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91579M

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

SUPERVISOR'S USE ONLY

Tuanaki, Kaupae 3, 2015

91579M Te whakahāngai i ngā tikanga pāwhaitua hei whakaoti rapanga

2.00 i te ahiahi Rāapa 25 Whiringa-ā-rangi 2015
Whiwhinga: Ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pāwhaitua hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pāwhaitua mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pāwhaitua mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tuhia ō mahinga KATOA.

Tirohia mēnā kei a koe te pukapuka Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Mēnā ka hiahia whārangi atu anō koe mō ō tuhinga, whakamahia ngā whārangi wātea kei muri o tēnei pukapuka, ka āta tohu ai i te tau tūmahi.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–27 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATAHI

(a) Whiriwhiria a $\int\left(\sqrt{x}+6\cos 2x\right)\mathrm{d}x$.

(b) Whakaotihia te whārite pārōnaki $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{x}$, ina ko $x = 1$, kāti ko $y = 3$.

(c) Mēnā ko $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{e}^{2x}}{4y}$ me $y = 4$ ina ko $x = 0$, kimihia te uara o y ina ko $x = 2$.

QUESTION ONE

ASSESSOR'S USE ONLY

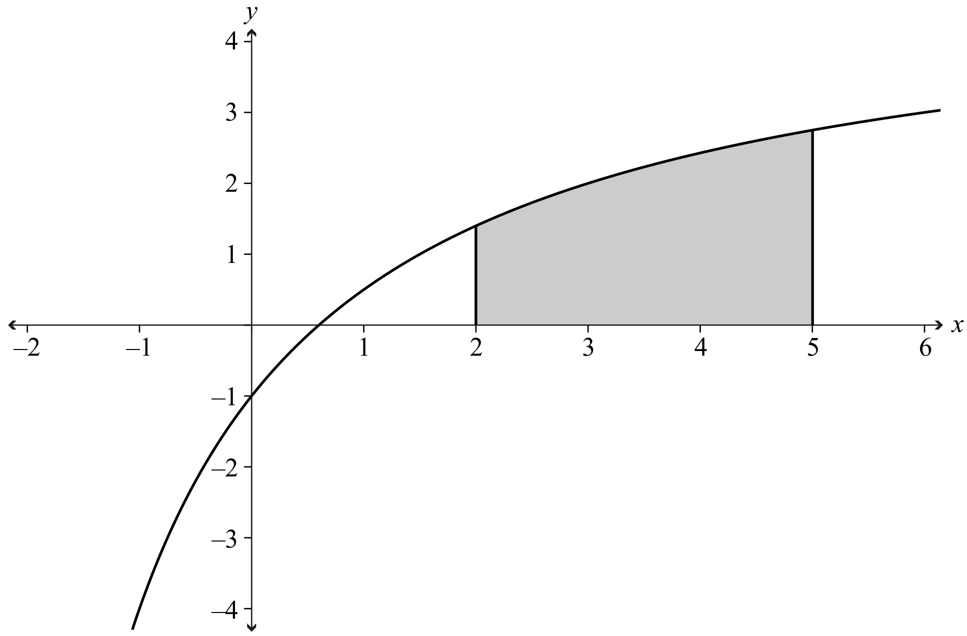
(a) Find $\int\left(\sqrt{x}+6\cos 2x\right)\mathrm{d}x$.

(b) Solve the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{x}$, given that when $x = 1$, $y = 3$.

(c) If $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{e}^{2x}}{4y}$ and $y = 4$ when $x = 0$, find the value of y when $x = 2$.

(d) Whakamahia te tikanga pāwhaitua hei tātai i te horahanga e rohea ana e te ānau $y = \dfrac{5x-3}{x+3}$ me ngā rārangi $y = 0$, $x = 2$ me $x = 5$.

Ka whakaaturia kaurukitia te horahanga i te hoahoa i raro nei.



