Assessment Schedule – 2023

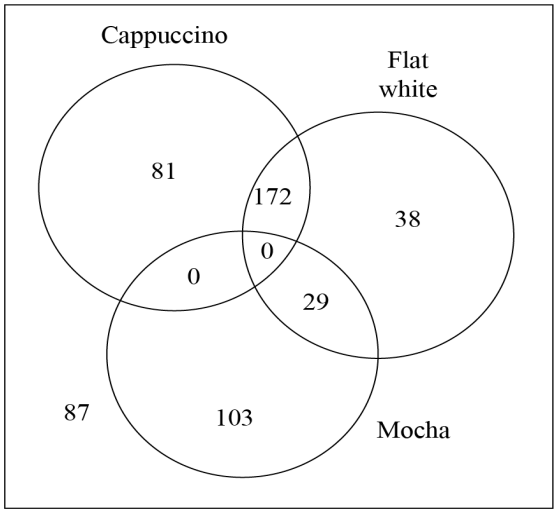
Mathematics and Statistics (Statistics): Apply probability concepts in solving problems (91585)

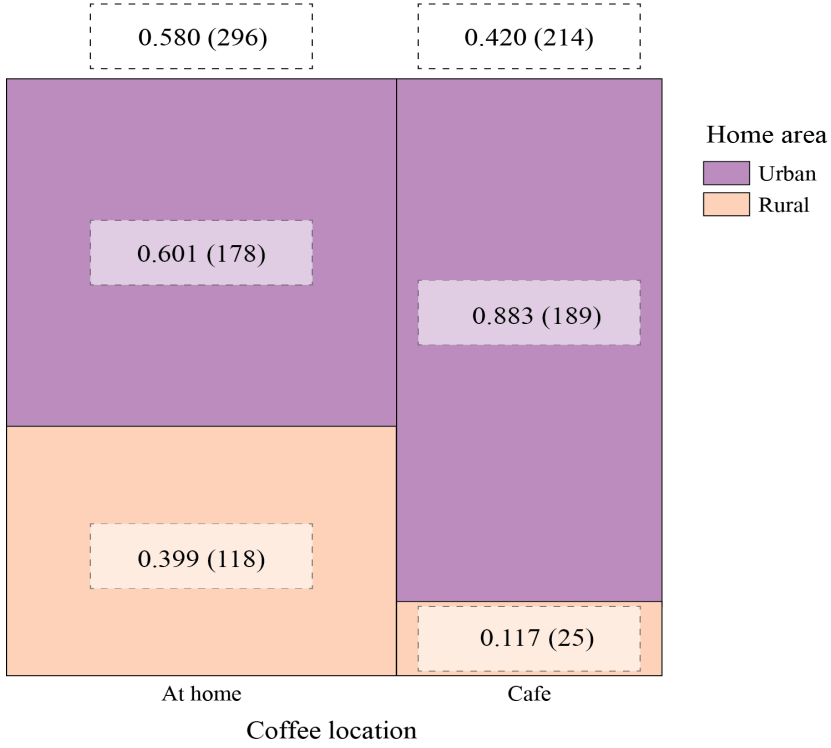
Evidence Statement

<table>
<tr><th>Q1</th><th>Expected Coverage</th><th>Achievement (u)</th><th>Achievement with Merit (r)</th><th>Achievement with Excellence (t)</th></tr>
<tr><td>(a)(i)</td><td>P(Year 9 or 11 | do not like coffee) = $\frac{81}{111}$ = 0.730
<table><tr><td>Like coffee</td><td>13</td><td>11</td><td>22</td><td>46</td></tr><tr><td>Do not like coffee</td><td>43</td><td>38</td><td>30</td><td>111</td></tr><tr><td>Totals</td><td>56</td><td>49</td><td>52</td><td>157</td></tr></table></td><td>• Correct probability.</td><td></td><td></td></tr>
<tr><td>(ii)</td><td>P(at least one of four like coffee | from Year 11)
= 1 – P(none of the four students like coffee)
$= 1 - \left(\frac{38}{49}\times\frac{37}{48}\times\frac{36}{47}\times\frac{35}{46}\right)$
= 1 – 0.348
= 0.652</td><td>• Partially correct strategy established (e.g. replacement or trying to work out without the “one minus” complement strategy and making some errors).</td><td>• Correct conditional probability for all four students.</td><td></td></tr>
<tr><td>(iii)</td><td>Assumption is that all four Year 11 students views on coffee are independent and their views of like / dislike of coffee do not affect each other.
This may not be valid given the small sample (Year 11 students at the same school).
OR
The relatively high chance that respondents may know each other.</td><td></td><td>• Correct assumption identified in context.</td><td>• Correct assumption identified in context.
AND
Correct discussion of the validity of the assumption in context.</td></tr>
</table>

