National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Exemplar for Internal Achievement Standard

Mathematics and Statistics Level 1

This exemplar supports assessment against:

Achievement Standard 91029

Apply linear algebra in solving problems

An annotated exemplar is an extract of student evidence, with a commentary, to explain key aspects of the standard. It assists teachers to make assessment judgements at the grade boundaries.

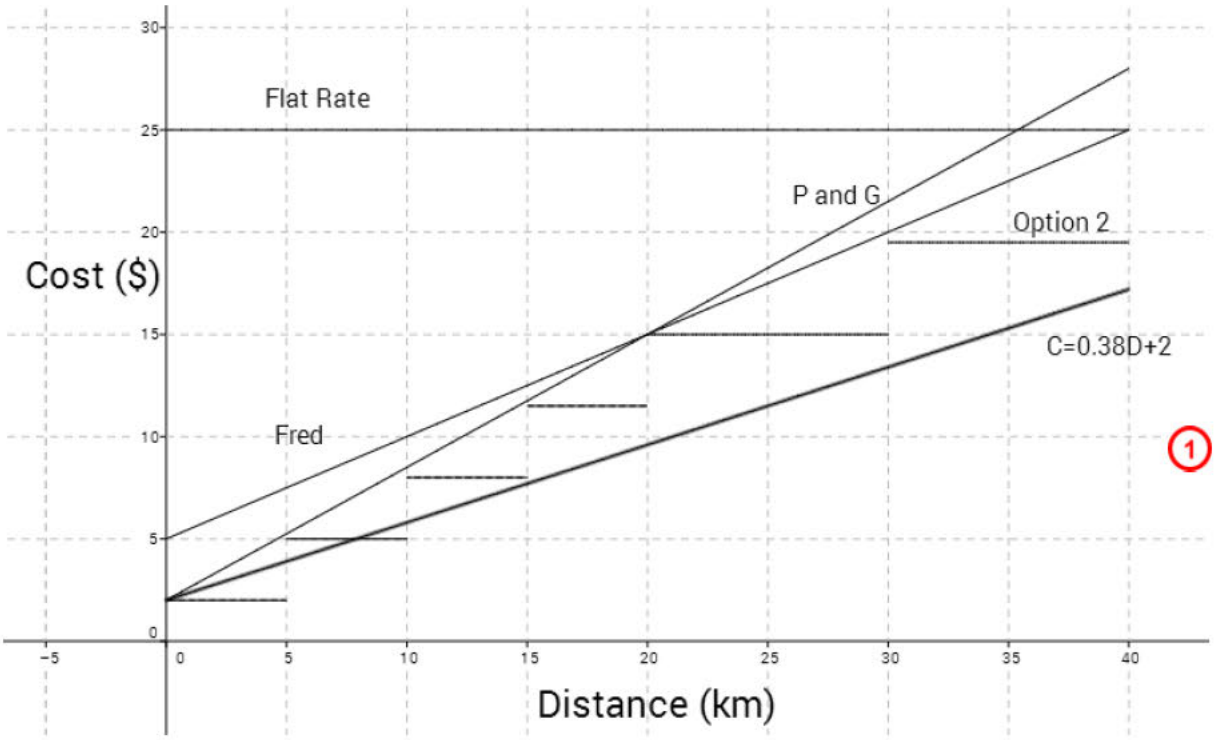
New Zealand Qualifications Authority

To support internal assessment

	Grade Boundary: Low Excellence
1.	For Excellence, the student needs to apply linear algebra, using extended abstract thinking, in solving problems. This involves one or more of: devising a strategy to investigate or solve a problem, identifying relevant concepts in context, developing a chain of logical reasoning, or proof, forming a generalisation and also using correct mathematical statements, or communicating mathematical insight. This student's evidence is a response to the TKI assessment resource 'Taxi Charges'. This student has identified relevant concepts in context to represent the three taxi companies' charges graphically (1), recommended which taxi company to use for two destinations (2), and recommended distances for which P&G Taxis is the cheapest (3). The student has also described and justified two options for Fred's Taxi Company to be the cheapest company (4) (5). Correct mathematical statements have been used throughout the response. For a more secure Excellence, the student would need to show greater depth in their thinking, for example, by discussing the reasons for selecting their options for Fred's Taxi Company to be the cheapest company. The student could also consider a range of distances for which the first of the suggestions for Fred's Taxis will be cheaper than the second.

