National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Exemplar for Internal Achievement Standard

Mathematics and Statistics Level 1

This exemplar supports assessment against:

Achievement Standard 91030

Apply measurement in solving problems

An annotated exemplar is an extract of student evidence, with a commentary, to explain key aspects of the standard. These will assist teachers to make assessment judgements at the grade boundaries.

New Zealand Qualification Authority

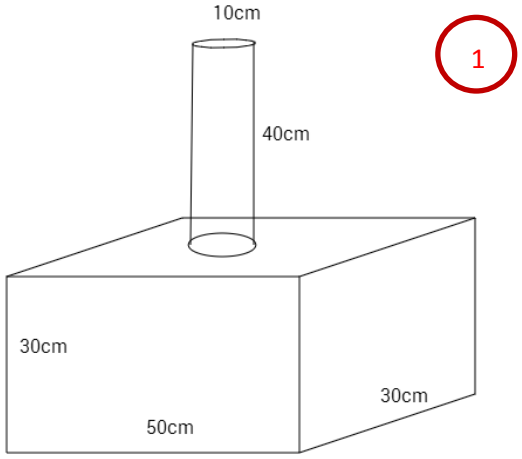
To support internal assessment from 2014

	Grade Boundary: Low Excellence
1.	For Excellence, the student needs to apply measurement, using extended abstract thinking, in solving problems. This involves one or more of: devising a strategy to investigate or solve a problem, identifying relevant concepts in context, developing a chain of logical reasoning, or proof, forming a generalisation and also using correct mathematical statements, or communicating mathematical insight. This student's evidence is a response to the TKI assessment resource 'Garden Sculpture'. This student has devised a strategy to investigate or solve a problem by designing a sculpture (1) finding the length of the surrounding fence (2), the area of the paved surround (3) the total volume of the sculpture (4) and the dimensions of the blocks from which the pieces would be carved (5). They have begun to investigate the minimum volume of the crate (6). The student has used correct mathematical statements throughout the response. For a more secure Excellence, the student would need to find the minimum volume of stone needed for the sculpture, and justify why the dimensions of the crate are a minimum. For example, they could do this by referring to the position of the pieces in the crate, and adjusting the height to provide some space between the pieces and the top of the crate.

Student 1: Low Excellence

Length of fence surrounding =
110+90+110+90 = 400cm

2



1

Area of sculpture base =50x30 = 1500cm^2
Area of paved area including sculpture base
110x90 = 9900cm^2
Area of paved surface
9900-1500 = 8400cm^2

3

Volume of cylinder
$\pi r^2 \times h = \pi 5^2 \times 40 = 3141.59cm^3$
or $\pi 0.05^2 \times 0.4 = 0.00314m^3$

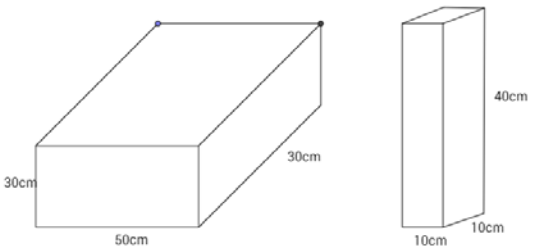
Volume of rectangular prism
50x30x30 =45 000cm^3
or 0.5x0.3x0.3=0.045m^3

4

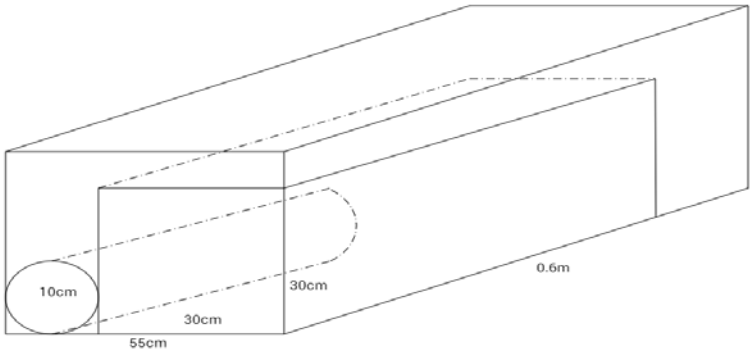
Total volume of sculpture
45000+3141.59 = 48 141.59cm^3
or 0.045+0.00314 = 0.04814m^3

