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91578M

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

SUPERVISOR'S USE ONLY

Tuanaki, Kaupae 3, 2015

91578M Te whakahāngai i ngā tikanga pārōnaki hei whakaoti rapanga

2.00 i te ahiahi Rāapa 25 Whiringa-ā-rangi 2015
Whiwhinga: Ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pārōnaki hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pārōnaki mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pārōnaki mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tuhia ō mahinga KATOA.

Tirohia mēnā kei a koe te pukapuka Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Mēnā ka hiahia whārangi atu anō koe mō ō tuhinga, whakamahia ngā whārangi wātea kei muri o tēnei pukapuka, ka āta tohu ai i te tau tūmahi.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–23 kei roto i tēnei pukapuka, ā, kāore tētahi o aua whārangi i te takoto kau.

ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATAHI

MĀ TE KAIMĀKA ANAKE

(a) Whiriwhiria te pārōnaki o $y = 6\tan(5x)$.

(b) Whiriwhirihia te rōnaki o te pātapa ki te pānga $y = (4x - 3x^2)^3$ i te pūwāhi (1,1).

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe hei whakaoti i tēnei rapanga.

(c) Kimihia ngā uara o x e piki ai te pānga $f(x) = 8x - 3 + \dfrac{2}{x+1}$.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe hei whakaoti i tēnei rapanga.

ASSESSOR'S USE ONLY

QUESTION ONE

(a) Differentiate $y = 6\tan(5x)$.

(b) Find the gradient of the tangent to the function $y = (4x - 3x^2)^3$ at the point (1,1).

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) Find the values of x for which the function $f(x) = 8x - 3 + \dfrac{2}{x+1}$ is increasing.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) Mō tēhea, ēhea uara rānei o x ko te pātapa ki te kauwhata o te pānga $f(x)=\dfrac{x+4}{x(x-5)}$ he whakarara ki te tuaka-x?

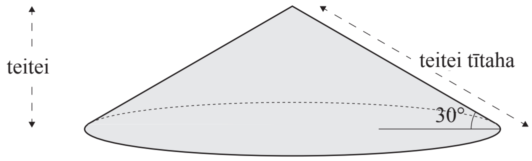
Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe hei whakaoti i tēnei rapanga.

ASSESSOR'S USE ONLY

(d) For what value(s) of x is the tangent to the graph of the function $f(x)=\dfrac{x+4}{x(x-5)}$ parallel to the x-axis?

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) Ina hauhaketia te tote i te Grassmere Saltworks ka puta he koeko i te wā e taka ana i te ara nekeneke.
Ko te tītaha o te koeko he 30° te koki ki te huapae.
Ko te pāpātanga o te whakarato a te ara nekeneke i te tote he 2 m^3 tote i te meneti.



I runga i ngā here manatārua, kāore e whakaaetia te whakaaturanga o tēnei rauemi i konei.

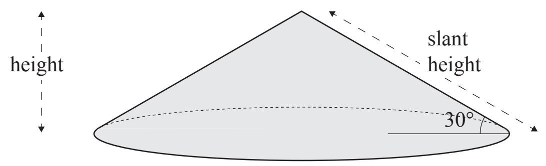
https://bronzblog.files.wordpress.com/2013/07/imgp1182.jpg

Kimihia te pāpātanga e piki ana te teitei tītaha ina he 10 m te pūtoro o te koeke.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe hei whakaoti i tēnei rapanga.

MĀ TE KAIMĀKA ANAKE

(e) Salt harvested at the Grassmere Saltworks forms a cone as it falls from a conveyor belt.
The slant of the cone forms an angle of 30° with the horizontal.
The conveyor belt delivers the salt at a rate of 2 m^3 of salt per minute.



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https://bronzblog.files.wordpress.com/2013/07/imgp1182.jpg

Find the rate at which the slant height is increasing when the radius of the cone is 10 m.

You must use calculus and show any derivatives that you need to find when solving this problem.

ASSESSOR'S USE ONLY

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUARUA

(a) Whiriwhiria te pārōnaki o $f(x)=\sqrt[5]{x-3x^2}$.

(b) Kimihia te rōnaki o te rārangi hāngai ki te ānau $y=x-\dfrac{16}{x}$ ki te pūwāhi ina ko $x = 4$.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe hei whakaoti i tēnei rapanga.