Student 1: Low Excellence

NZQA Intended for teacher use only



1

	City Centre	Port	Bethlehem
Fred's Taxi Company	13.5	17	27.5
P and G Taxi Company	13.05	17.6	31.25
Flatrate Taxi Company	25	25	25

Best company to use to travel to the City Centre and the Port is Fred's Taxi Company because it the cheapest rate to the Port and the second cheapest to the City Centre. 2

It would be cheaper to use P and G for any distance less than 20km. 3

Fred could change his charge per km and also his fixed charge.

An example would be C = 0.38D + 2. C = cost, D = distance in km. 4

Another way Fred could make his company the cheapest would be to offer a fixed rate per distance.

$2 up to 5km	$5 up to 10km	$8 up to 15km
$11.50 up to 20km	$15 up to 30km	$19.50 up to 40km
$24 up to 60 km	$30 for anything above 60 km	

My graph shows that these are cheaper options because the graphs of these two options are always below the graphs of the first three options.

5

Fred will be cheaper with both but the second is better because Fred would make more money with it in most cases up to 40km.

	Grade Boundary: High Merit
2.	For Merit, the student needs to apply linear algebra, using relational thinking, in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts and representations, demonstrating understanding of concepts, forming and using a model and also relating findings to a context, or communicating thinking using appropriate mathematical statements. This student's evidence is a response to the TKI assessment resource 'Taxi Charges'. This student has connected different concepts and representations to represent the three taxi companies' charges algebraically (1), recommended which taxi company to use for two destinations (2), and recommended distances for which P&G Taxis is the cheapest (3). The student has also recommended new charge rates for Fred's Taxis (4). Appropriate mathematical statements have been used throughout the response. To reach Excellence, the student needs to alter option 1 so that it is always cheaper and explain their thinking about the new charges more clearly, for example with graphs.

Student 2: High Merit

NZQA Intended for teacher use only

1. Fred's Taxi Company C=0.5D+5
   P&G Taxi Company C=0.65D+2
   Flatrate Taxi Company C=60km=25

2. City Centre Port
   Fred's Taxi Company C = 0.50(17) + 5 = 8.5 + 5 = $13.50 (city centre)
   C = 0.50(24) + 5 = 12 + 5 = $17 (port)

   P and G Taxi Company C = 0.65(17) + 2 = 11.05 + 2 = $13.05 (city centre)
   C = 0.65(24) + 2 = 15.6 + 2 = $17.60 (port)

   Flatrate Taxi Company C = $25.00 (both port and city)

   I would recommend to use P and G Taxi Company to travel to the city centre. I would recommend Fred's taxi Company to travel to port.

3.

P&G	Freds	Flatrate
C = 0.65(20)+2	C = 0.5(20) + 5	C = $25.00
C = 13 + 2	C = 10 + 5	
C = $15.00	C = $15.00	
C = 0.65(19) + 2	C = 0.5(19) + 5	C = $25.00
C = 12.35 + 2	C = 9.5 + 5	
C = $14.35	C = $14.50	

P&G is the cheapest for up to 20km

4. In order for Fred to make his company the cheapest for people to use to travel to any destination he will need to change his flat rate to be cheapest for short trips and change his charge per kilometre in order to be cheapest over longer trips.

