Assessment Schedule - 2022
Mathematics and Statistics: Apply algebraic methods in solving problems (91261)

## Evidence

| Q | Evidence | Achievement | Merit | Excellence |
| :---: | :---: | :---: | :---: | :---: |
| ONE <br> (a) | $\begin{aligned} & \frac{2 x-3}{x+4}=3 \\ & 2 x-3=3 x+12 \\ & x=-15 \end{aligned}$ | Correct solution. |  |  |
| (b)(i) | $\begin{aligned} & 6 x^{3} y-15 x^{2} \sqrt{y}=3 x^{2} \sqrt{y}(2 x \sqrt{y}-5) \\ & \text { Accept } 3 x^{2} y\left(2 x-\frac{5}{\sqrt{y}}\right) \end{aligned}$ | Obtains $3 x^{2}(2 x y-5 \sqrt{y})$ | Correct expression. |  |
| (ii) | $\begin{aligned} \frac{6 x^{2}-x-12}{3 x^{2}-5 x-12} & =\frac{(2 x-3)(3 x+4)}{(3 x+4)(x-3)} \\ & =\frac{2 x-3}{x-3} \end{aligned}$ <br> Don't penalise hashing a correct answer | Correct simplified fraction. |  |  |
| (c)(i) | Sum of orange corners: $\mathrm{A}+\mathrm{A}+24=2 \mathrm{~A}+24$ $[\mathrm{A}+\mathrm{B}]$ <br> Sum of blue corners: $\mathrm{A}+21+\mathrm{A}+3=2 \mathrm{~A}+24 \quad[(\mathrm{~A}+3)+(\mathrm{B}-3)]$ <br> Therefore sum of orange corners $=$ sum of blue corners, no matter where you start the square. | Correct algebraic evidence but no conclusion. | Two sums compared and conclusion explicitly drawn. |  |
| (ii) | Product of orange corners: $\mathrm{A}(\mathrm{~A}+24)=\mathrm{A}^{2}+24 \mathrm{~A}$ <br> Product of blue corners: $(\mathrm{A}+21)(\mathrm{A}+3)=\mathrm{A}^{2}+24 \mathrm{~A}+63$ <br> If these products are equal: $\mathrm{A}^{2}+24 \mathrm{~A}+63=\mathrm{A}^{2}+24 \mathrm{~A} * *$ <br> So $63=0$ <br> Which is impossible. <br> Or a statement that 63 cannot equal zero. <br> OR <br> An argument based on the orange corners being A and B , and the blue corners being $\mathrm{A}+3$ and $\mathrm{B}-3$, leading to $\begin{aligned} & 3 \mathrm{~B}-3 \mathrm{~A}-9=0 \\ & \mathrm{~B}-\mathrm{A}=3 \# \# \end{aligned}$ <br> This cannot true if B is on a different row, and, as this is not true, the products cannot be equal. <br> [or equivalent arguments with different valid expressions for the corners] |  | Correct <br> algebraic evidence up to line **. <br> OR <br> Simplified relationship between A and B (line \#\#) but no conclusion | Correct and complete algebraic reasoning. <br> OR <br> Correct algebraic evidence with conclusion. |



| NØ | N1 | N2 | A3 | A4 | M5 | M6 | E7 | E8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No response; <br> no relevant <br> evidence. | A valid <br> attempt at one <br> question. | 1 u | 2 u | 3 u | 1 r | 2 r | 1 t | 2 t |