5

Dimensions of blocks required
Cylinder
Rectangular prism=10x10x40=4 000cm^3
Base
Rectangular prism=50x30x30=45 000cm^3

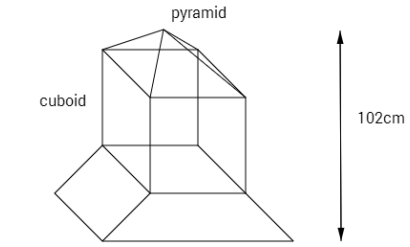


The pieces could be packed side by side so the minimum height would be 30cm
So minimum dimensions 55cmx60cmx30cm
and minimum volume = 99 000 cm^3=0.099m^3
but I think there will be plenty of space at the back and on the side



6

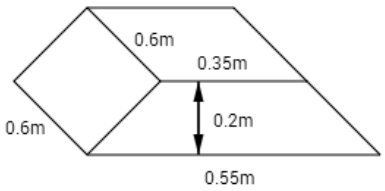
	Grade Boundary: High Merit
2.	For Merit, the student needs to apply measurement, using relational thinking in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts and representations, demonstrating understanding of concepts, forming and using a model and also relating findings to a context, or communicating thinking using appropriate mathematical statements. This student's evidence is a response to the TKI assessment resource 'Garden Sculpture'. This student has carried out a logical sequence of steps by designing a sculpture (1), finding the length of the surrounding fence (2), the area of the paved surround (3), the total volume of the sculpture (4), the dimensions of the blocks from which the pieces would be carved (5) and the minimum volume of stone needed (6). The student has communicated thinking using appropriate mathematical statements. To reach Excellence, the student would need to investigate the minimum volume of a crate that the pieces of the sculpture will fit into neatly, for example, by considering that the pieces could be separated and adjusting the dimensions of the base so that there is some space between the pieces and the sides of the crate.



1

Dimensions of crate:
55cm long, 60cm wide, 102cm high

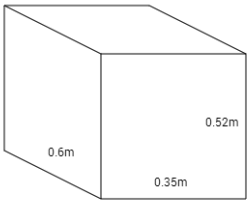
In this case I am assuming that the sculpture cannot be disassembled.



Part A: base shape- trapezoidal prism
length 0.55m, top length 0.35m, width 0.6m, height 0.2m
Volume of stone in part A:

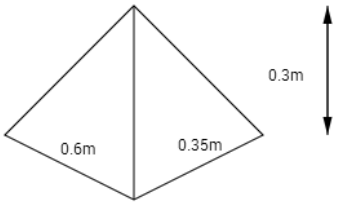
$$= \left(\frac{0.35 + 0.55}{2} \right) \times 0.2 \times 0.6$$

$$= \frac{0.9}{2} \times 0.2 \times 0.6 = 0.45 \times 0.2 \times 0.6 = 0.054 m^3$$



Part B: shape – cuboid

Volume = 0.6x0.2x0.6=0.1092m^3



Part C: shape – pyramid
Height=0.3m Base area = 0.35x0.6=0.21m^2
Volume of pyramid

$$= \frac{1}{3} \times 0.21 \times 0.3 = 0.021 m^3$$

Total volume of sculpture
= 0.054+0.1092+0.021=0.1842m^3

4

2

Length of fence = (1.15+1.2) x 2 = 4.7m
Area of paving = 1.15x1.2 – 0.6x0.55 = 1.05m^2 because the base does not sit on the paving.

