Assessment Schedule - 2021
Mathematics and Statistics: Apply algebraic methods in solving problems (91261)
Evidence

| $\begin{gathered} \mathbf{Q} \\ \text { ONE } \end{gathered}$ | Expected coverage | Achievement (u) | Merit (r) | Excellence (t) |
| :---: | :---: | :---: | :---: | :---: |
| (a)(i) | $27 y^{5}$ | Correct expression. |  |  |
| (a)(ii) | $2 y^{9}$ | Correct expression. |  |  |
| (b) | Deduce that $(3 x+2)(x-4)=0$ $\text { OR }\left(x+\frac{2}{3}\right)(x-4)=0$ <br> So <br> Original equation was $3 x^{2}-10 x-8=0$ | Correct equation but coefficients not whole numbers OR incorrect relevant factorised form but consistent expanded form. | Correct equation. |  |
| (c) | $\begin{aligned} & x^{2}-3 k x+2 k^{2}=0 \\ & (x-k)(x-2 k)=0 \end{aligned}$ <br> So solutions are $k$ and 2 k , [hence one is twice the other]. <br> OR $x^{2}-3 k x+2 k^{2}=0$ $\Delta=(-3 k)-4(1)\left(2 k^{2}\right)=k^{2}$ <br> Hence, $x=\frac{-(-3 k) \pm \sqrt{k^{2}}}{2}$ $\begin{aligned} & =\frac{3 k \pm k}{2} \\ = & \frac{4 k}{2} \text { or } \frac{2 k}{2} \end{aligned}$ <br> i.e. one root is $2 k$ and the other is $k$ and hence one is twice the other. | Equation factorised. <br> OR <br> Discriminant evaluated. | Correct conclusion with valid working. |  |


| (d) | Curve Two: $y=(x-1)(x+1)-2$ <br> So $y=x^{2}-3$ <br> Curve One: $x^{2}=y^{2}+1$, <br> so substitute into Curve Two to give: $\begin{aligned} & y=y^{2}+1-3=y^{2}-2 \\ & y^{2}-y-2=0 \\ & (y-2)(y+1)=0 \end{aligned}$ <br> Either $y=2$ so $x^{2}=5$ and $x= \pm \sqrt{5}$, so points of intersection are $( \pm \sqrt{5}, 2)= \pm 2.24,2)$ <br> Or $y=-1$ so $x^{2}=2$ and $x= \pm \sqrt{2}$, so points of intersection are $( \pm \sqrt{2},-1)=( \pm 1.41,-1)$. <br> FYI: | Correct expansion and simplification of Curve Two. | Forms one equation in one variable. | T1: Correct identification of two solutions only or of four $x$ values. <br> T2: Complete correct solution with correct mathematical statements. |
| :---: | :---: | :---: | :---: | :---: |


| N0 | N1 | N2 | A3 | A4 | M5 | M6 | E7 | E8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No response; no relevant evidence. | Part Q correct | 1 of u | 2 of u | 3 of u | 1 of r | 2 of r | T1 | T2 |


| $\begin{gathered} \mathbf{Q} \\ \text { TWO } \end{gathered}$ | Expected coverage | Achievement (u) | Merit (r) | Excellence (t) |
| :---: | :---: | :---: | :---: | :---: |
| (a) | $\frac{(x-4)(x+3)}{4(x+3)}=\frac{x-4}{4}$ or equivalent | Correct simplified fraction (not bashed). |  |  |
| (b) | $\begin{aligned} & \frac{5 x(x+2)-(x-3)(x-4)}{(x-3)(x+2)} \\ & =\frac{4 x^{2}+17 x-12}{(x-3)(x+2)} \end{aligned}$ <br> or equivalent | Line 1 set up, or equivalent. | Obtains correct fraction. |  |
| (c)(i) | $\begin{aligned} \mathrm{D}=11, \text { so }\left(1+\frac{R}{100}\right)^{11} & =2 \\ \left(1+\frac{R}{100}\right) & =\sqrt[11]{2}=1.065 \\ R & =6.5(\%) \end{aligned}$ | Sets up equation and uses the $11^{\text {th }}$ root. <br> OR <br> CAO | Obtains correct solution. |  |
| (c)(ii) | $\begin{aligned} & \left(1+\frac{R}{100}\right)^{D}=2 \\ & \log \left[\left(1+\frac{R}{100}\right)^{D}\right]=\log 2 \\ & D \log \left(1+\frac{R}{100}\right)=\log 2 \end{aligned}$ $D=\frac{\log (2)}{\log \left(1+\frac{R}{100}\right)}$ | Logs taken of both sides (one of lines 2 or 3 must be shown). | Given expression derived correctly (one of lines 2 or 3 must be shown). |  |
| (c)(iii) | $\begin{gathered} \frac{72}{R}=\frac{\log (2)}{\log \left(1+\frac{R}{100}\right)} \\ 72 \log \left(1+\frac{R}{100}\right)=R \log (2) \\ \log \left(\left(1+\frac{R}{100}\right)^{72}\right)=\log \left(2^{R}\right) \\ \left(1+\frac{R}{100}\right)^{72}=2^{R} \\ 2^{R}-\left(1+\frac{R}{100}\right)^{72}=0 \end{gathered}$ | Sets equations equal. | Processes powers (line 3) | T1: Obtains given equation with incorrect mathematical statements. <br> T2: Obtains final given equation with correct mathematical statements. <br> PL: It may be reqd to make $T 1=$ (c) (ii), $T 2=(c)(i i i)$ |

