Assessment Schedule - 2023
Mathematics and Statistics: Apply algebraic methods in solving problems (91261)

## Evidence

| Q | Evidence | Achievement | Merit | Excellence |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { ONE } \\ & \text { (a)(i) } \end{aligned}$ | $\frac{2}{n}$ | - Correct simplified expression with positive index, as given. |  |  |
| (ii) | $\begin{aligned} \left(\frac{n^{3}}{16 n^{6}}\right)^{-0.5} & =\sqrt{16 n^{3}} \\ & =4 n^{\frac{3}{2}} \end{aligned}$ | - Correct interpretation of negative power. <br> OR <br> Interprets power of 0.5 as square root. <br> OR <br> Obtains unsimplified equivalent fraction or numerical coefficients such as $2^{2}$ or $\frac{1}{0.25}$. |  |  |
| (b) | $\left.\begin{array}{rl} \frac{2\left(10 x^{2}-11 x+3\right)}{x(5 x-3)} & =\frac{2(2 x-1)(5 x-3)}{x(5 x-3)} \\ & =\frac{2(2 x-1)}{x} \\ & =4-\frac{2}{x} \end{array}\right\}=4 \text { and } B=-2 \text {. }$ | - Correct factorisation of numerator. | - Correct values for A and B (if not explicitly stated, line 3 is required). |  |
| (c) | Initially $t=0$ $\begin{aligned} & 900=40+k \mathrm{e}^{0} \\ & k=860 \\ & 450=40+860 \mathrm{e}^{-0.5 t} \\ & 860 \mathrm{e}^{-0.5 t}=410 \\ & \mathrm{e}^{-0.5 t}=0.477 \end{aligned}$ <br> Take log of both sides: $\begin{aligned} & \ln \left(\mathrm{e}^{-0.5 t}\right)=\ln 0.477 \\ & -0.5 t=\frac{\ln 0.477}{\ln \mathrm{e}} \\ & t=1.48 \text { years } \end{aligned}$ | - $\mathrm{t}=0$ substituted. | - $k$ correctly found using $t=0$. | - Correctly solved. |

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| (d) | $\begin{aligned} & x^{2}-k(2 x+29)+32 k=0 \\ & x^{2}-2 k x+3 k=0 \\ & \text { using } b^{2}-4 a c=0 \\ & 4 k^{2}-12 k=0 \\ & k=0 \text { and } k=3 \end{aligned}$ <br> So, $k=3$, so quadratic can be written as $y=\frac{x^{2}}{6}+16$ <br> When $x=0, y=16$ | - First step in solving simultaneous equations: substitution for $x$ or $y$, or equivalent, to give an equation in one variable only OR $y$-intercept correc | - Setting discriminant $=0$ for relevant equation (allow minor error) <br> OR <br> Calculus used correctly to obtain both $k$ and $y$ intercept correctly | $\mathrm{T}: k$ correct. <br> TT: both k and $y$ intercept correct. |
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| NØ | N1 | N2 | A3 | A4 | M5 | M6 | E7 | E8 |
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| No response; <br> no relevant <br> evidence. | A valid <br> attempt at one <br> question. | 1 u | 2 u | 3 u | 1 r | 2 r | 1 t | 2 t |

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| Q | Evidence | Achievement | Merit | Excellence |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { TWO } \\ & \text { (a)(i) } \end{aligned}$ | $\begin{aligned} & 3 m+1=2^{4} \\ & 3 m=15, m=5 \end{aligned}$ | - Correct solution. |  |  |
| (ii) | $\begin{aligned} & \log _{x} 64=2 \\ & x^{2}=64 \\ & x= \pm 8 \end{aligned}$ <br> As base cannot be negative, $x=8$ <br> OR $\begin{aligned} & x^{6}=64^{3} \\ & x=\sqrt[6]{262144} \\ & x= \pm 8 \end{aligned}$ <br> As base cannot be negative, $x=8$ | - Written in an index form. OR $x=8$ obtained with no consideration of $x=-8$. | - Correct answer with justification or evidence of negative value being disregarded. |  |
| (b) | $\begin{aligned} & \frac{5^{7 x+6}}{5^{-2 x}}=\left(5^{3}\right)^{p} \\ & 5^{7 x+6-(-2 x)}=5^{3 p} \\ & 9 x+6=3 p \\ & p=3 x+2 \end{aligned}$ <br> OR $\begin{aligned} p & =\log _{125}\left(\frac{5^{7 x+6}}{25^{-x}}\right) \\ & =\log _{125}\left(\frac{5^{7 x+6}}{5^{-2 x}}\right) \\ & =\log _{125}\left(5^{9 x+6}\right) \\ & =\log _{5}\left(\frac{5^{9 x+6}}{3}\right) \\ & =3 x+2 \end{aligned}$ | - Conversion to either $5^{3 p}$ or $5^{-2 x}$. <br> OR $x=\frac{p-2}{3}$ <br> OR <br> Log expression up to line 1 , which is only one possible log approach. | - Correct answer (simplification not required) |  |
| (c) | $\begin{aligned} 6+\log _{b}\left(b^{-3}\right)+\log _{b}\left(b^{\frac{1}{2}}\right) & =6-3 \log (b)+\frac{1}{2} \log _{b}(b) \\ & =6-3+\frac{1}{2} \\ & =3 \frac{1}{2} \end{aligned}$ | - Combine logs into 1 log term, e.g. $\log _{b}(b-2.5)$. | - Rewriting both log terms bringing down the power. | - Correct value, even if the candidate goes direct to the numerical values |