3

Calculating the minimum volume of stone needed.
Smallest cuboid to cover trapezoidal prism: Volume = 0.55x0.2x0.6 = 0.066m^3
Smallest cuboid to cover cuboid: Volume = 0.6x0.35x0.52 = 0.1092m^3
Smallest cuboid to cover pyramid: Volume =0.35x0.6x0.3=0.063m^3

5

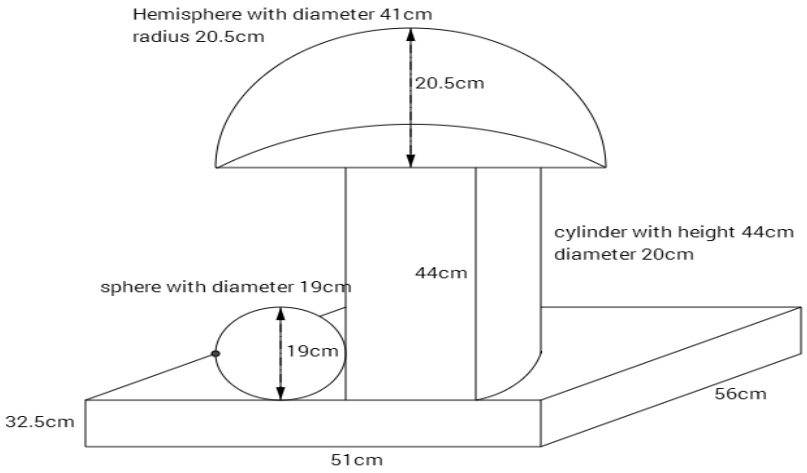
Total minimum volume of stone for sculpture = 0.066 + 0.1092 + 0.063 = 0.2382m^3 of stone
Note; 0.2832-0.01842 = 0.054m^3 is amount of space wasted.

However as stone blocks from the local quarry come in cuboid blocks the minimum volume of stone needs to be established by carving each geometrical shape from 3 different cuboid blocks. Because of this the total stone needed to construct this sculpture is 0.2382m^3 and 0.054m^3 of this stone is wasted.

6

	Grade Boundary: Low Merit
3.	For Merit, the student needs to apply measurement, using relational thinking in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts and representations, demonstrating understanding of concepts, forming and using a model and also relating findings to a context, or communicating thinking using appropriate mathematical statements. This student's evidence is a response to the TKI assessment resource 'Garden Sculpture'. This student has carried out a logical sequence of steps by designing a sculpture (1) finding the length of the surrounding fence (2), the total volume of the sculpture (3), the dimensions of the blocks from which the pieces would be carved (4) and the minimum volume of stone needed for the sculpture (5). The student has communicated thinking using appropriate mathematical statements. For a more secure Merit, the student would need to correctly calculate the area of paving around the sculpture, begin to consider how the blocks might be placed in a crate to minimize the volume, and round appropriately throughout their work.

Student 3: Low Merit



1

Volume of sculpture

Cuboid = 32.5x51x56 = 92820cm^3

Sphere $= \frac{4}{3}\pi \times 9.5^3 = 3591.364002cm^3$

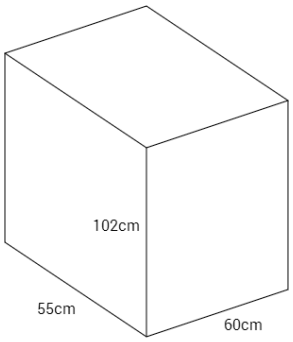
Hemisphere $= \frac{1}{2}\left(\frac{4}{3}\pi \times 20.5^3\right) = 18043.47561cm^3$

Cylinder = $\pi \times 10^2 x44 = 13823.00768cm^3$

Total volume = 128277.8473cm^3 = 0.1282778473m^3

3

Volume of box



=0.55x0.6x1.02
=0.3366m^3
=336600cm^3

Volume left over from box
= 336600-128277.8473
=208322.1527cm^3

Length of fence
=(111+116)x2 = 454cm = 4.54m

2

Area of paving
=111x116 = 12876cm^2 = 1.2876m^2

One cuboid = 51x56x32.5 = 92820cm^3 for base
One cuboid = 20x20x44 = 17600cm^3 for cylinder
One cuboid = 19x19x19 = 6859cm^3 for sphere
One cuboid = 41x41x20.5 = 34460.5cm^3 for hemisphere

4

Maximum total volume required
= 92820+17600+6859+34460.5
= 151739.5 cm^3
= 0.1517395 m^3

5

	Grade Boundary: High Achieved
4.	For Achieved, the student needs to apply measurement in solving problems. This involves selecting and using a range of methods, demonstrating knowledge of measurement concepts and terms and also communicating solutions which would usually require only one or two steps. This student's evidence is a response to the TKI assessment resource 'Garden Sculpture'. This student has designed a sculpture (1) found the area of the paved surround (2), the volumes of the pieces that make up the sculpture (3) and the total volume of the sculpture (4). To reach Merit, the student would need to find the length of the fence, together with the dimensions and volumes of the blocks required to make all of the pieces.

