Pilot Assessment Schedule - 2023
Mathematics and Statistics: Demonstrate mathematical reasoning (91947)

## Evidence

| Q | Evidence | Achievement | Merit | Excellence |
| :---: | :---: | :---: | :---: | :---: |
| ONE <br> (a) | $\begin{aligned} T & =\pi \times \sqrt{\frac{2.5 \times 0.9659}{9.81}} \\ & =\pi \times \sqrt{0.2462} \\ & = \pm \pi \times 0.4961 \\ & = \pm 1.5587 \end{aligned}$ <br> $\pm$ Not required. <br> Allow C.A.O. | - Correct answer. |  |  |
| (b)(i) | Area of one label $\begin{aligned} & =2 \pi \times 4.5 \times 15 \\ & =424.115 \mathrm{~cm}^{2} \end{aligned}$ <br> Total Area of all labels $\begin{aligned} & =12 \times 424.115 \\ & =5089.38 \mathrm{~cm}^{2} \end{aligned}$ <br> Allow any sensible rounding. <br> Allow $1620 \pi \mathrm{~cm}^{2}$. | - Area of one label. <br> OR <br> Total Area of all labels but with a minor error. | - Correct answer. |  |
| (ii) | Volume of one tin $=15 \pi p^{2}$ <br> Volume of 12 tins $=180 \pi p^{2}$ <br> Volume of the box $=720 p^{2}$ <br> Volume of space $\begin{aligned} & =720 p^{2}-180 \pi p^{2} \\ & =180 p^{2}(4-\pi) \end{aligned}$ <br> Proportion Space $\begin{aligned} & =\frac{180 p^{2}(4-\pi)}{720 p^{2}} \\ & =\frac{(4-\pi)}{4} \end{aligned}$ | - Volume of one $\operatorname{tin}=15 \pi p^{2}$ <br> OR <br> Volume of one box $=720 p^{2}$ | - Volume of space $=180 p^{2}(4-\pi)$ OR <br> Proportion space with a minor error. | - Correct working. |
| (c)(i) | $\begin{aligned} & \mathrm{AB}^{2}=70^{2}+140^{2} \\ & \mathrm{AB}^{2}=24500 \\ & \mathrm{AB}=\sqrt{24500} \\ & \mathrm{AB}=156.52 \mathrm{~km} \end{aligned}$ <br> Not required to show that angle $\mathrm{APB}=90^{\circ}$ | - Correct answer, with appropriate working. |  |  |
| (ii) | $\begin{aligned} & \tan \angle \mathrm{ABP}=\frac{70}{140} \\ & \angle \mathrm{ABP}=\tan ^{-1}\left(\frac{70}{140}\right) \\ & \angle \mathrm{ABP}=26.57^{\circ} \end{aligned}$ <br> Required bearing $\begin{aligned} & =180^{\circ}+120^{\circ}+26.57^{\circ} \\ & =326.57^{\circ} \end{aligned}$ <br> Allow other valid methods. | - Finding, with appropriate working that $\angle \mathrm{ABP}=26.57^{\circ}$ OR CAO | - Correct bearing. |  |


| (iii) | $\text { Using speed }=\frac{\text { distance }}{\text { time }}$ <br> For ship W: $k=\frac{70}{\text { Time }_{\mathrm{W}}}$ $\operatorname{Time}_{\mathrm{W}}=\frac{70}{k}$ <br> For ship V: $S_{\mathrm{V}}=\frac{140}{\text { Time }_{\mathrm{V}}}$ $\begin{aligned} & \text { Time }_{\mathrm{V}}=\frac{70}{S_{\mathrm{V}}} \\ & \text { But Time } \\ & \mathrm{W} \\ & + \text { Time }_{\mathrm{V}}=4 \\ & \frac{70}{k}+\frac{140}{S_{\mathrm{V}}}=4 \\ & \frac{140}{S_{\mathrm{V}}}=4-\frac{70}{k} \\ & \frac{140}{S_{\mathrm{V}}}=\frac{4 k-70}{k} \\ & S_{\mathrm{V}}=\frac{140 k}{4 k-70} \end{aligned}$ <br> Allow equivalent solutions. | - Expression for time of ship W. <br> OR <br> Expression for time of ship $V$. <br> OR $y=\frac{140}{4}$ <br> AND $k=\frac{70}{4}$ | - Forming the equation $\frac{70}{k}+\frac{140}{S_{\mathrm{V}}}=4$ <br> OR <br> Correct expression for $S_{\mathrm{v}}$, but with a minor error. <br> OR $v=2 k$ | - Correct expression for $S_{\mathrm{v}}$. |
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| NØ | N1 | N2 | A3 | $\mathbf{A 4}$ | M5 | M6 | E7 | E8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No response; <br> no relevant <br> evidence. | ONE <br> question <br> attempted <br> towards <br> solution. | 1 u | 2 u | 3 u | 1 r | 2 r | t 1 | t 2 |


