.g'}{g^2}$ mēnā rānei $y = \dfrac{u}{v}$ kāti $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Te Ture Pānga Hiato, te Ture Mekameka rānei

$\left(f\left(g\right)\right)' = f'\left(g\right).g'$

mēnā rānei $y = f(u)$ ā, $u = g(x)$ kāti $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}u}\cdot\dfrac{\mathrm{d}u}{\mathrm{d}x}$

NGĀ TIKANGA TAU

Te Ture Taparara

$$\int_a^b f(x)\ \mathrm{d}x \approx \frac{1}{2}h\left[y_0 + y_n + 2(y_1 + y_2 + \ldots + y_{n-1})\right]$$

ina $h = \dfrac{b-a}{n}$ ā, $y_r = f(x_r)$

Te Ture a Simpson

$$\int_a^b f(x)\ \mathrm{d}x \approx \frac{1}{3}h\left[y_0 + y_n + 4(y_1 + y_3 + \ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})\right]$$

ina $h = \dfrac{b-a}{n}$, $y_r = f(x_r)$, ā, he taurua te n.

[1] whakarea

Product Rule

$(f.g)' = f.g' + g.f'$ or if $y = uv$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = u\dfrac{\mathrm{d}v}{\mathrm{d}x} + v\dfrac{\mathrm{d}u}{\mathrm{d}x}$

Quotient Rule

$\left(\dfrac{f}{g}\right)' = \dfrac{g.f' - f.g'}{g^2}$ or if $y = \dfrac{u}{v}$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Composite Function or Chain Rule

$\big(f(g)\big)' = f'(g).g'$

or if $y = f(u)$ and $u = g(x)$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}u}\cdot\dfrac{\mathrm{d}u}{\mathrm{d}x}$

NUMERICAL METHODS

Trapezium Rule

$$\int_a^b f(x)\ \mathrm{d}x \approx \frac{1}{2}h\left[y_0 + y_n + 2(y_1 + y_2 + ... + y_{n-1})\right]$$

where $h = \dfrac{b-a}{n}$ and $y_r = f(x_r)$

Simpson's Rule

$$\int_a^b f(x)\ \mathrm{d}x \approx \frac{1}{3}h\left[y_0 + y_n + 4(y_1 + y_3 + ... + y_{n-1}) + 2(y_2 + y_4 + ... + y_{n-2})\right]$$

where $h = \dfrac{b-a}{n}$, $y_r = f(x_r)$ and n is even.

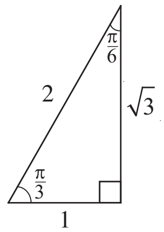
TE PĀKOKI

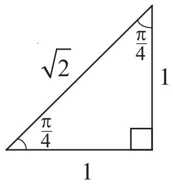
$$\text{cosec}\ \theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$





Te Ture Aho

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Te Ture Whenu

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Ngā Whārite ka Pono Ahakoa ngā Uara Ka Whakaurua Atu

$$\cos^2\theta + \sin^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \text{cosec}^2\theta$$

Ngā Otinga Whānui

Mēnā $\sin\theta = \sin\alpha$ kāti $\theta = n\pi + (-1)^n\alpha$

Mēnā $\cos\theta = \cos\alpha$ kāti $\theta = 2n\pi \pm \alpha$

Mēnā $\tan\theta = \tan\alpha$ kāti $\theta = n\pi + \alpha$

ko te n, he tau tōpū ahakoa

Ngā Koki Hiato

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

Ngā Koki Rearua

$$\sin 2A = 2\sin A\cos A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A\end{aligned}$$

Ngā Otinga Whakarau

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

Ngā Otinga Tāpiri

$$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$$

TE INE

Te Tapatoru

Horahanga $= \frac{1}{2}ab\sin \mathrm{C}$

Te Taparara

Horahanga $= \frac{1}{2}(a+b)h$

Te Pewanga

Horahanga $= \frac{1}{2}r^2\theta$

Te roa o te pewa $= r\theta$

Te Rango

Rōrahi $= \pi r^2 h$

Horahanga mata kōpiko $= 2\pi rh$

Te Koeko

Rōrahi $= \frac{1}{3}\pi r^2 h$

Horahanga mata kōpiko $= \pi rl$ ina ko te l te teitei o te tītaha

Te Poi

Rōrahi $= \frac{4}{3}\pi r^3$

Horahanga mata $= 4\pi r^2$

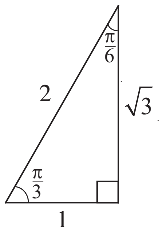
TRIGONOMETRY

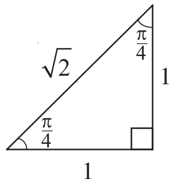
$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$





Sine Rule

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine Rule

$c^2 = a^2 + b^2 - 2ab\cos C$

Identities

$\cos^2\theta + \sin^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$

General Solutions

If $\sin\theta = \sin\alpha$ then $\theta = n\pi + (-1)^n\alpha$

If $\cos\theta = \cos\alpha$ then $\theta = 2n\pi \pm \alpha$

If $\tan\theta = \tan\alpha$ then $\theta = n\pi + \alpha$

where n is any integer

Compound Angles

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

Double Angles

$\sin 2A = 2\sin A\cos A$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A\\ &= 2\cos^2 A - 1\\ &= 1 - 2\sin^2 A\end{aligned}$$

Products

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A\sin B = \sin(A+B) - \sin(A-B)$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

Sums

$\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

$\sin C - \sin D = 2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

$\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

$\cos C - \cos D = -2\sin\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

MEASUREMENT

Triangle

Area $= \dfrac{1}{2}ab\sin C$

Trapezium

Area $= \dfrac{1}{2}(a+b)h$

Sector

Area $= \dfrac{1}{2}r^2\theta$

Arc length $= r\theta$

Cylinder

Volume $= \pi r^2 h$

Curved surface area $= 2\pi rh$

Cone

Volume $= \dfrac{1}{3}\pi r^2 h$

Curved surface area $= \pi rl$ where $l =$ slant height

Sphere

Volume $= \dfrac{4}{3}\pi r^3$

Surface area $= 4\pi r^2$

English translation of the wording on the front cover

L3–CALCMF

Level 3 Calculus, 2015

2.00 p.m. Wednesday 25 November 2015

FORMULAE AND TABLES BOOKLET for 91577, 91578 and 91579

Refer to this booklet to answer the questions in your Question and Answer booklets.

Check that this booklet has pages 2–7 in the correct order and that none of these pages is blank.

YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.