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| (d) | $4^{x}-10=3 \times 4^{x}$ | Obtains $4^{2 x}$ | • Solved for ' $u$ '. | • Correct value. |
| :---: | :--- | :--- | :--- | :--- |
| $4^{2 x}-3 \times 4^{x}-10=0$ | OR |  |  |  |
| Let $u=4^{x}$ | $16^{x}$ |  |  |  |
| $u^{2}-3 u-10=0$ | OR |  |  |  |
| $(u+2)(u-5)=0$ | $\left.\left(4^{x}\right)^{2}\right)$ |  |  |  |
| $4^{x}=-2$ or $4^{x}=5$ | $3 G 4^{x}$ |  |  |  |
| Negative value not valid so: |  |  |  |  |
| $\log 4^{x}=\log 5$ |  |  |  |  |
| $x=\frac{\log 5}{\log 4}$ |  |  |  |  |
| $x=1.16$ |  |  |  |  |
| Accept $\log _{4}(5)$ |  |  |  |  |


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| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { THREE } \\ & (\mathrm{a})(\mathrm{i}) \end{aligned}$ | $\begin{aligned} & (5 x-2)(x+3)=0 \\ & x=\frac{2}{5} \text { or }-3 \end{aligned}$ | - Both values correct. |  |  |
| (ii) | $\frac{(3 x-4)(x-2)}{(3 x+4)(3 x-4)}=\frac{x-2}{3 x+4}$ | - Correct expression. |  |  |
| (b) | Does not touch the $x$-axis so: $\begin{aligned} & b^{2}-4 a c<0 \\ & 8^{2}-4(2) p<0 \\ & 64-8 p<0 \\ & \text { Accept } p \geq 8 \end{aligned}$ <br> Could also use completing the square. | - Set up inequality (line $3)$. <br> - OR <br> - $p=8$ <br> - OR <br> - $p<8$. | - Correct answer. |  |
| (c) | $\begin{aligned} & \frac{x^{2}+2 x+k}{(x+5)(x+2)}=\frac{x-3}{x+2} \\ & x^{2}+2 x+k=(x-3)(x+5) \\ & x^{2}+2 x+k=x^{2}+2 x-15 \end{aligned}$ <br> Therefore, $k=-15$ <br> Or equivalent approach. | - Makes progress towards solution by eliminating denominators, or equivalent. | - Value found. |  |
| (d) | Equation $\begin{aligned} & y=a(x+1.25)(x-1.25) \\ & y=a\left(x^{2}-1.5625\right) \\ & x=0, y=-3 \\ & -3=-1.5625 a \\ & a=1.92 \\ & y=1.92(x+1.25)(x-1.25) \\ & x=1.1, y=-0.678 \end{aligned}$ <br> No, boat will not float. <br> OR <br> Using vertex form: $\begin{aligned} & y=a(x-h)^{2}+k \\ & y=a x^{2} \\ & 3=a(1.25)^{2} \\ & a=1.92 \\ & y=1.92(1.1)^{2} \\ & y=2.3232 \end{aligned}$ <br> The edge of the canal is only 0.6768 m below the water surface, so the boat won't float. <br> Accept other correct variations of these approaches. | - General equation, fitting the context, formed in any correct format. | - Coefficient of $x^{2}$ found.. | - Depth of canal at appropriate width, or width of canal at 1 m depth, calculated. <br> AND <br> Statement that boat will not float or similar comment. |

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