## Assessment Schedule - 2021

Mathematics and Statistics (Statistics): Apply probability concepts in solving problems (91585)
Evidence Statement

| Q | Expected Coverage | Achievement (u) | Merit (r) | Excellence (t) |
| :---: | :---: | :---: | :---: | :---: |
| ONE <br> (a) | $\frac{1}{5} \times \frac{1}{4}=\frac{1}{20}(=0.05)$ <br> No, they were not correct in their prediction. | - Correct decision with numerical support.. |  |  |
| (b) | $\begin{aligned} & \mathrm{P}(\text { adult })=\mathrm{P}(\mathrm{~EB} \cap \text { adult })+\mathrm{P}(\mathrm{~EB} \cap \text { adult }) \\ & =\left(\frac{1}{5} \times \frac{3}{4}\right)+\left(\frac{4}{5} \times \frac{2}{3}\right) \\ & =\frac{41}{60}(=0.6833) \end{aligned}$ | - Correct answer. |  |  |
| (c) | $\mathrm{P}(\mathrm{~EB} \mid \text { Adult })=\frac{9}{41}(=0.2195) \quad \mathrm{P}(\mathrm{GA} \mid \text { Adult })=\frac{32}{41}(=0.7805)\left[>\frac{9}{41}\right]$ <br> OR $\mathrm{P}(\mathrm{~EB} \cap \text { Adult })=\frac{3}{20}(=0.15) \quad \mathrm{P}(\mathrm{GA} \cap \text { Adult })=\frac{8}{15}(=0.5333) \quad[>0.15]$ <br> OR <br> Calculate likelihood ratio $=3.56$ <br> Therefore, it is more likely for an Adult ticket to be purchased as a General Admission ticket. | - At least ONE relevant probability calculated. | - TWO probabilities correct and compared, with correct conclusion made. |  |


| (d) | Using 2021 proportions: $\mathrm{P}(\mathrm{~EB} \mid \mathrm{A})==\frac{\frac{1}{5} \times \frac{3}{4}}{\frac{1}{5} \times \frac{3}{4}+\frac{4}{5} \times \frac{2}{3}}=\frac{9}{41}=0.22$ <br> Using 2022 proportions: $\mathrm{P}(\mathrm{~EB} \mid \mathrm{A})==\frac{\frac{3}{5} \times \frac{3}{4}}{\frac{3}{5} \times \frac{3}{4}+\frac{2}{5} \times \frac{2}{3}}=\frac{27}{43}=0.63[>0.22]$ <br> This change increases the proportion of Adult tickets purchased that are sold as EB tickets. | - States that the proportion of adult tickets that are sold as EB tickets will increase supported by the one conditional probability. | - States that the proportion of adult tickets that are sold as EB tickets will increase supported by the two conditional probabilities. |
| :---: | :---: | :---: | :---: |
| (e) | Let the proportion of EB tickets $=p$ $\mathrm{P}(\mathrm{~EB} \mid \mathrm{C})=\frac{p \times \frac{1}{4}}{\left(p \times \frac{1}{4}\right)+\left((1-p) \times \frac{1}{3}\right)}=0.30$ <br> Solve to find the proportion of EB tickets that need to be sold, $p$ is $4 / 11$ ( $=0.3636$ ) | - Correct equation for $\mathrm{P}(\mathrm{EB} \mid \mathrm{C})$ setup. | - Correct solution for $\mathrm{P}(\mathrm{EB})$. |


| NØ | N1 | N2 | A3 | A4 | M5 | M6 | E7 | E8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No response; no relevant evidence. | Making progress / attempt at one part of the question. | 1 of $u$ | 2 of u | 3 of u | 1 of r | 2 of r | 1 of t | 2 of t |