QUESTION TWO

ASSESSOR'S USE ONLY

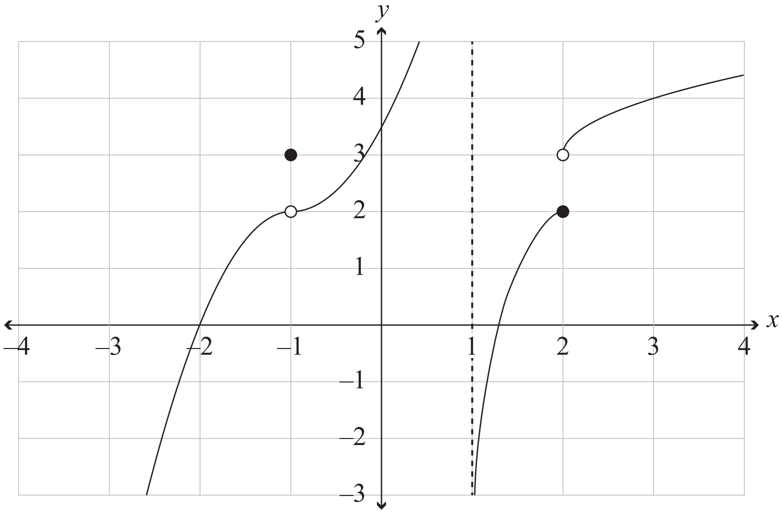
(a) Differentiate $f(x)=\sqrt[5]{x-3x^2}$.

(b) Find the gradient of the normal to the curve $y=x-\dfrac{16}{x}$ at the point where $x = 4$.

You must use calculus and show any derivatives that you need to find when solving this problem.

MĀ TE KAIMĀKA ANAKE

(c) E tohu ana te kauwhata i raro nei i te pānga $y = f(x)$.



Mō te pānga i runga ake:

(i) Kimihia te (ngā) uara mō x e ū ki ēnei whakaritenga e whai ake:

1. kāore i te tautuhia a $f(x)$: ______

2. kāore e taea te kimi pārōnaki mō $f(x)$: ______

3. $f''(x) > 0$: ______

(ii) He aha te uara o $f(-1)$? ______

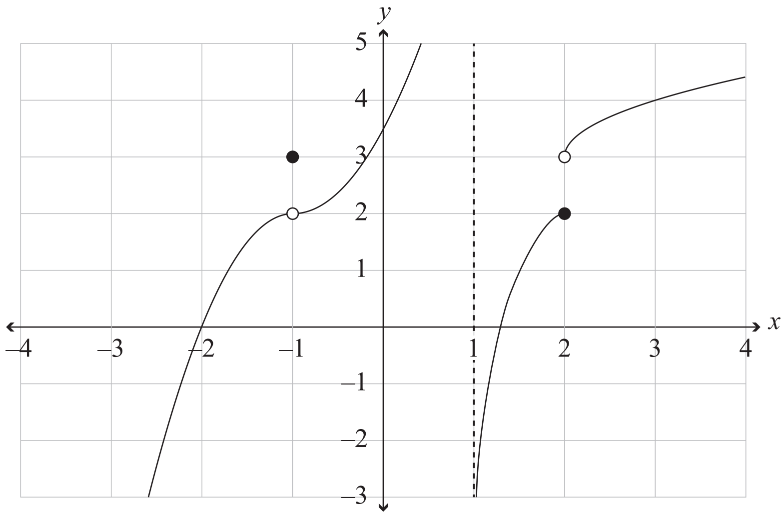
Āta kōrero mai mēnā kāore rawa he uara.

(iii) He aha te uara o $\lim\limits_{x \to 2} f(x)$? ______

Āta kōrero mai mēnā kāore rawa he uara.

(c) The graph below shows the function $y = f(x)$.

ASSESSOR'S USE ONLY



For the function above:

(i) Find the value(s) of x that meet the following conditions:

1. $f(x)$ is not defined: ______

2. $f(x)$ is not differentiable: ______

3. $f''(x) > 0$: ______

(ii) What is the value of $f(-1)$? ______

State clearly if the value does not exist.

(iii) What is the value of $\lim\limits_{x \to 2} f(x)$? ______

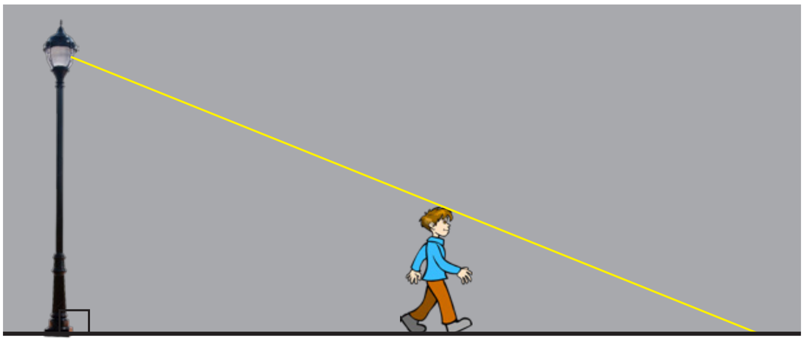
State clearly if the value does not exist.

(d) He 5 m i runga ake i te papa tētahi tūrama tiriti, ā, he papatahi te whenua.