| Q | Evidence | Achievement | Merit | Excellence |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { TWO } \\ & \text { (a)(i) } \end{aligned}$ | $\begin{aligned} \angle \mathrm{HAB} & =\frac{(8-2) \times 180^{\circ}}{8} \\ & =135^{\circ} \\ \angle \mathrm{ZAB} & =v=\frac{135^{\circ}}{2}=67.5^{\circ} \end{aligned}$ <br> Do not allow assumption of $v=67.5^{\circ}$. | - Clear and justified working to show that $v=67.5^{\circ}$. |  |  |
| (ii) | In triangle ZAX: $\begin{aligned} & \sin 67.5=\frac{X Z}{120} \\ & X Z=120 \times \sin 67.5 \\ & X Z=110.87 \mathrm{~cm} \end{aligned}$ <br> Also $\cos 67.5=\frac{\mathrm{AX}}{120}$ $A X=120 \times \cos 67.5$ $\mathrm{AX}=45.92 \mathrm{~cm}$ <br> Area of triangle ZAX: $\begin{aligned} & =\frac{1}{2} \times 45.92 \times 110.87 \\ & =2545.69 \mathrm{~cm}^{2} \end{aligned}$ <br> Area of whole octagon: $\begin{aligned} & =16 \times 2515.69 \\ & =40730.99 \mathrm{~cm}^{2} \end{aligned}$ <br> Allow other valid methods. | - Finding, with appropriate working that $\mathrm{XZ}=110.87 \mathrm{~cm}$. OR <br> Finding, with appropriate working that $\mathrm{AX}=45.92 \mathrm{~cm}$. OR Consistent area of triangle. <br> OR <br> Area of octagon, but with a minor error. | - Correct answer for the area of the whole octagon. |  |
| (iii) | For the $n$-sided table: $\begin{aligned} & \angle \mathrm{HAB}=\frac{(n-2) \times 180^{\circ}}{n} \\ & \angle \mathrm{ZAB}=v=\frac{(n-2) \times 180^{\circ}}{2 n} \end{aligned}$ <br> In triangle ZAX: $\begin{aligned} & \sin \left(\frac{(n-2) \times 180}{2 n}\right)=\frac{\mathrm{XZ}}{\mathrm{p}} \\ & \mathrm{XZ}=p \sin \left(\frac{(n-2) \times 180}{2 n}\right) \end{aligned}$ <br> Also $\mathrm{AX}=p \cos \left(\frac{(n-2) \times 180}{2 n}\right)$ <br> Area of triangle ZAX: $=\frac{1}{2} \times p^{2} \times \sin \left(\frac{(n-2) \times 180}{2 n}\right) \times \cos \left(\frac{(n-2) \times 180}{2 n}\right)$ <br> Area of triangle ZAB: $=p^{2} \times \sin \left(\frac{(n-2) \times 180}{2 n}\right) \times \cos \left(\frac{(n-2) \times 180}{2 n}\right)$ <br> Area of whole polygon: $=n p^{2} \times \sin \left(\frac{(n-2) \times 180}{2 n}\right) \times \cos \left(\frac{(n-2) \times 180}{2 n}\right)$ | - Finding expression for angle $v$. <br> OR <br> Finding expression for height $h$. <br> OR <br> Finding expression for AX. | - Finding expression for area of triangle ZAX. OR Finding consistent expression for area of whole polygon. <br> OR <br> Finding expression for area of whole polygon, with one error | - Finding a correct expression for the area of the whole polygon table. Allow one minor error. |