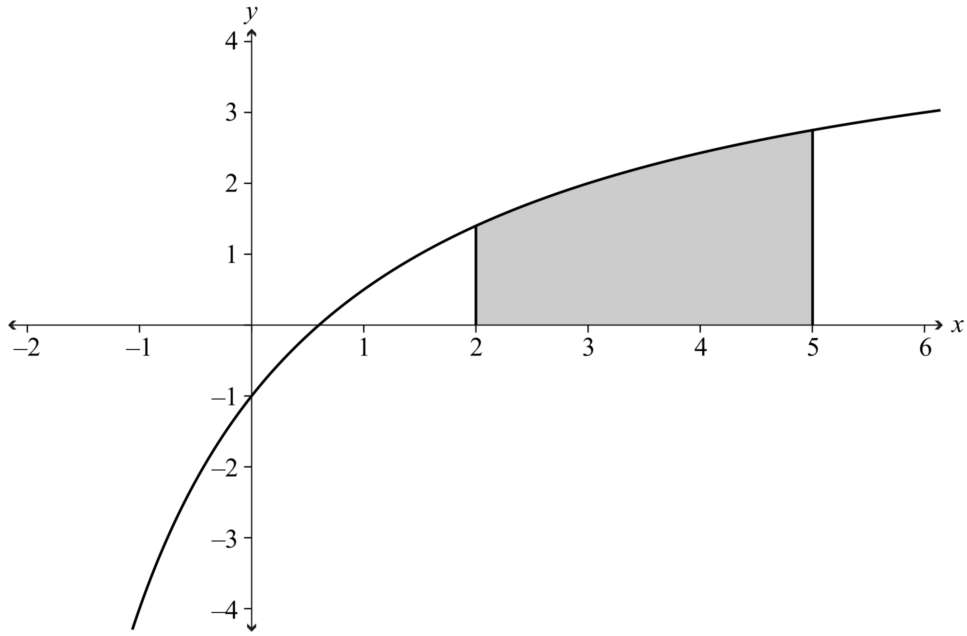
Whakaaturia ō mahinga katoa.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

MĀ TE KAIMĀKA ANAKE

(d) Use integration to find the area enclosed between the curve $y = \dfrac{5x-3}{x+3}$ and the lines $y = 0$, $x = 2$ and $x = 5$.

The area is shown shaded in the diagram below.



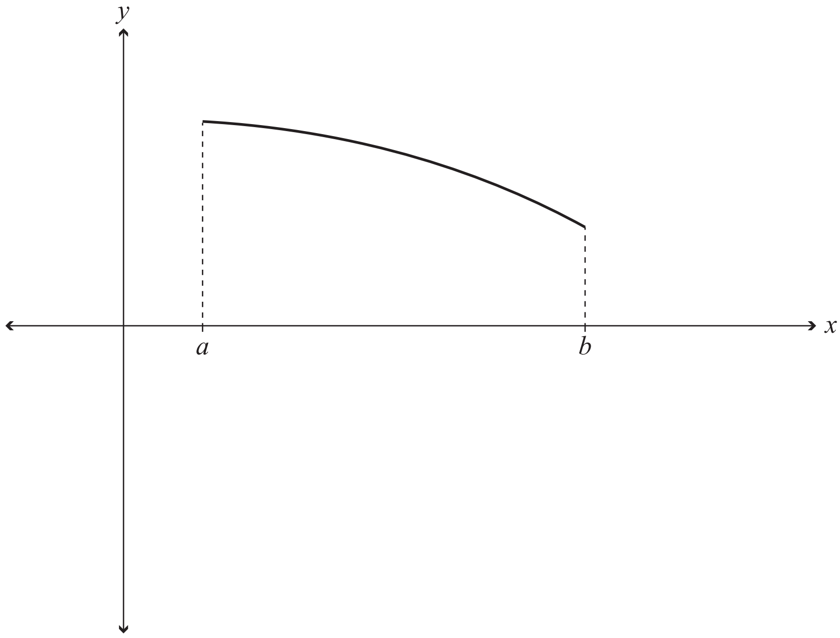
Show your working.

You must use calculus and give the results of any integration needed to solve this problem.

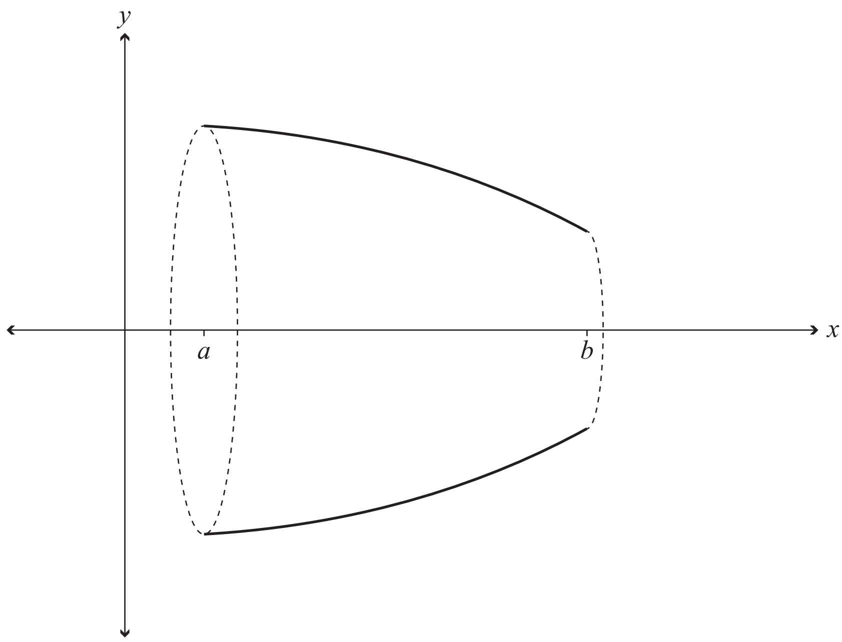
ASSESSOR'S USE ONLY

(e)

Me kī ka tautuhia te ānau mā te pānga y = $f(x)$, ka rohea mā te $x = a$ me $x = b$.