Kei te whakahipa atu mā raro tētahi tama 1.5 m te tāroaroa, mai i te pūwāhi i raro tonu i te tūrama tiriti i te 2 mita i te hēkona.

He aha te pāpātanga e huri ana te roa o tana ātārangi ina tae atu ia ki te 8 m te tawhiti mai i te pūwāhi i raro tonu i te tūrama?

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe hei whakaoti i tēnei rapanga.

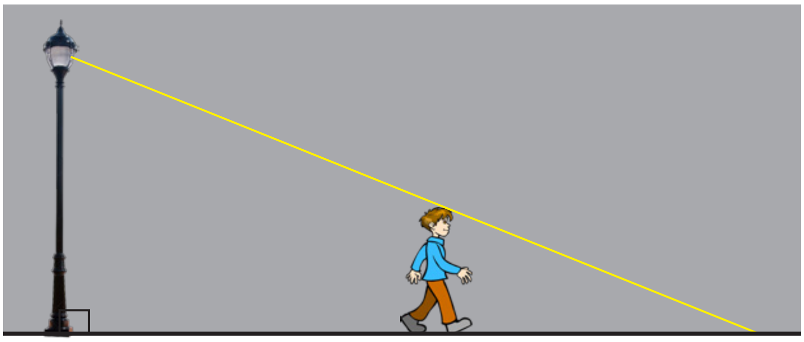


(d) A street light is 5 m above the ground, which is flat.

A boy, who is 1.5 m tall, is walking away from the point directly below the streetlight at 2 metres per second.

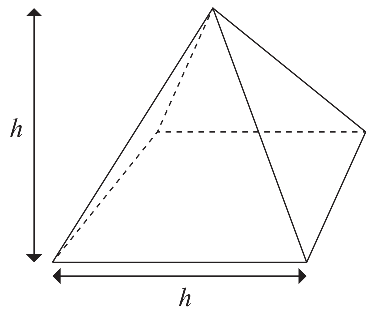
At what rate is the length of his shadow changing when the boy is 8 m away from the point directly under the light?

You must use calculus and show any derivatives that you need to find when solving this problem.



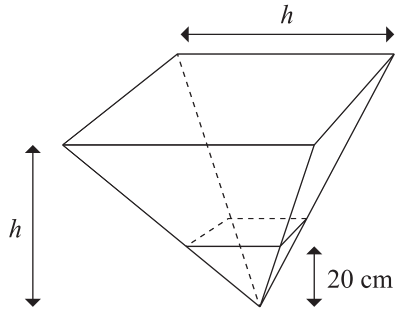
ASSESSOR'S USE ONLY

(e) Ka hangaia tētahi ipu wai ki te āhua o tētahi koeko-tapawhā rite. He ōrite te teitei o te koeko ki te roa o ia taha o tana pūtake.



Ka tapahia mai i runga o te koeko te teitei poutū o te 20 cm, ā, ka tāpirihia a runga papatahi hou.

Ka huria kōarotia te koeko, ā, ka putua atu he wai ki roto ki te pāpātanga o te 3000 cm^3 ia meneti.

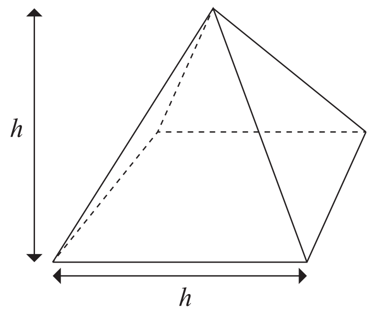


Kimihia te pāpātanga e piki ana te horahanga o te mata o te wai ina he 15 cm te hōhonu o te wai.

$$\text{Rōrahi o te koeko} = \frac{1}{3} \times \text{horahanga pūtake} \times \text{teitei}$$

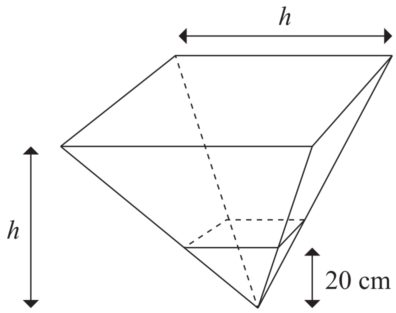
Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe hei whakaoti i tēnei rapanga.

(e) A water container is constructed in the shape of a square-based pyramid. The height of the pyramid is the same as the length of each side of its base.



A vertical height of 20 cm is then cut off the top of the pyramid, and a new flat top added.

The pyramid is then inverted and water is poured in at a rate of 3000 cm^3 per minute.



Find the rate at which the surface area of the water is increasing when the depth of the water is 15 cm.

$$\text{Volume of pyramid} = \frac{1}{3} \times \text{base area} \times \text{height}$$

You must use calculus and show any derivatives that you need to find when solving this problem.