(iv)	P(like coffee | Year 9) = $\frac{13}{56} = 0.232$ P(like coffee | Year 11) = $\frac{11}{49} = 0.224$ P(like coffee | Year 13) = $\frac{22}{52} = 0.423$ Yes, comparing Y9 or Y11 with Y13 students, or No, comparing Y9 with Y11 students. *Accept argument based on comparison of conditional probabilities or use of ratios.*	• Correctly calculating at least two conditional probabilities that would allow a valid comparison to be made.	• Correctly calculating at least two conditional probabilities that would allow a valid comparison to be made. AND Correct conclusion for this group of students.	
(v)	The results from each school would need to be separated out, and for each year group across both islands. The results of P(like coffee | year group) would need to be the same value (or given very low sample size very close to being the same value).		• Partially correct explanation that shows some understanding of idea of independence in context.	• Full and correct explanation that shows complete understanding.
(b)(i)	P(coffee will increase in price) $= \frac{17}{41} = 0.4146$ or $\frac{18}{41} = 0.4390$ Model developed by: P(price will increase in next two successive quarters) $= \left(\frac{17}{41}\right)^2 = 0.172$ or $\left(\frac{18}{41}\right)^2 = 0.193$	• One correct meaningful probability from the visualisation but must have the denominator of 41.	• Value of 0.172 or 0.193 shown with correct working. AND An explanation about the lack of validity of assumed independence.	• Correct illustration of the model probability value and an explanation about the lack of validity of assumed independence, justified correctly, e.g. either through looking at the clumpy nature of the visualisation or the contextual element such as recent inflationary pressure in the economy.
(ii)	For the model to be valid, it is assumed each quarter's price fluctuation is independent from the previous values, which given recent price and inflationary pressures is not valid. Just by looking at the visualisation, it is possible to see that price changes from quarter to quarter over the last decade are too clumped together to be independent of each other.			

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Making progress / attempt at one part of the question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

Q2	Expected Coverage	Achievement (u)	Achievement with Merit (r)	Achievement with Excellence (t)
(a)(i)	 P(preferred a cappuccino) $= \frac{253}{510} = 0.496$	• Venn diagram (or other correct appropriate representation) with values entered from results supplied. Values of 87 and 172 can be missing or incorrect.	• Correct proportion.	
(ii)	P(cappuccino only | preferred cappuccino) $= \frac{81}{253} = 0.320$ P(mocha only | preferred mocha) $= \frac{103}{132} = 0.780$ $\frac{0.320}{0.780} = 0.410$ Claim is **not supported** [as results from the survey are not the 50% more claimed]. [It is more than 50% less likely for people who preferred cappuccino to select any cappuccino, compared to those who preferred mocha selecting only mocha.]	• ONE correct or consistent probability calculated. Note: $\frac{0.780}{0.320} = 2.4375$ gets u only.	• BOTH probabilities calculated consistently and correct relative calculation. Note: 41% more likely gets r only.	• Correctly reasoned response to claim.
(b)(i)	P(coffee at a cafe | live in an urban area) = 0.515	• Correct conditional probability stated from eikosogram.		

(ii)	P(coffee at a cafe | live in an urban area) = 0.515 from above. P(coffee at a cafe | live in a rural area) = 0.175 We can be confident that an urban-based person is much more likely to have a coffee in a cafe compared to a rural person. The difference in proportions would be unlikely to change significantly due to **small sampling variation** from the large sample.	• A correct pair of conditional probabilities stated from eikosogram. OR A correctly reasoned argument.	• Correctly reasoned argument that show the differences between rural and urban areas, with reference to sampling variation.	•
(iii)	Accept counts (must be rounded) or proportions-based approach . 	• At least one cell correct.	• At least four values correct as counts or proportions.	• All six values correct either as counts or proportions.

Calculations using proportions: (or could represent raw data in a contingency table)

P(Home) = 0.825 × 0.28 + 0.485 × 0.72 = 0.5802

P(Café) = 0.175 × 0.28 + 0.515 × 0.72 = 0.4198

$P(\text{Urban} \mid \text{Home}) = \frac{0.485 \times 0.72}{0.5802} = 0.601$ $P(\text{Urban} \mid \text{Cafe}) = \frac{0.515 \times 0.72}{0.4198} = 0.883$

$P(\text{Rural} \mid \text{Home}) = \frac{0.825 \times 0.28}{0.5802} = 0.399$ $P(\text{Rural} \mid \text{Cafe}) = \frac{0.715 \times 0.28}{0.4198} = 0.117$

Calculations using numbers (out of 510):