| Q | Expected Coverage | Achievement (u) | Merit (r) | Excellence (t) |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { TWO } \\ & \text { (a)(i) } \end{aligned}$ | $\frac{2}{17}(0.1176)$ | - Correct probability. |  |  |
| (ii) | Idyll and Escapade share 5 acts $\rightarrow \frac{5}{37}(=0.1351)$ <br> Idyll and Serendipity share 13 acts $\rightarrow \frac{13}{37}(=0.3514)$ <br> Escapade and Serendipity share 8 acts $\rightarrow \frac{8}{37}(=0.2162)$ <br> Idyll and Serendipity [have the greatest probability of seeing the same acts $\}$. | - ONE proportion correctly calculated. | - Correct conclusion with supporting calculation(s) |  |
| (iii) | Number of acts unique to Idyll $=2 \rightarrow \frac{2}{37}(=0.0541)$ <br> Number of acts unique to Escapade $=4 \rightarrow \frac{4}{37}(=0.1081)$ <br> Number of acts unique to Serendipity $=11 \rightarrow \frac{11}{37}(=0.2973)$ <br> The attendee should attend Serendipity [as it has the largest probability of seeing a unique act]. | - Correct conclusion. |  |  |


| (b)(i) <br> (ii) | $\mathrm{P}(\text { correct })=\frac{80+25+5}{214}=\frac{110}{214}=51.4 \%$ <br> [In addition to having a low rate of correct predictions (51\%),] we note the following: <br> - only 25 / $72(=0.347)$ of the people predicted to attend Escapade actually <br> - only 5 / $40(=0.125)$ of the people predicted to attend Serendipity actually attended <br> - $80 / 102(=0.784)$ of the people predicted to attend Idyll actually attended. <br> We also note the following: <br> - only 25 / 65 (= 0.385 ) of the people attending Escapade were predicted to attend <br> - only 5 / $52(=0.096)$ of the people attending Serendipity were predicted to attend <br> - $80 / 97(=0.825)$ of the people attending Idyll were predicted to attend. So, the model has a low rate of correct predictions, both overall, and for two of the three festivals. <br> Accept other valid reasoning. | - Correct answer. | - At least TWO correct proportions calculated. <br> AND <br> Proportions used appropriately to support reasoning about a potential issue with the model. | - Merit <br> AND <br> States that the model has a low rate of correct. |
| :---: | :---: | :---: | :---: | :---: |
| (c) | A simulation would allow Patrick to see the variation in (or distribution of) the number or proportion of cola drinks in samples of size 300, based on the assumption that the proportion of cola drinks is $\frac{1}{3}$. <br> Patrick could then compare what was observed ( 85 cola drinks or a probability of 0.2833 ) to this simulated distribution to consider the likelihood of the observed result (or a lower proportion) happening. |  | - Discussion of how a simulation would allow Patrick to see that there is variation associated with estimates of cola drink numbers / proportions, and to make a decision on the number / proportion of cola drinks, without numerical support given. | - A clear discussion of how a simulation would allow Patrick to see that there is variation associated with estimates of cola drink numbers / proportions, and to make a decision on the number/proportion of cola drinks, with numerical support given. |


| NØ | N1 | N2 | A3 | A4 | M5 | M6 | E7 | E8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No response; no relevant evidence. | Making progress / attempt at one part of the question. | 1 of u | 2 of u | 3 of $u$ | 1 of r | 2 of r | 1 of t | 2 of t |