ASSESSOR'S USE ONLY

TŪMAHI TUATORU

(a) Mō tēhea, ēhea uara rānei o x ko te pātapa ki te kauwhata o te pānga $f(x) = 5\ln(2x - 3)$ he 4 te rōnaki?

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe hei whakaoti i tēnei rapanga.

(b) Mēnā $f(x) = \dfrac{x}{e^{3x}}$, kimihia te (ngā) uara o x kia puta ko $f'(x) = 0$.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe hei whakaoti i tēnei rapanga.

(c) Ka tautuhia ā-tawhā tētahi ānau mā ngā whārite $x = 3\cos t$ me $y = \sin 3t$.

Kimihia te rōnaki o te rārangi hāngai ki te ānau i te pūwāhi ina ko $t = \dfrac{\pi}{4}$.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe hei whakaoti i tēnei rapanga.

ASSESSOR'S USE ONLY

QUESTION THREE

(a) For what value(s) of x does the tangent to the graph of the function $f(x) = 5\ln(2x-3)$ have a gradient of 4?

You must use calculus and show any derivatives that you need to find when solving this problem.

(b) If $f(x) = \dfrac{x}{e^{3x}}$, find the value(s) of x such that $f'(x) = 0$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) A curve is defined parametrically by the equations $x = 3\cos t$ and $y = \sin 3t$.

Find the gradient of the normal to the curve at the point where $t = \dfrac{\pi}{4}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

MĀ TE KAIMĀKA ANAKE

(d) Ko te whārite mō te nekehanga o tētahi korakora ka tukuna mā te whārite pārōnaki

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\mathrm{k}^2 x$$

ina ko x te pananga o te korakora mai i te pūtaketanga i te wā t, ā, ko k te aumou tōrunga.

(i) Me whakaatu ko $x = \mathrm{A}\cos \mathrm{k}t + \mathrm{B}\sin \mathrm{k}t$, ina ko A me B ngā aumou, he otinga ki te whārite nekehanga.

(ii) I te pūtaketanga te korakora i te tuatahi, ā, e neke ana i te tere o te 2k.

Kimihia ngā uara o A me B i te otinga $x = \mathrm{A}\cos \mathrm{k}t + B\sin \mathrm{k}t$.

(d) The equation of motion of a particle is given by the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\mathrm{k}^2x$$

where x is the displacement of the particle from the origin at time t, and k is a positive constant.

(i) Show that $x = \mathrm{A}\cos \mathrm{k}t + \mathrm{B}\sin \mathrm{k}t$, where A and B are constants, is a solution of the equation of motion.

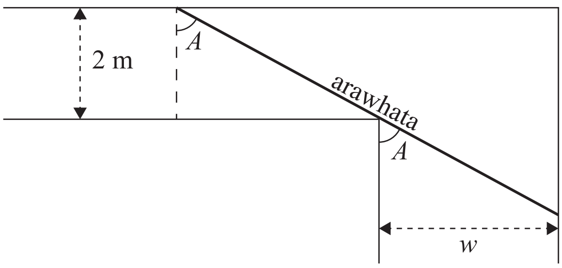
(ii) The particle was initially at the origin and moving with velocity 2k.

Find the values of A and B in the solution $x = \mathrm{A}\cos \mathrm{k}t + B\sin \mathrm{k}t$.

ASSESSOR'S USE ONLY

(e) He 2 m te whānui o tētahi kauhanga.

I te pito ka huri i te 90° ki tētahi atu kauhanga.



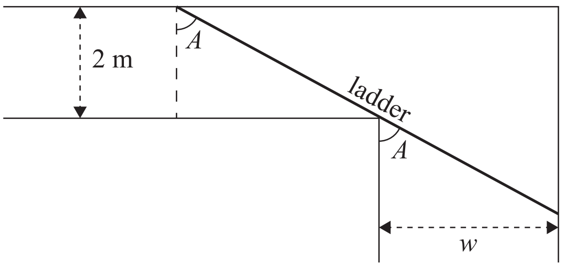
He aha te whānui iti rawa, w, o te kauhanga tuarua mēnā ka taea tētahi arawhata 5 m te roa te heri huapae huri i te kokonga?

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe hei whakaoti i tēnei rapanga.

MĀ TE KAIMĀKA ANAKE

(e) A corridor is 2 m wide.

At the end it turns 90° into another corridor.



What is the minimum width, w, of the second corridor if a ladder of length 5 m can be carried horizontally around the corner?

You must use calculus and show any derivatives that you need to find when solving this problem.

ASSESSOR'S USE ONLY

He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

English translation of the wording on the front cover

Level 3 Calculus, 2015

91578M Apply differentiation methods in solving problems

2.00 p.m. Wednesday 25 November 2015
Credits: Six

91578M

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–23 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.