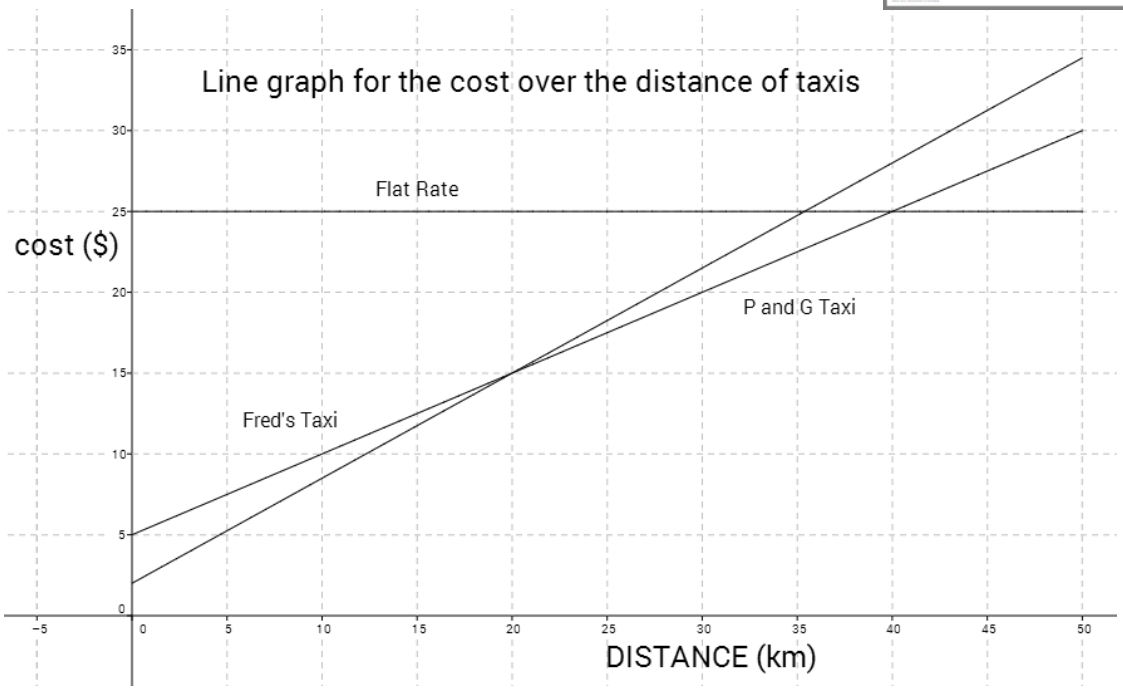
   I would recommend that Fred change to one of these two options
   1. C = 0.35D + 3 (covers both short and long distance trips).
   2. C = 24 (over 35km and up to 60km) and C = 0.60D+2 (under 35km trips)

   Fred could then charge the second rate if his client wants to travel over 60km.(only charge for every extra km travelled and add the $24.00 after)
   In the long run Fred would make more money if he used option 2.

	Grade Boundary: Low Merit
3.	For Merit, the student needs to apply linear algebra, using relational thinking, in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts and representations, demonstrating understanding of concepts, forming and using a model and also relating findings to a context, or communicating thinking using appropriate mathematical statements. This student's evidence is a response to the TKI assessment resource 'Taxi Charges'. This student has connected different concepts and representations to represent the three taxi companies' charges graphically (1), recommended which taxi company to use for one destination (2), and recommended, with justification, distances for which P&G Taxis is the cheapest (3). The student has also described one option for Fred's Taxi Company to be the cheapest company (4). Appropriate mathematical statements have been used. For a more secure Merit, the student would need to recommend which company is cheaper for a second destination and investigate further possible new charges for Fred's Taxi Company.

Student 3: Low Merit

NZQA Intended for teacher use only



1

Fred's Taxi Company	P And G Taxi Company	Flatrate Taxi Company
C = .5D+5	C = .65D + 2	C = 25
$0.50 / km	$0.65 / km	$25 fixed price
$5.00 fixed price	$2.00 fixed price	

C = cost D = kilometres

1. City Centre = 17km

Fred's Taxi:	C = .5D + 5	C = .5(17) + 5	C = $13.5
P and G Taxi	C = .65D + 2	C = .65(17) + 5	C = $13.05

P and G Taxi Company will be cheaper than Fred's when going to the city centre.

2

2. It would be cheapest to use P and G Taxi Company when travelling at a distance from 1km to 20 km. If you travel 20km or and above, it would be the same or a higher price than Fred's.

Fred's Taxi:	C = .5D + 5	C = .5(20) + 5	C = $15
P and G Taxi	C = .65D + 2	C = .65(20) + 5	C = $15

These are the same

Fred's Taxi:	C = .5D + 5	C = .5(19) + 5	C = $14.50
P and G Taxi	C = .65D + 2	C = .65(19) + 5	C = $14.35

P and G is the cheapest

Fred's Taxi:	C = .5D + 5	C = .5(21) + 5	C = $15.5
P and G Taxi	C = .65D + 2	C = .65(21) + 5	C = $15 .65

Fred's is the cheapest

3

3. For Fred to be the cheapest use C = .5D + 1. This would be cheaper than his original rate and it would be cheaper than the P and G Taxi Company.