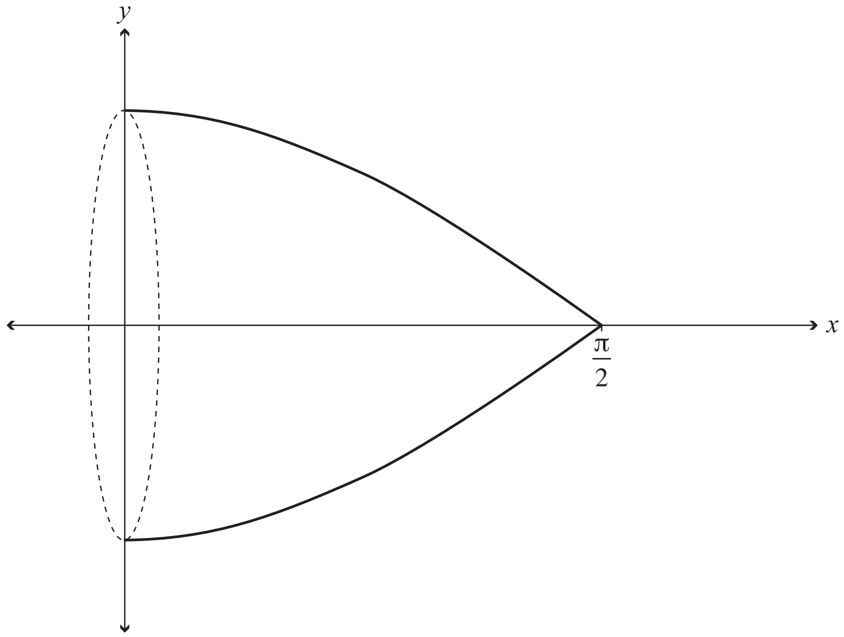
Ka hurihuria tēnei wāhanga o te ānau i te tuaka-x, e ai ki raro nei.



Ko te rōrahi ka puta i tēnei hurihuringa ka tukuna mā te tātai

$$\text{Rōrahi} = \pi\int_a^b \left(f(x)\right)^2 \mathrm{d}x$$

E whakaatu ana te kauwhata i raro nei i te pānga $y = \cos x$, i waenga $x = 0$ me $x = \frac{\pi}{2}$, e hurihuri ana i te tuaka-x.

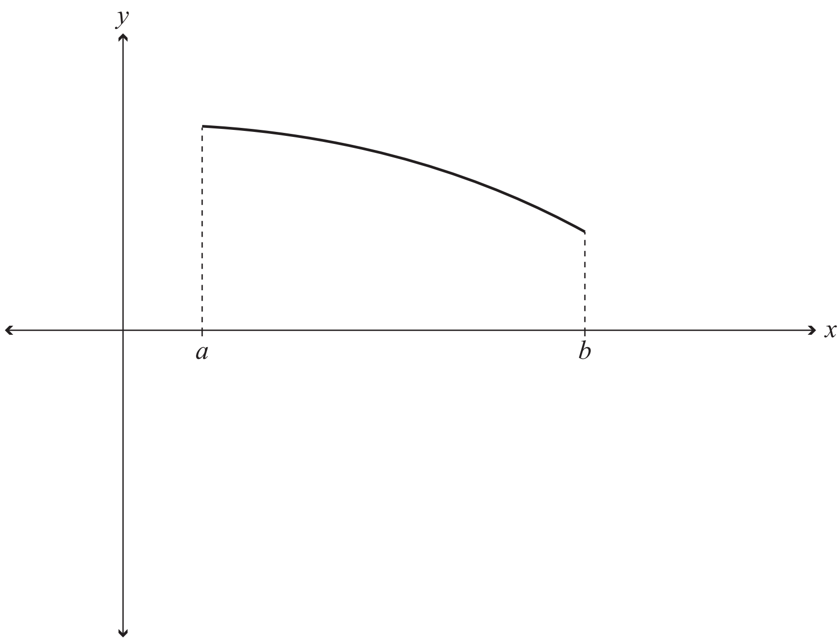


Kimihia te rōrahi ka puta i tēnei hurihuringa.