| Q | Evidence | Achievement | Merit | Excellence |
| :---: | :---: | :---: | :---: | :---: |
| TWO <br> (a) | $\begin{aligned} & \left(x-\frac{1}{3}\right)\left(x+\frac{2}{7}\right) \\ & =(3 x-1)(7 x+2) \\ & =21 x^{2}-x-2 \\ & \mathrm{a}=21, \mathrm{~b}=-1, \mathrm{c}=-2 \end{aligned}$ | Correct values of $\mathrm{a}, \mathrm{b}$, and c . |  |  |
| (b)(i) <br> (ii) | $(-12)^{2}-4(2)(7)=88$ $\begin{aligned} & \text { So }(-12)^{2}-4(2)(k)[=0] \\ & 8 k=144 \\ & k=18 \end{aligned}$ <br> accept use of inequality | Correct discriminant OR substitution made (line 1) | Correct value of k. |  |
| (c) | $\begin{aligned} & \sqrt{2 x+3}=3 x \\ & 2 x+3=9 x^{2} \\ & 9 x^{2}-2 x-3=0 \\ & x=0.6991 \text { or } x=-0.4768(4 \mathrm{sf}) \end{aligned}$ | Obtains correct quadratic. | Obtains both correct solutions. |  |
| (d)(i) | $\begin{aligned} & \mathrm{f} x^{2}+\mathrm{g} x+\mathrm{h}=\mathrm{h} x^{2}+\mathrm{g} x+\mathrm{f} \\ & (\mathrm{f}-\mathrm{h}) x^{2}+(\mathrm{h}-\mathrm{f})=0 \\ & (\mathrm{f}-\mathrm{h})\left(x^{2}-1\right)=0 \\ & x^{2}=1 \\ & x=1 \text { or } x=-1 \end{aligned}$ <br> Accept $\pm \sqrt{\frac{-(h-f)}{(f-h)}}$ or equivalent. |  | Correct working to obtain one solution only. | Both correct solutions obtained. |


| (ii) | Roots of $\mathrm{Q}(x)$ are $x=\frac{-\mathrm{g} \pm \sqrt{\mathrm{g}^{2}-4 \mathrm{fh}}}{2 \mathrm{f}}$ <br> Roots of $\mathrm{Q}^{*}(x)$ are $x=\frac{-\mathrm{g} \pm \sqrt{\mathrm{g}^{2}-4 \mathrm{hf}}}{2 \mathrm{~h}}$ <br> If $A=\frac{-g-\sqrt{g^{2}-4 f h}}{2 f}$, then one of the roots of $\mathrm{Q}^{*}(x)$ will be $k A=k\left(\frac{-g-\sqrt{g^{2}-4 h f}}{2 h}\right)=\frac{-g-\sqrt{g^{2}-4 \mathrm{fh}}}{2 h}$ $\text { so } k=\frac{f}{h}$ <br> OR <br> If roots of $\mathrm{Q}(x)$ are A and B , $A B=\frac{\mathrm{h}}{\mathrm{f}}$ <br> If roots of $\mathrm{Q}^{*}(x)$ are kA and kB , $(k A)(k B)=\frac{f}{h}$ <br> So it follows that: $\mathrm{k}^{2} \mathrm{AB}=\mathrm{k}^{2} \frac{\mathrm{~h}}{\mathrm{f}}=\frac{\mathrm{f}}{\mathrm{~h}}$ <br> and $\mathrm{k}^{2}=\frac{\mathrm{f}^{2}}{\mathrm{~h}^{2}}$ and $\mathrm{k}=( \pm) \frac{\mathrm{f}}{\mathrm{h}}$ <br> OR <br> If roots of $\mathrm{Q}(x)$ are A and B , $\mathrm{A}+\mathrm{B}=\frac{-\mathrm{g}}{\mathrm{f}}$ <br> If roots of $\mathrm{Q}^{*}(\mathrm{x})$ are kA and kB , $\mathbf{k A}+\mathbf{k B}=\frac{-\mathrm{g}}{\mathrm{f}}$ <br> So it follows that: $\mathrm{k}(\mathrm{~A}+\mathrm{B})=\mathrm{k}\left(\frac{-\mathrm{g}}{\mathrm{f}}\right)=\frac{-\mathrm{g}}{\mathrm{~h}} \mathrm{k}$ <br> and $k=\frac{f}{h}$ | Correct expressions for all 4 roots obtained (may be combined) <br> OR $A B=\frac{h}{f}$ <br> OR $\mathrm{A}+\mathrm{B}=\frac{-\mathrm{g}}{\mathrm{f}}$ | Correct <br> expression <br> involving k as function of f,g and / or h OR <br> Finds $k=\frac{h}{f}$ <br> [this results from saying the root of $\mathrm{Q}(x)$ is kA and that of $\mathrm{Q}^{*}(x)$ is A] | Finds $k=\frac{f}{h}$ |
| :---: | :---: | :---: | :---: | :---: |


| NØ | N1 | N2 | A3 | A4 | M5 | M6 | E7 | E8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No response; <br> no relevant <br> evidence. | A valid <br> attempt at one <br> question. | 1 u | 2 u | 3 u | 1 r | 2 r | 1 t | 2 t |