|  | Alternative Method <br> In triangle ZAX: $\mathrm{AX}=p \cos \left(\frac{(n-2) \times 180}{2 n}\right)$ <br> Then Pythagoras' theorem in triangle ZAX gives: $\begin{aligned} & \mathrm{XZ}^{2}=p^{2}-p^{2} \cos ^{2}\left(\frac{(n-2) \times 180}{2 n}\right) \\ & \mathrm{XZ}=\sqrt{p^{2}-p^{2} \cos ^{2}\left(\frac{(n-2) \times 180}{2 n}\right)} \\ & \mathrm{XZ}=p \sqrt{1-\cos ^{2}\left(\frac{(n-2) \times 180}{2 n}\right)} \end{aligned}$ <br> Area of triangle ZAB $\begin{gathered} =p^{2} \times \sqrt{1-\cos ^{2}\left(\frac{(n-2) \times 180}{2 n}\right)} \\ \times \cos \left(\frac{(n-2) \times 180}{2 n}\right) \end{gathered}$ <br> Area of whole polygon $\begin{gathered} =p^{2} \times \sqrt{1-\cos ^{2}\left(\frac{(n-2) \times 180}{2 n}\right)} \\ \times \cos \left(\frac{(n-2) \times 180}{2 n}\right) \end{gathered}$ <br> Or equivalent solution. |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| (b)(i) | $\begin{aligned} & \text { Perimeter }=100 \mathrm{~cm} \\ & 2 x+2 y=100 \\ & x+y=50 \\ & y=50-x \end{aligned}$ <br> OR $\begin{aligned} & y^{2}=x^{2}+10^{2} \\ & y=\sqrt{x^{2}+100} \end{aligned}$ <br> OR equivalent. | - Finding any correct equation involving $x$ and $y$. | - Finding $y$ in terms of $x$. |  |
| (ii) | Pythagoras Theorem: $\begin{aligned} & x^{2}+10^{2}=(50-x)^{2} \\ & x^{2}+100=2500-100 x+x^{2} \\ & 100 x=2400 \\ & x=24 \mathrm{~cm} \end{aligned}$ <br> Area of triangle $\frac{1}{2} \times 48 \times 10=240 \mathrm{~cm}^{2}$ | - Expanding RHS to (\#1). <br> OR <br> Consistent simplification to equation in its simplest form. <br> OR <br> CAO | - Finding $x=24 \mathrm{~cm}$. <br> OR <br> Area of triangle but with a minor error. | - Area of triangle found. |


| NØ | N1 | N2 | A3 | A4 | M5 | M6 | E7 | E8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No response; <br> no relevant <br> evidence. | ONE <br> question <br> attempted <br> towards <br> solution. | TWO <br> questions <br> attempted <br> towards <br> solution. | 1 u | 2 u | 1 r | 2 r | t 1 | t 2 |


| Q | Evidence | Achievement | Merit | Excellence |
| :---: | :---: | :---: | :---: | :---: |
| THREE <br> (a)(i) | $\begin{aligned} & y=5 x+15 \\ & \text { Accept } 5 x+15 \end{aligned}$ | Correct answer. |  |  |
| (ii) | $y=2 x^{2}-2 x$ <br> Allow any equivalent version. | Coefficient of $x^{2}$ or $x$ correct. <br> AND <br> Recognition of a quadratic equation. |  |  |
| (iii) | $\begin{align*} & 2 x^{2}-2 x=5 x+15 \\ & 2 x^{2}-7 x-15=0 \\ & (2 x+3)(x-5)=0 \\ & 2 x+3=0 \text { AND } x-5=0 \\ & x=\frac{-3}{2} \text { AND } x=5 \end{align*}$ <br> Allow any equivalent form. <br> Allow consistency from (i) and (ii) as long as quadratic equation is formed. | Formaing the quadratic equation in three terms (\# 1) <br> OR <br> Consistent factorisation of quadratic at Level 6. | Both values of $x$. |  |
| (b) | Accurate graph drawn of $y=10 x+24$ <br> Accurate graph drawn of $y=3 x^{2}-14 x-120$ <br> Intersection points identified $x=12 \text { and } x=-4$ <br> Allow the algebraic rearrangement to $3 x^{2}-24 x-144=0$ or $x^{2}-8 x-48=0$ and then a graphical method using either of these graphs <br> Do not allow C.A.O. <br> Must have evidence of a graphical method for the award of grade $r$ or grade t. | Accurate graph of any of $\begin{aligned} & y=3 x^{2}-14 x-120 \\ & y=3 x^{2}-24 x-144 \\ & y=x^{2}-8 x-48 \end{aligned}$ <br> OR <br> Both intersection points identified algebraically. <br> OR <br> Evidence of a systematic process involving tables and method of trial and improvement for the two solutions. | Both intersection points identified, but not accurately. | Both intersection points identified accurately. <br> AND <br> with evidence of use of an accurate graph. |
| (c) | Area $=20$ $\begin{align*} & \frac{1}{2} \times x \times(2 x+x+7)=20 \\ & x(3 x+7)=40 \\ & 3 x^{2}+7 x-40=0 \\ & (3 x-8)(x+5)=0 \end{align*}$ <br> Either $3 x-8=0$ or $x+5=0$ | Setting up relevant equation (\#1) <br> OR <br> Consistent factorisation | Reaching stage (\#2) <br> OR <br> Consistent solutions | $x=\frac{8}{3}$ with evidence that $x=-5$ has been ignored. |

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## Solution to Question Three (b)




| NØ | N1 | N2 | A3 | A4 | M5 | M6 | E7 | E8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No response; <br> no relevant <br> evidence. | ONE point <br> made <br> incompletely. | 1 u | 2 u | 3 u | 1 r | 2 r | 1 t | 2 t |

## Cut Scores

| Not Achieved | Achievement | Achievement with Merit | Achievement with Excellence |
| :---: | :---: | :---: | :---: |
| $0-6$ | $7-12$ | $13-18$ | $19-24$ |

