Assessment Schedule – 2023

Mathematics and Statistics: Apply algebraic methods in solving problems (91261) Evidence

Q	Evidence	Achievement	Merit	Excellence
ONE (a)(i)	$\frac{2}{n}$	• Correct simplified expression with positive index, as given.		
(ii)	$\left(\frac{n^3}{16n^6}\right)^{-0.5} = \sqrt{16n^3} = 4n^{\frac{3}{2}}$	• Correct interpretation of negative power. OR Interprets power of 0.5 as square root. OR Obtains unsimplified equivalent fraction or numerical coefficients such as 2^2 or $\frac{1}{0.25}$.		
(b)	$\frac{2(10x^2 - 11x + 3)}{x(5x - 3)} = \frac{2(2x - 1)(5x - 3)}{x(5x - 3)}$ $= \frac{2(2x - 1)}{x}$ $= 4 - \frac{2}{x}$ $A = 4 \text{ and } B = -2$	• Correct factorisation of numerator.	• Correct values for A and B (if not explicitly stated, line 3 is required).	
(c)	Initially $t = 0$ $900 = 40 + ke^{0}$ k = 860 $450 = 40 + 860e^{-0.5t}$ $860e^{-0.5t} = 410$ $e^{-0.5t} = 0.477$ Take log of both sides: $\ln(e^{-0.5t}) = \ln 0.477$ $-0.5t = \frac{\ln 0.477}{\ln e}$ t = 1.48 years	• t = 0 substituted.	• <i>k</i> correctly found using <i>t</i> = 0.	Correctly solved.

(d)	$x^{2} - k(2x + 29) + 32k = 0$ $x^{2} - 2kx + 3k = 0$ using $b^{2} - 4ac = 0$ $4k^{2} - 12k = 0$ k = 0 and $k = 3So, k = 3, so quadratic can be written asy = \frac{x^{2}}{6} + 16When x = 0, y = 16$	 First step in solving simultaneous equations: substitution for <i>x</i> or <i>y</i>, or equivalent, to give an equation in one variable only OR <i>y</i>-intercept correc 	 Setting discriminant = 0 for relevant equation (allow minor error) OR Calculus used correctly to obtain both k and y- intercept correctly 	T: <i>k</i> correct. TT: both k and <i>y</i> - intercept correct.
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NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	A valid attempt at one question.	1u	2u	3u	lr	2r	lt	2t

Q	Evidence	Achievement	Merit	Excellence
TWO (a)(i)	$3m+1=2^4$ 3m=15, m=5	• Correct solution.		
(ii)	$log_{x} 64 = 2$ $x^{2} = 64$ $x = \pm 8$ As base cannot be negative, x = 8 OR $x^{6} = 64^{3}$ $x = \sqrt[6]{262144}$ $x = \pm 8$ As base cannot be negative, x = 8	• Written in an index form. OR x = 8 obtained with no consideration of $x = -8$.	• Correct answer with justification or evidence of negative value being disregarded.	
(b)	$\frac{5^{7x+6}}{5^{-2x}} = (5^3)^p$ $5^{7x+6-(-2x)} = 5^{3p}$ 9x+6 = 3p p = 3x+2 OR $p = \log_{125} \left(\frac{5^{7x+6}}{25^{-x}}\right)$ $= \log_{125} \left(\frac{5^{7x+6}}{5^{-2x}}\right)$ $= \log_{125} (5^{9x+6})$ $= \log_5 \left(\frac{5^{9x+6}}{3}\right)$ = 3x+2	• Conversion to either 5^{3p} or 5^{-2x} . OR $x = \frac{p-2}{3}$ OR Log expression up to line 1, which is only one possible log approach.	• Correct answer (simplification not required)	
(c)	$6 + \log_b (b^{-3}) + \log_b (b^{\frac{1}{2}}) = 6 - 3\log(b) + \frac{1}{2}\log_b(b)$ $= 6 - 3 + \frac{1}{2}$ $= 3\frac{1}{2}$	• Combine logs into 1 log term, e.g. $\log_b(b - 2.5)$.	• Rewriting both log terms bringing down the power.	• Correct value, even if the candidate goes direct to the numerical values

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(d)	$4^{x} - 10 = 3 \times 4^{x}$ $4^{2x} - 3 \times 4^{x} - 10 = 0$ Let $u = 4^{x}$ $u^{2} - 3u - 10 = 0$ $(u + 2)(u - 5) = 0$ $4^{x} = -2 \text{ or } 4^{x} = 5$ Negative value not valid so: $\log 4^{x} = \log 5$ $x = \frac{\log 5}{\log 4}$ $x = 1.16$	• Obtains 4^{2x} OR 16^x OR $(4^x)^2$) OR $3G4^x$	• Solved for 'u'.	• Correct value.
	x = 1.16 Accept $\log_4(5)$			

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No response; no relevant evidence.	A valid attempt at one question.	1u	2u	3u	lr	2r	lt	2t

Q	Evidence	Achievement	Merit	Excellence
THREE (a)(i)	(5x-2)(x+3) = 0 $x = \frac{2}{5}$ or -3	• Both values correct.		
(ii)	$\frac{(3x-4)(x-2)}{(3x+4)(3x-4)} = \frac{x-2}{3x+4}$	• Correct expression.		
(b)	Does not touch the x-axis so: $b^2 - 4ac < 0$ $8^2 - 4(2)p < 0$ 64 - 8p < 0 Accept $p \ge 8$ Could also use completing the square.	 Set up inequality (line 3). OR p = 8 OR p < 8. 	• Correct answer.	
(c)	$\frac{x^{2} + 2x + k}{(x+5)(x+2)} = \frac{x-3}{x+2}$ $x^{2} + 2x + k = (x-3)(x+5)$ $x^{2} + 2x + k = x^{2} + 2x - 15$ Therefore, $k = -15$ Or equivalent approach.	• Makes progress towards solution by eliminating denominators, or equivalent.	• Value found.	
(d)	Equation y = a(x+1.25)(x-1.25) $y = a(x^2 - 1.5625)$ x = 0, y = -3 -3 = -1.5625a a = 1.92 y = 1.92(x+1.25)(x-1.25) x = 1.1, y = -0.678 No, boat will not float. OR Using vertex form: $y = a(x-h)^2 + k$ $y = ax^2$ $3 = a(1.25)^2$ a = 1.92 $y = 1.92(1.1)^2$ y = 2.3232 The edge of the canal is only 0.6768 m below the water surface, so the boat won't float. Accept other correct variations of these approaches.	• General equation, fitting the context, formed in any correct format.	• Coefficient of x^2 found	 Depth of canal at appropriate width, or width of canal at 1m depth, calculated. AND Statement that boat will not float or similar comment.

(e)	$2\pi rh + 2\pi r^{2} = 8rh + 4r^{2}$ $\pi rh + \pi r^{2} = 4rh + 2r^{2}$ $\pi h + \pi r = 4h + 2r$ $\pi h - 4h = 2r - \pi r$ $h(\pi - 4) = r(2 - \pi)$ $h = r\frac{(2 - \pi)}{(\pi - 4)}$ $= \frac{r(\pi - 2)}{(4 - \pi)}$ i.e. $h = 1.33r$	• Sets up initial equation. OR gives r correctly in terms of h. box is incorrect OR If surface area of and SA = Ar + Bhr, where B \neq 0, accept consistent, possibly unsimplified, answer of $h = \frac{r\left(\pi - \frac{A}{2}\right)}{\left(\frac{B}{2} - \pi\right)}$????? indecipherable	T: Correct expression but not simplified. TT: Correct expression.

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