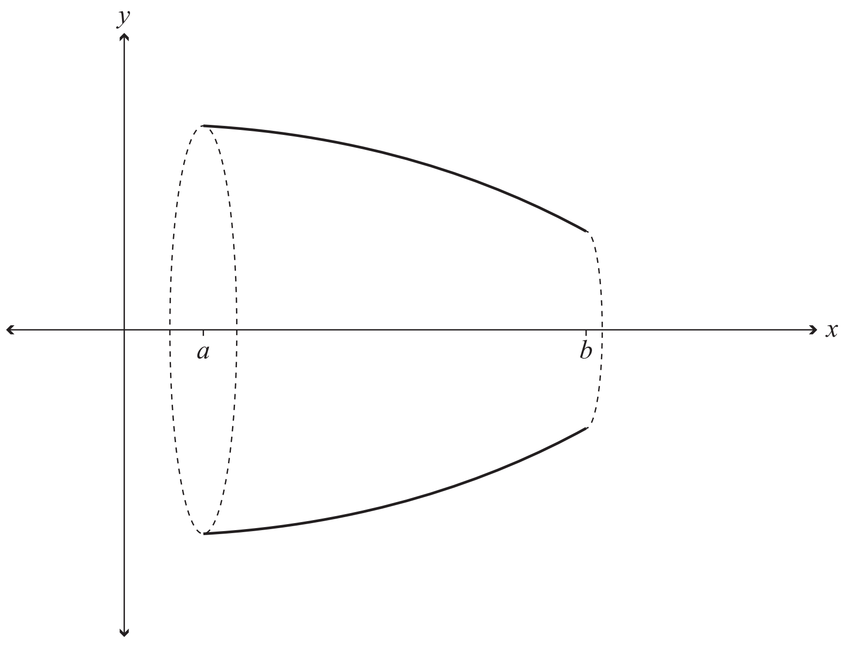
Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

(e)

Consider the curve defined by the function y = $f(x)$, bounded by $x = a$ and $x = b$.



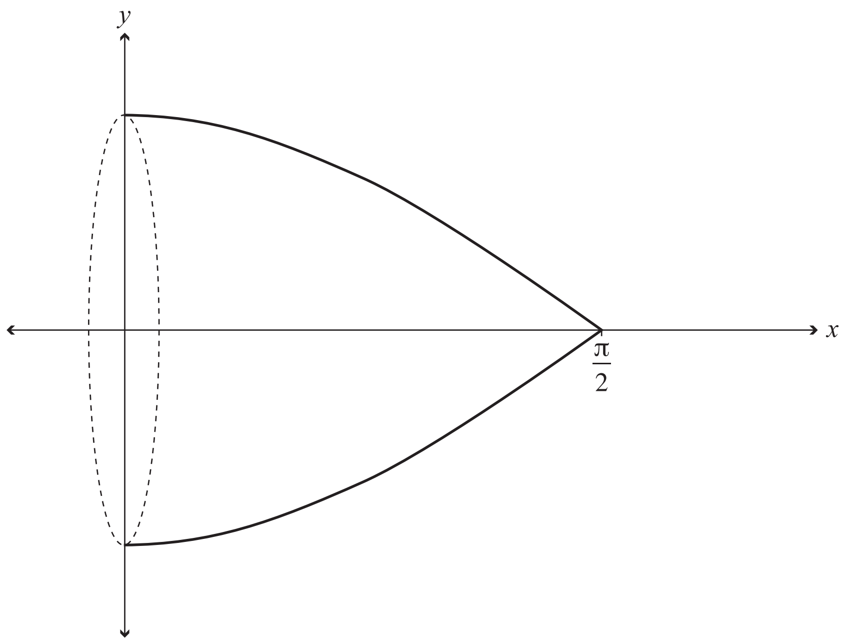
This portion of the curve is rotated around the x-axis, as shown below.



The volume created by this rotation is given by the formula

$$\text{Volume} = \pi \int_a^b \left(f(x)\right)^2 \mathrm{d}x$$

The graph below shows the function $y = \cos x$, between $x = 0$ and $x = \frac{\pi}{2}$, rotated around the x-axis.



Find the volume created by this rotation.

You must use calculus and give the results of any integration needed to solve this problem.

ASSESSOR'S USE ONLY

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUARUA

(a) Whiriwhiria a $\int\left(3-\frac{5}{x^2}\right)\mathrm{d}x$.

(b) Whakamahia ngā uara i raro ki te kimi i tētahi āwhiwhitanga ki $\int_1^{2.5} f(x)\,\mathrm{d}x$, mā te whakamahi i te Ture Taparara.

x	1	1.25	1.5	1.75	2	2.25	2.5
$f(x)$	0.3	0.7	1.65	1.9	2.35	1.7	1.1

ASSESSOR'S USE ONLY

QUESTION TWO

(a) Find $\int \left(3 - \frac{5}{x^2}\right) \mathrm{d}x$.