Total Rural = 0.28 × 510 = 143 Home given Rural = 0.825 × 143 = 118

Café given Rural = 0.175 × 143 = 25

Total Urban = 0.72 × 510 = 367 Home given Urban = 0.485 × 367 = 178

Café given Urban = 0.515 × 367 = 189

Total Home = 118 + 178 = 296 Total Café = 25 + 189 = 214

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Making progress / attempt at one part of the question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

<table>
<tr><th>Q3</th><th>Expected Coverage</th><th>Achievement (u)</th><th>Achievement with Merit (r)</th><th>Achievement with Excellence (t)</th></tr>
<tr><td>(a)</td><td>
<table>
<tr><td></td><td>Fair Trade brand</td><td>Not Fair Trade brand</td><td></td></tr>
<tr><td>Extra strong</td><td>5</td><td>3</td><td>8</td></tr>
<tr><td>Not extra strong</td><td>7</td><td>0</td><td>7</td></tr>
<tr><td></td><td>12</td><td>3</td><td>15</td></tr>
</table>
Given that there is a value of 0 at the required intersection, this provides evidence that not fair trade and not extra strength are mutually exclusive.</td><td>• Table or other correct representation constructed.</td><td>• Correct conclusion with evidence.</td><td></td></tr>
<tr><td>(b)(i)</td><td>P(H H H) or P(T T T) = 0.125 + 0.125 = 0.25</td><td>• Correct probability.</td><td></td><td></td></tr>
<tr><td>(ii)</td><td>
<table>
<tr><th>Friend A</th><th>Friend B</th><th>Friend C</th><th>Paying friend</th></tr>
<tr><td>H</td><td>H</td><td>T</td><td>C</td></tr>
<tr><td>H</td><td>H</td><td>H</td><td>Flip again</td></tr>
<tr><td>H</td><td>T</td><td>H</td><td>B</td></tr>
<tr><td>T</td><td>H</td><td>H</td><td>A</td></tr>
<tr><td>T</td><td>T</td><td>H</td><td>C</td></tr>
<tr><td>T</td><td>T</td><td>T</td><td>Flip again</td></tr>
<tr><td>T</td><td>H</td><td>T</td><td>B</td></tr>
<tr><td>H</td><td>T</td><td>T</td><td>A</td></tr>
</table>
Each friend can expect to pay the same number of times as they all have two outcomes each.</td><td>• Evidence that each friend will have to pay an equal number of times.</td><td>• Correct decision with support of statistical reasoning.</td><td></td></tr>
</table>

(iii)	P(flip 5 or fewer times on one occasion) $= \frac{3}{4} + \frac{1}{4} \times \frac{3}{4} + \left(\frac{1}{4}\right)^2 \times \frac{3}{4} + \left(\frac{1}{4}\right)^3 \times \frac{3}{4} + \left(\frac{1}{4}\right)^4 \times \frac{3}{4} = \frac{1023}{1024} = 0.999023$ OR $= 1 - \left(\frac{1}{4}\right)^5 = \frac{1}{1024}$ P(flip more than 5 times on one occasion) = 1 – P(flip 5 or fewer times on one occasion) = 0.000977 or = $\left(\frac{1}{4}\right)^5 = \frac{1}{1024}$ Expected number of meetings out of 100 that require more than 5 flips = 100 × 0.000977 = 0.0977 After 100 coffee meet ups, you would expect this to occur at less than one tenth of a meeting. So, it is **not an unusual occurrence** that the friends don't need to flip more than 5 times [even after two years' worth of coffee meet ups].	• Some correct probability statements such as P(1 flip) or P(2 flips).	• Correct probability calculated for more than 5 flips.	• Calculate the expected number of occurrences out of 100 meetings. AND Stating that this is not an unusual occurrence about not needing more than 5 flips.
(iv)	A simulation would allow the group of friends to see the **variation** through the **distribution** of the number (or proportion) of occasions each friend had to pay for the coffees in sets of sample size 100 based on the model of each friend paying $\frac{1}{3}$ of the time. Friend A could then compare the observed value of 49 (or 0.49) to this simulated distribution to consider the likelihood of the observed number (or proportion) happening. A decision could be made then as to whether the 49 occasions after 100 coffee meet ups are indeed unusual.	• Discussion of how a simulation would allow the friends to see that there is variability associated with estimates of number of occasions each friend would have to pay.	• A clear discussion of how a simulation, **without** parameters stated, would allow the friends to see that they need to consider sampling variation to make a decision on whether Friend A has been unlucky.	• A clear discussion of how a simulation, **with** parameters stated, would allow the friends to see that they need to consider sampling variation to make a decision on whether Friend A has been unlucky.

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Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 14	15 – 18	19 – 24