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| $\mathbf{N 0}$ | $\mathbf{N 1}$ | $\mathbf{N 2}$ | $\mathbf{A 3}$ | $\mathbf{A 4}$ | $\mathbf{M 5}$ | M6 | E7 | E8 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| No response; no <br> relevant evidence. | Part Q <br> correct | 1 of u | 2 of $u$ | 3 of $u$ | 1 of r | 2 of r | T1 | T2 |

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| $\mathbf{Q}$ <br> THREE | Expected coverage | Achievement (u) | Merit (r) | Excellence (t) |
| :---: | :---: | :---: | :---: | :---: |
| (a) | Volume of cuboid $=p^{3}$ in simplest form. | Correct answer. |  |  |
| (b) | $\begin{aligned} & (p-4)(p+5)(p-3) \\ & =(p-4)\left(p^{2}+2 p-15\right) \\ & =p^{3}-2 p^{2}-23 p+60 \end{aligned}$ | Correct simplified expression. |  |  |
| (c) | Volume of cuboid $\begin{aligned} & =p(p+a)(p-a) \\ & =p^{3}-p a^{2} \end{aligned}$ <br> If this is the same as the volume of the cube, $\begin{aligned} & p^{3}-p a^{2}=p^{3} \\ & \text { so } p a^{2}=0 \end{aligned}$ <br> Thus $a=0$ is the only solution (since $p \neq 0$ ). OR that there are no [non-trivial] solutions. <br> Hence the cuboid never has the same volume as the cube unless you leave the sides unchanged. Or equivalent. | Correct working to the point of equating the two volumes. | Makes correct conclusion with valid working and reasoning. |  |
| (d)(i) | SA of cuboid $\begin{aligned} & =2[(p-a)(p+a)+10(p-a)+10(p+a)] \\ & =2\left[p^{2}-a^{2}+20 p\right] \\ & \text { Thus } 2 p^{2}-2 a^{2}+40 p=6 p^{2} \\ & \qquad \begin{aligned} 0 & =4 p^{2}-40 p+2 a^{2} \\ 0 & =2 p^{2}-20 p+a^{2} \end{aligned} \end{aligned}$ | Finds expression for SA of cuboid and expands it (accept no use of factor of 2 for all sides). | Derives given equation clearly. |  |
| (d)(ii) | For there to be solutions, $\begin{aligned} 400-8 a^{2} & >0 \\ 50 & >a^{2} \end{aligned}$ <br> Hence $0<a<7.1$. <br> If $a=7$, we solve $2 c^{2}-20 c+49=0$ and obtain $p=5.707$ or $p=4.293$. <br> However, one of the sides of the cuboid is $p-a$, meaning that it will be negative with either value of $p$ obtained here. <br> If $a=6$, we solve $2 p^{2}-20 p+36=0$ <br> and obtain $p=7.646$ or $p=2.354$. <br> The larger of these 2 values is a valid solution so the largest possible whole number value of $a$ is 6 . <br> The cube has sides of 7.646 cm , and the cuboid is $1.646 \times 13.646 \times 10 \mathrm{~cm}$. | Evaluates discriminant. | Obtains inequality (or implies one) for $a$ or $a^{2}$. | T1: Obtains $a=7$ and dimensions of 5.707 for the cube but does not go on from there. <br> T2: Obtains correct dimensions for the cube and the cuboid. |


| N0 | N1 | N2 | A3 | A4 | M5 | M6 | E7 | E8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No response; no relevant evidence. | Part Q correct | 1 of u | 2 of u | 3 of u | 1 of r | 2 of r | T1 | T2 |

Cut Scores

| Not Achieved | Achievement | Achievement with Merit | Achievement <br> with Excellence |
| :---: | :---: | :---: | :---: |
| $0-7$ | $8-13$ | $14-19$ | $20-24$ |