(b) Use the values given in the table below to find an approximation to $\int_1^{2.5} f(x)\,\mathrm{d}x$, using the Trapezium Rule.

x	1	1.25	1.5	1.75	2	2.25	2.5
$f(x)$	0.3	0.7	1.65	1.9	2.35	1.7	1.1

MĀ TE KAIMĀKA ANAKE

(c) Ka ohorere te whakatere a tētahi ahanoa[1], i te neke ki tētahi tere aumou i te tuatahi. Mai i te tīmatanga o tana whakatere ka taea te whakatauira te nekehanga o te ahanoa mā te whārite pārōnaki

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{50t^2 - 80\sqrt{t}}{5\sqrt{t}} \quad \text{mō } 0 \le t \le 20$$

ina ko v te tere o te ahanoa i te m s^{-1}

ā, ko t te wā ā-hēkona i muri mai i te whakaterenga o te ahanoa.

Mēnā ko te tere tuatahi o te ahanoa he 6 m s^{-1}, kimihia te tere o te ahanoa ina ko $t = 4$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

[1] mea

(c) An object originally moving at a constant velocity suddenly starts to accelerate. From the start of the object's acceleration the motion of the object can be modelled by the differential equation

$$\frac{dv}{dt} = \frac{50t^2 - 80\sqrt{t}}{5\sqrt{t}} \quad \text{for } 0 \le t \le 20$$

where v is the velocity of the object in m s^{-1}

and t is the time in seconds after the object starts to accelerate.

If the original velocity of the object was 6 m s^{-1}, find the velocity of the object when t = 4.

You must use calculus and give the results of any integration needed to solve this problem.

ASSESSOR'S USE ONLY

(d) I te tāone o Clarkeville, he pānga riterite i waenga i te pāpātanga e huri ai te taupori, P, o te tāone i tētahi wā me te taupori o te tāone i taua wā anō.

(i) Tuhia tētahi whārite pārōnaki e whakatauira ana i tēnei āhuatanga.

(ii) I te tīmatanga o te tau 2000, he 12 000 te taupori o te tāone.

I te tīmatanga o te 2010, he 16 000 te taupori o te tāone.

Whakaotia te whārite pārōnaki i (i) ki te kimi i te taupori o te tāone ā te tīmatanga o te tau 2025.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

ASSESSOR'S USE ONLY

(d) In the town of Clarkeville, the rate at which the population, P, of the town changes at any instant is proportional to the population of the town at that instant.

(i) Write a differential equation which models this situation.

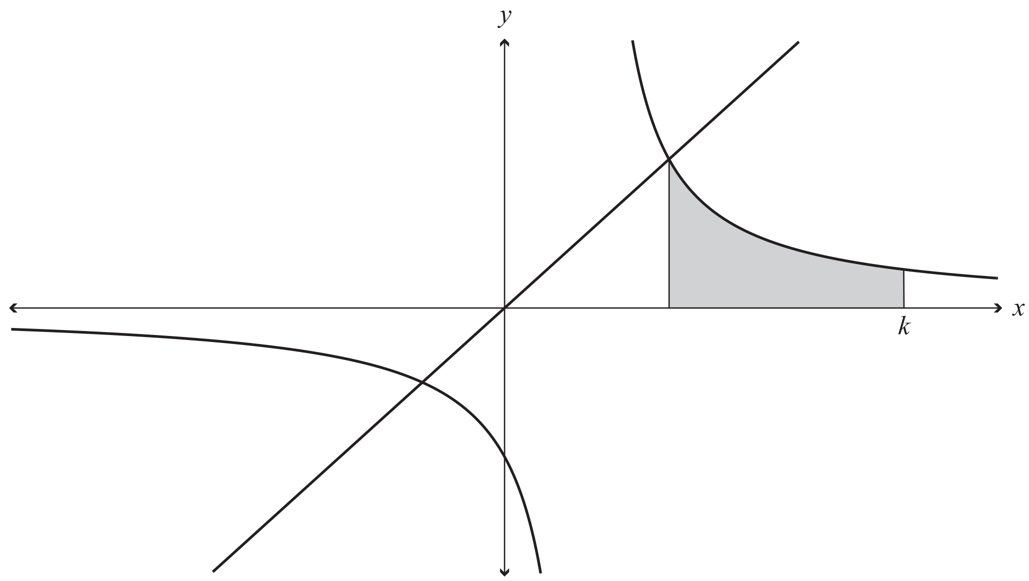
(ii) At the start of 2000, the population of the town was 12 000.

At the start of 2010, the population of the town was 16 000.

Solve the differential equation in (i) to find the population the town will have at the start of 2025.

You must use calculus and give the results of any integration needed to solve this problem.

(e) E whakaaturia ana ngā kauwhata o $y = \dfrac{2}{x-1}$ me $y = x$ ki ngā tuaka i raro nei.