4

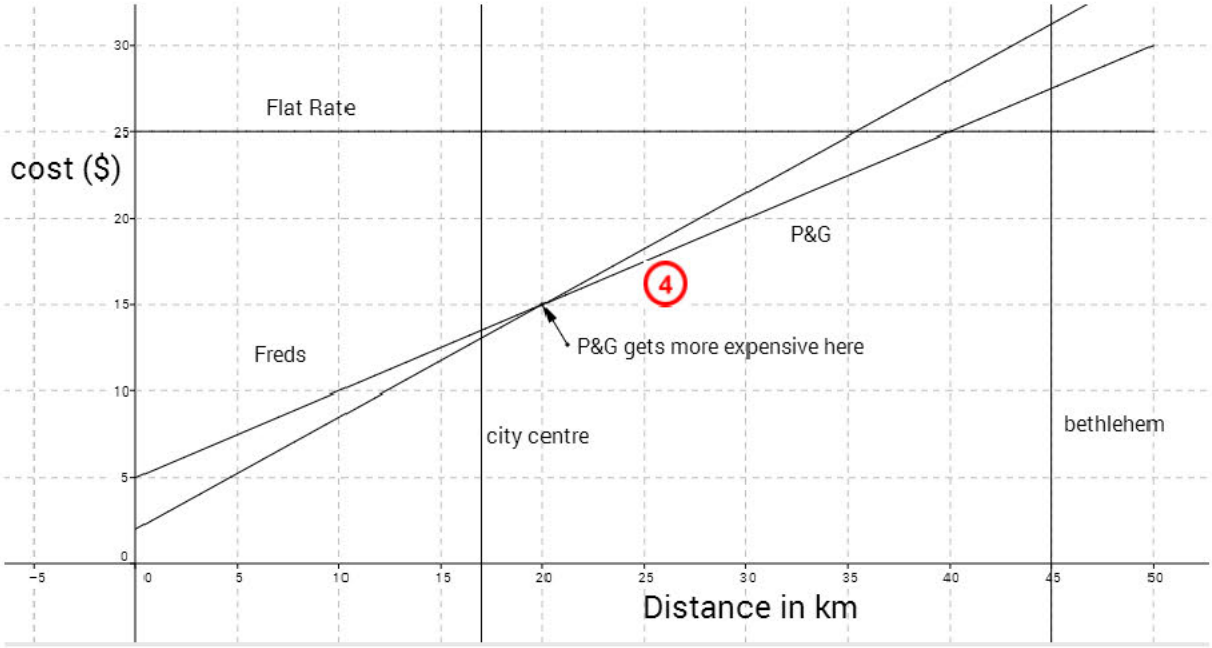
	Grade Boundary: High Achieved
4.	For Achieved, the student needs to apply linear algebra in solving problems. This involves selecting and using a range of methods, demonstrating knowledge of algebraic concepts and terms, and communicating solutions which usually require only one or two steps. This student's evidence is a response to the TKI assessment resource 'Taxi Charges'. This student has represented the three taxi companies' charges graphically (1), formed a linear model (2), recommended which taxi company to use for two destinations (3) and used simultaneous equations (implied by reading off the graph) to recommend distances for which P&G Taxis is the cheapest (4). To reach Merit, the student would need to clearly communicate their thinking with appropriate mathematical statements. For example, linking their decisions to the evidence visible in the graphs.

Student 4: High Achieved

1. Freds taxi company = C = 5+0.5D
   P and G Taxi company = C = 0.65D + 2
   Flatrate Taxi Company = C = 25 (2)

2. I would recommend taking P and G to the city centre and Flatrate to Bethlehem. (3)

3. 20 or less km's, P and G is cheapest.



(1) (4)

	Grade Boundary: Low Achieved
5.	For Achieved, the student needs to apply linear algebra in solving problems. This involves selecting and using a range of methods, demonstrating knowledge of algebraic concepts and terms, and communicating solutions which usually require only one or two steps. This student's evidence is a response to the TKI assessment resource 'Taxi Charges'. This student has represented the three taxi companies' charges graphically (1), formed a linear model (2) and used formulae to recommend which taxi company to use for two destinations (3). For a more secure Achieved, the student would need to improve the quality of the communication.

Student 5: Low Achieved

1. Freds = 5 + .5D
   Pand G = 2 + .65C
   Flatrate = 25

2

2.