NCEA Level 3 Mathematics and Statistics (Statistics) (91585) 2021 — page 5 of 7

| Q | Expected Coverage |  |  |  |  | Achievement (u) | Merit (r) | Excellence (t) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| THREE <br> (a)(i) |  | Dancing | Socialising | Other | Total | - Correct probability calculated. |  |  |
|  | Less than 25 years old | 510 | 120 | 120 | 750 |  |  |  |
|  | 25 years or older | 54 | 160 | 46 | 250 |  |  |  |
|  | $\mathrm{P}(\text { less than } 25 \text { years and interested in socialising })=\frac{120}{1000}=\frac{3}{25}(=0.12)$ |  |  |  |  |  |  |  |
| (b) | $\begin{aligned} & \mathrm{P}(\text { interested in dancing } \mid \text { less than } 25)=\frac{510}{750}\left(=\frac{51}{75}\right)(=0.68) \\ & \mathrm{P}(\text { interested in dancing } \mid 25 \text { or older })=\frac{54}{250}\left(=\frac{27}{125}\right)(=0.216) \\ & \frac{\mathrm{P}(\text { interested in dancing } \mid \text { less than } 25)}{\mathrm{P}(\text { interested in dancing } \mid 25 \text { or older })}=\frac{\frac{51}{75}}{\frac{27}{125}}=\frac{85}{27}=3.148 \end{aligned}$ <br> Data supports this claim [as 3.148 is greater than 2]. |  |  |  |  | - ONE relevant probability correct. | - BOTH conditional probabilities calculated.. <br> AND <br> Correct conclusion as to data supporting the claim. |  |

\begin{tabular}{|c|c|c|c|c|}
\hline (c)(i)

(ii) \& \begin{tabular}{l}
$$
\begin{aligned}
& \mathrm{P}(\text { less than } 25 \cap \text { interested in dancing })=\frac{510}{1000}=0.51 \\
& \mathrm{P}(\text { less than } 25) \times \mathrm{P}(\text { interested in dancing })=\frac{750}{1000} \times \frac{564}{1000}=0.423 \\
& {[\neq 0.51]}
\end{aligned}
$$ <br>
OR
$$
\left.\mathrm{P}(\text { dancing } \mid<25)=\frac{510}{750}(=0.68) \quad \mathrm{P}(\text { dancing })=\frac{564}{1000}(=0.564)[\neq 0.68]\right)
$$ <br>
OR
$$
\left.\mathrm{P}(<25 \mid \text { dancing })=\frac{510}{564}(=0.904) \quad \mathrm{P}(<25)=\frac{750}{1000}(=0.75)[\neq 0.904]\right)
$$ <br>
The two events ['festival attendee is less than 25 years old' and 'festival attendee is primarily interested in dancing'] are not independent. <br>
Accept other valid tests for independence. <br>
The proportion of attendees interested in dancing varies by age group.

 \& - ONE relevant probability correct. \& - All figures calculated correctly and correct conclusion. \& 

- All figures calculated correctly and correct conclusion. <br>
AND <br>
Clear explanation of the relationship between the two events.
\end{tabular} <br>

\hline (d) \& | Reasons why care should be taken when using this data, for example: |
| :--- |
| - Data given is for only the first 1000 purchasers. Predictions made based on this data may not be correct, as these early purchasers may have different reasons for attending than purchasers who bought a ticket at the venue. The estimate of the proportion of attendees less than 25 years old and primarily interested in dancing could be more or less for the early purchasers. |
| - Data given is for online purchasers. Predictions made based on this data may not be correct, as these online purchasers may have different reasons for attending than later purchasers. The estimate of the proportion of attendees less than 25 years old and primarily interested in dancing could be more or less for the early purchasers. |
| Accept other valid possible reasons. | \& \& - ONE reason identified and clearly explained. OR TWO reasons identified without sufficient detail. \& - TWO reasons identified with implication about impact on estimate of proportion of attendees less than 25 years old and primarily interested in dancing. <br>

\hline
\end{tabular}

NCEA Level 3 Mathematics and Statistics (Statistics) (91585) 2021 — page 7 of 7

| NØ | N1 | N2 | A3 | A4 | M5 | M6 | E7 | E8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No response; no relevant evidence | Making progress / attempt at one part of the question. | 1 of u | 2 of u | 3 of u | 1 of r | 2 of r | 1 of t | 2 of t |

Cut Scores

| Not Achieved | Achievement | Achievement with Merit | Achievement with Excellence |
| :---: | :---: | :---: | :---: |
| $0-7$ | $8-13$ | $14-18$ | $19-24$ |