NCEA Level 2 Mathematics and Statistics (91261) 2022 - page 5 of 6 - Version 5

| Q | Evidence | Achievement | Merit | Excellence |
| :---: | :---: | :---: | :---: | :---: |
| THREE <br> (a)(i) | $\sqrt{49 y^{36}}=7 y^{18}$ | Correct response. |  |  |
| (ii) | $\begin{aligned} & x \log (2)=\log (2022) \\ & x=10.98 \end{aligned}$ <br> Accept $\log _{2}(2022)$ | Correct solution. |  |  |
| (b) | $\begin{aligned} \log (3 a)+ & 2 \log \left(\frac{a}{6}\right) \\ & =\log (3 a)+\log \left(\left(\frac{a}{6}\right)^{2}\right) \\ & =\log \left(3 a\left(\frac{a}{6}\right)^{2}\right) \\ & =\log \left(\frac{a^{3}}{12}\right) \end{aligned}$ | Fraction not correctly simplified but otherwise correct. | Correct expression obtained with fraction correctly simplified. |  |
| (c)(i) | $\begin{aligned} & \log _{2}(x-\mathrm{a})-\log _{2}(x+\mathrm{a})=\mathrm{c} \\ & \log _{2} \frac{x-\mathrm{a}}{x+\mathrm{a}}=\mathrm{c} \\ & \frac{x-\mathrm{a}}{x+\mathrm{a}}=2^{\mathrm{c}} \\ & x-\mathrm{a}=2^{\mathrm{c}}(x+\mathrm{a})=x 2^{\mathrm{c}}+\mathrm{a} 2^{\mathrm{c}} \\ & x\left(1-2^{\mathrm{c}}\right)=\mathrm{a}+\mathrm{a} 2^{\mathrm{c}}=\mathrm{a}\left(1+2^{\mathrm{c}}\right) \\ & \text { so, } x=\mathrm{a} \frac{1+2^{\mathrm{c}}}{1-2^{\mathrm{c}}} \end{aligned}$ | Log expressions combined correctly. | Correct exponential equation obtained (line $3)$. | Correct mathematical statements lead to the required expression. |
| (ii) | Using the expression from (c) part (i) <br> Firstly, if $x$ is not defined, there will be no solutions, so that means that $1-2^{\text {c }}$ $\neq 0$, so $2^{c} \neq 1$, and $c \neq 0$. Hence c cannot be zero. <br> Secondly, if $\mathrm{a}=0$, then $x=0$, but then the logs will be undefined. Hence, a cannot be zero. <br> [Although, in the original equation, if $\mathrm{a}=0$ and $\mathrm{c}=0$, any strictly positive $x$ value is a solution, but the expression for $x$ is undefined] <br> Thirdly, for the original equation to be defined, both $x-\mathrm{a}>0$ and $\mathrm{x}+\mathrm{a}>0$ (accept one or the other, or both). |  | One constraint identified with reasoning. | Two constraints identified with reasoning. |


| NØ | N1 | N2 | $\mathbf{A 3}$ | $\mathbf{A 4}$ | M5 | M6 | E7 | E8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No response; <br> no relevant <br> evidence. | A valid <br> attempt at one <br> question. | 1 u | 2 u | 3 u | 1 r | 2 r | 1 t |  |

Cut Scores

| Not Achieved | Achievement | Achievement with Merit | Achievement with Excellence |
| :---: | :---: | :---: | :---: |
| $0-6$ | $7-12$ | $13-18$ | $19-24$ |