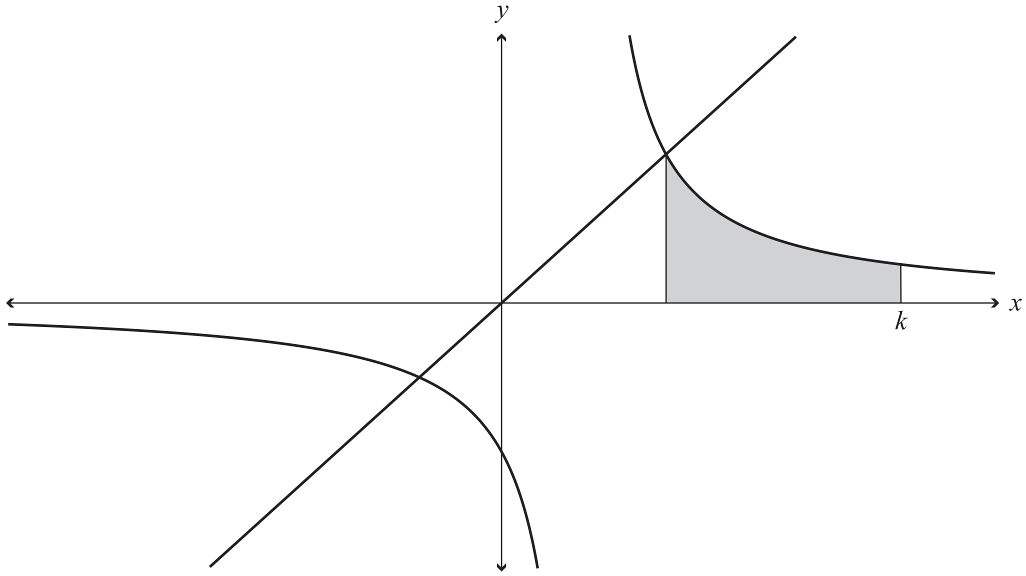


Ko te wāhi kauruku he 4 wae pūrua te horahanga.

Whiriwhiria te uara o k.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

(e) The graphs of $y = \dfrac{2}{x-1}$ and $y = x$ are shown on the axes below.



The shaded region has an area of 4 units squared.

Find the value of k.

You must use calculus and give the results of any integration needed to solve this problem.

ASSESSOR'S USE ONLY

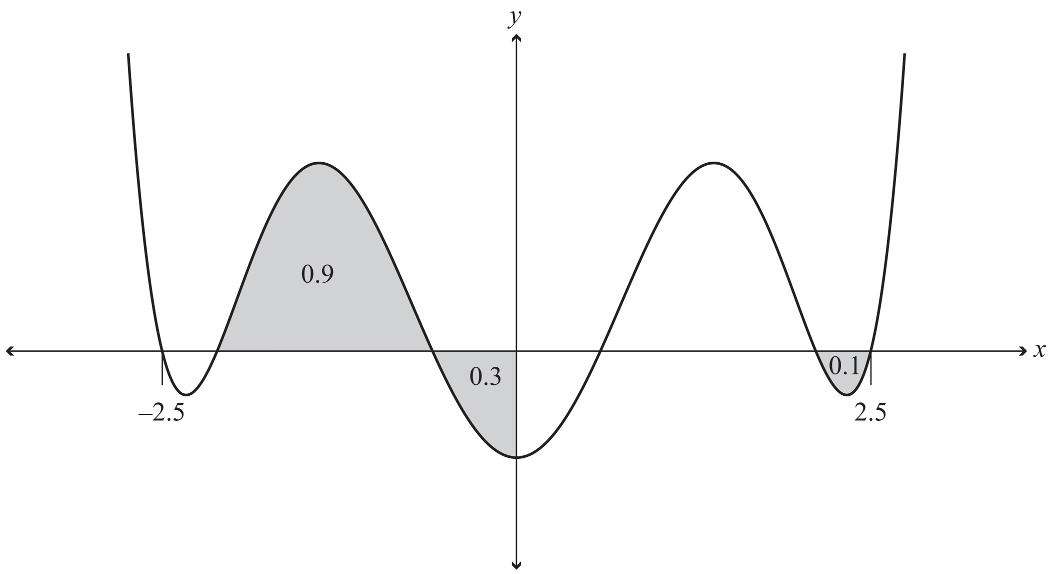
MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATORU

(a) Whiriwhiria a $\int\left((x+4)^2+8e^{4x}\right)dx$.

(b) Ko te kauwhata o te pānga $y = f(x)$ i raro nei he hangarite huri noa i te tuaka-y.

Kua whakaaturia ngā horahanga o ngā wāhi kauruku.



Whiriwhiria a $\int_{-2.5}^{2.5} f(x)\,dx$.