City Centre
Freds = $13.50
Pand G = $13.05
Flatrate = $25

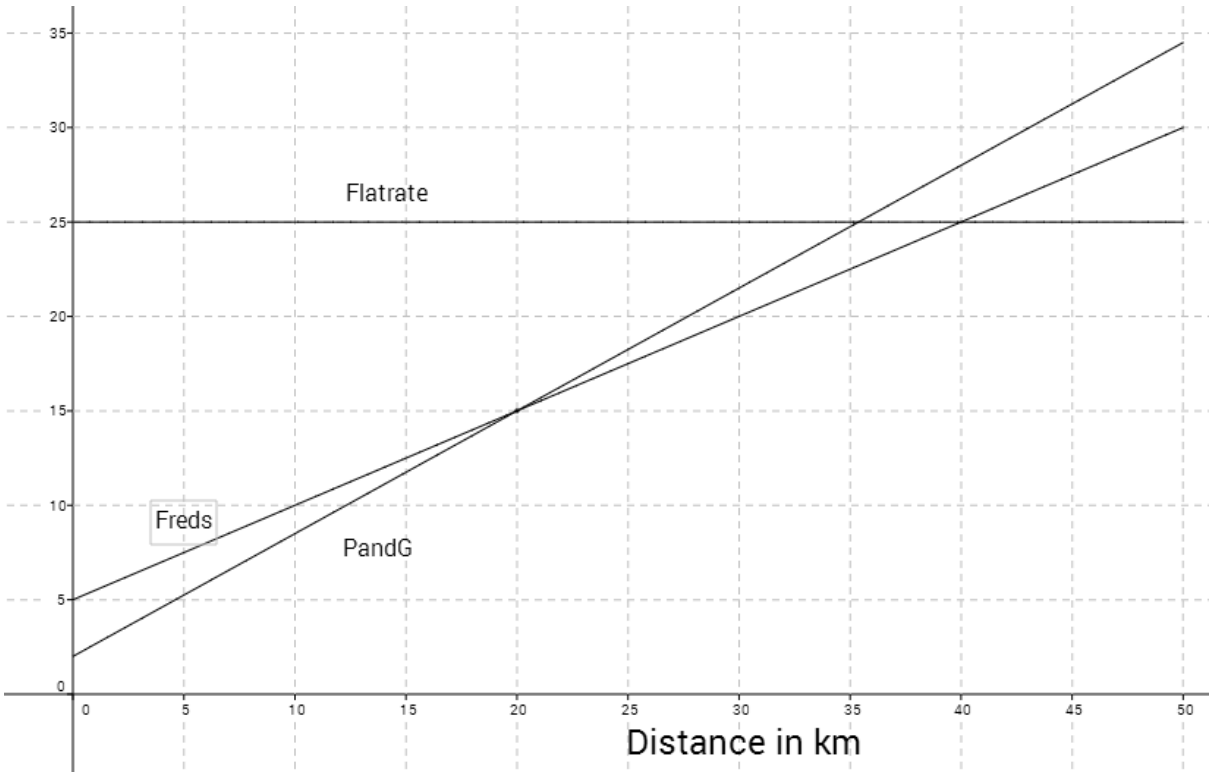
P and G is the cheapest

Bethlehem
Freds = $27.5
Pand G = $31.25
Flatrate = $25

Flatrate is the cheapest

3

3. Pand G cheapest from 0-18 km



1

	Grade Boundary: High Not Achieved
6.	For Achieved, the student needs to apply linear algebra in solving problems. This involves selecting and using a range of methods, demonstrating knowledge of algebraic concepts and terms, and communicating solutions which usually require only one or two steps. This student's evidence is a response to the TKI assessment resource 'Taxi Charges'. This student has formed a linear model (1) and correctly used formulae to recommend which taxi company to use for two destinations (2). To reach Achieved, the student would need to provide evidence of selecting and using another method in solving the problem.

Student 6: High Not Achieved

Fred	P and G	Flatrate
C = 5 + 0.50D	C = 0.65D + 2	C = 25 (up to 60km)

1

Cheapest to go to		
Bethlehem	City Centre	Port
Fred = $27.50	Fred = $13.50	Fred = $17
P&G = $31.25	P&G = $13.05	P&G = $17.60
Flatrate = $25.00	Flatrate = $25.00	Flatrate = $25

2

1. If I needed to hire a taxi to get to Bethlehem I would choose Flatrate taxi Company.
2. If I needed to hire a taxi to go to City Center I would choose P and G Taxi Company.