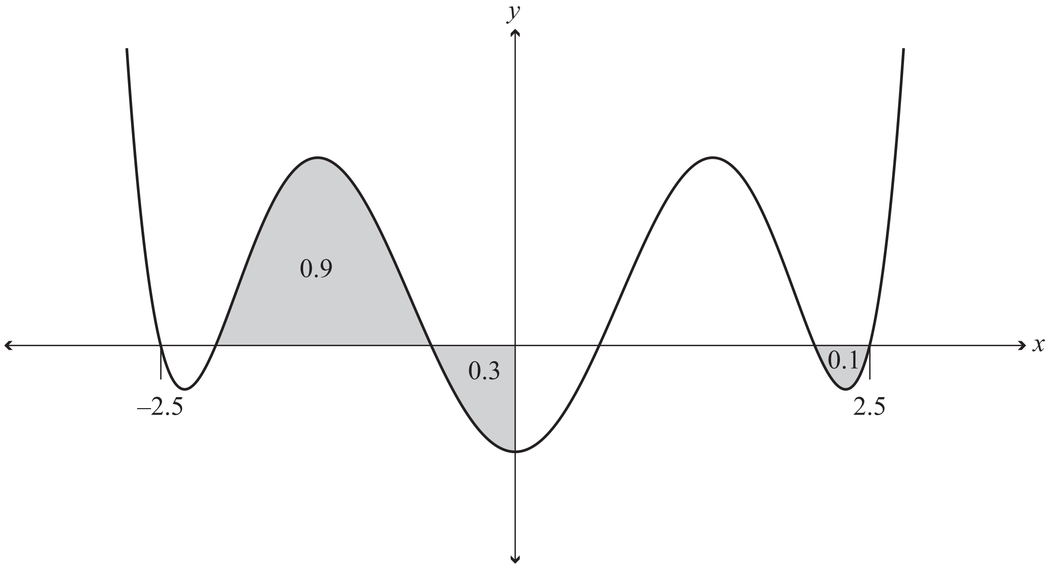
QUESTION THREE

ASSESSOR'S USE ONLY

(a) Find $\int\left((x+4)^2+8e^{4x}\right)dx$.

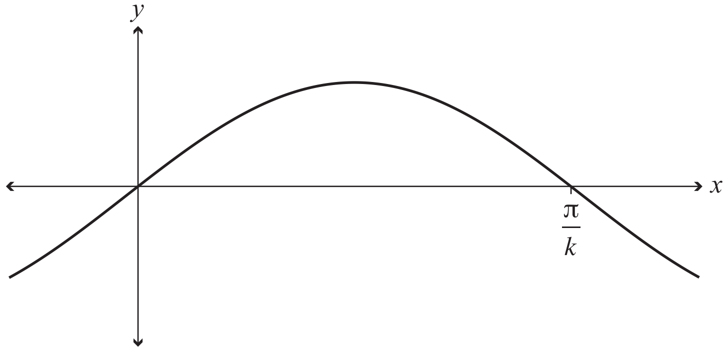
(b) The graph of the function $y = f(x)$ below is symmetrical about the y-axis.

The areas of the shaded regions are given.



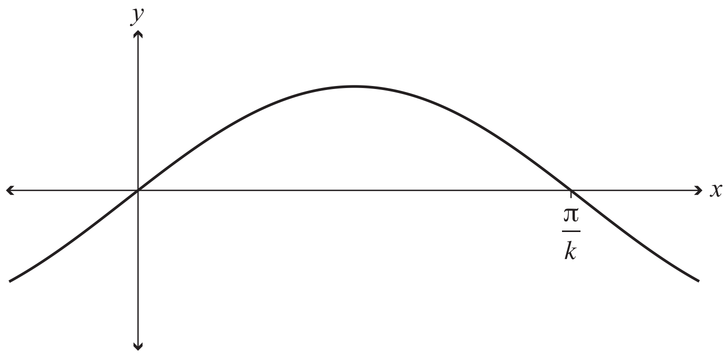
Find $\int_{-2.5}^{2.5} f(x)\,dx$.

(c) Kimihia he kīanga e ai ki k mō te wāhi e rohea ana e te pānga $y = \sin kx$ me te tuaka-x, i waenga $x = 0$ me $x = \dfrac{\pi}{k}$.



Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

(c) Find an expression in terms of k for the area bounded by the function $y = \sin kx$ and the x-axis, between $x = 0$ and $x = \frac{\pi}{k}$.



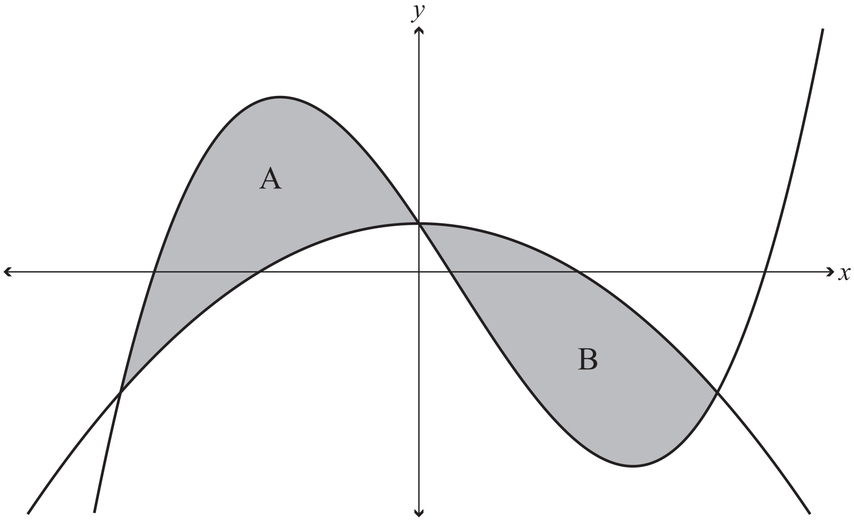
You must use calculus and give the results of any integration needed to solve this problem.

ASSESSOR'S USE ONLY

MĀ TE KAIMĀKA ANAKE

(d) E whakaaturia ana i raro ko ngā kauwhata o $f(x) = -x^2 + 2$ me $g(x) = x^3 - x^2 - kx + 2$.

E haukoti ana ngā kauwhata me te waihanga i ngā rohe kati, A me B.



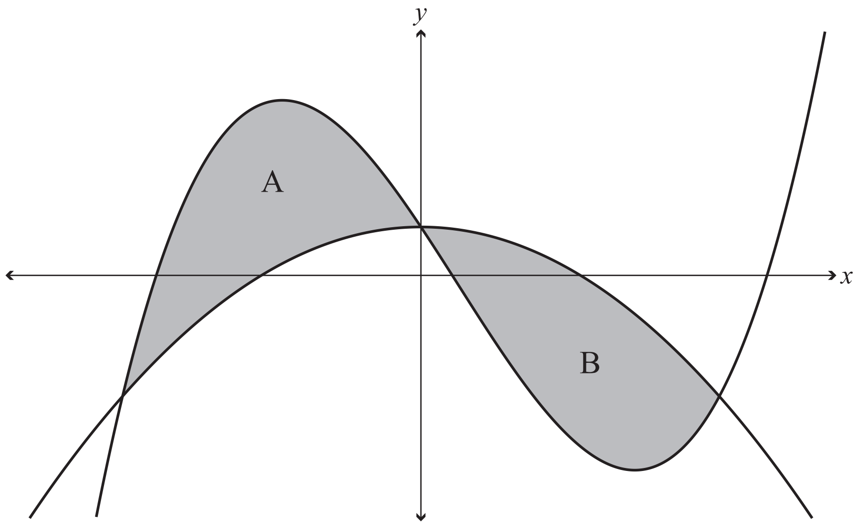
Whakaaturia he ōrite te horahanga o ēnei rohe e rua.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

ASSESSOR'S USE ONLY

(d) The graphs of $f(x) = -x^2 + 2$ and $g(x) = x^3 - x^2 - kx + 2$ are shown below.

The graphs intersect and create two closed regions, A and B.



Show that these two regions have the same area.

You must use calculus and give the results of any integration needed to solve this problem.

(e) Ka tīmata tētahi ahanoa mai i te okioki.

Ko te whakaterenga o te ahanoa ka tukuna mā te tātai $a = B\left(e^{kt}\right)^2$

ina ko a te whakaterenga o te ahanoa i m s^{-2}

ā, ko t te wā, ā-hēkona, mai i te whakahaeretanga o te ahanoa.

Whakaaturia ko te wā e tae atu ai te ahanoa ki te tere v_0 he

$$t = \frac{1}{2k}\ln\left(\frac{2v_0k + B}{B}\right)$$

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

MĀ TE KAIMĀKA ANAKE

(e) An object starts from rest.

The object's acceleration is given by the formula $a = B\left(e^{kt}\right)^2$

where a is the acceleration of the object in m s^{-2}

and t is the time, in seconds, from when the object started moving.

Show that the time that it takes the object to reach velocity v_0 is

$$t = \frac{1}{2k}\ln\left(\frac{2v_0k + B}{B}\right)$$

You must use calculus and give the results of any integration needed to solve this problem.

ASSESSOR'S USE ONLY

He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

English translation of the wording on the front cover

Level 3 Calculus, 2015

91579M Apply integration methods in solving problems

2.00 p.m. Wednesday 25 November 2015
Credits: Six

91579M

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.