Student 4: High Achieved

Part 1

Volume of A = 1/2x10x40x30=6000cm^3

Volume of B = 40x50x30=60 000cm^3

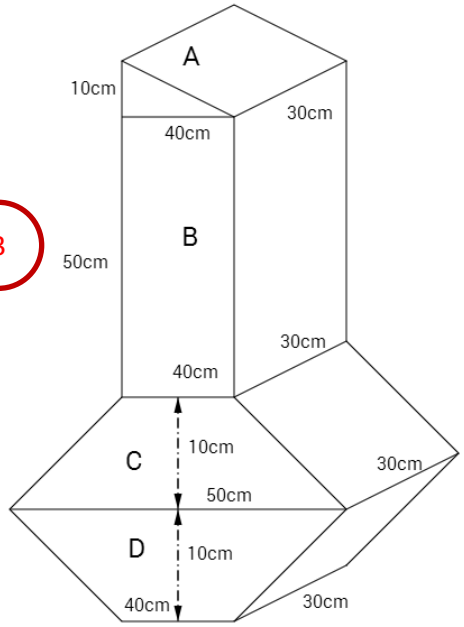
Volume of C = ½(40+50)x10x30=13 500cm^3

Volume of D = ½(40+50)x10x30=13 500cm^3

Volume of sculpture = A + B + C + D

= 93 000cm^3
= 0.093m^3

3



1

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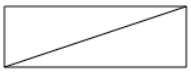
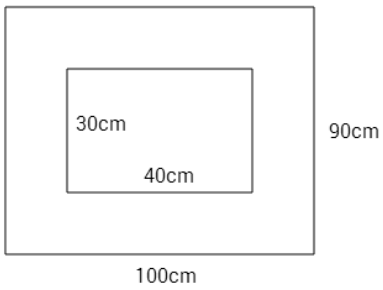
Part 2
For shape A the minimum volume of stone is 12 000cm^3. Because it is a triangular prism the volume must be doubled to form two triangles so a rectangular box is formed.



For shape B the shape is already a rectangular object so the volume of stone required is its volume = 60 000cm^3



Area = 100 x 90 = 9 00cm^2
Area of paving = 0.9 – 0.12 = 0.78m^2

2

	Grade Boundary: Low Achieved
5.	For Achieved, the student needs to apply measurement in solving problems. This involves selecting and using a range of methods, demonstrating knowledge of measurement concepts and terms and also communicating solutions which usually require only one or two steps. This student's evidence is a response to the TKI assessment resource 'Garden Sculpture'. This student has designed a sculpture (1), found the length of the surrounding fence (2), the area of the paved surround (3), the volumes of the pieces that make up the sculpture (4) and the total volume of the sculpture (5). For a more secure Achieved, the student would need to improve the communication of the solution. For example, they could do this by indicating what each calculation represents and including measurements on the diagram of the sculpture.

Student 5: Low Achieved

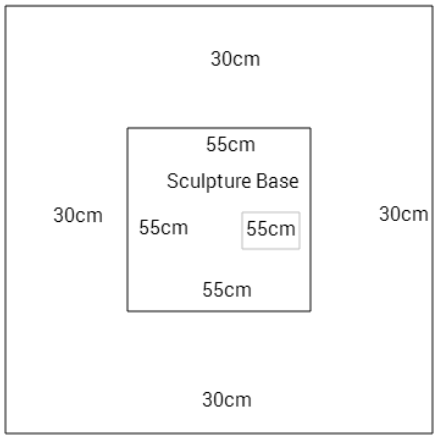
1.15m x 1.15m = 1.3225m^2

0.55m x 0.55m = 0.3025m^2

1.3225 – 0.3025 = 1.02

Paved area = 1.02m^2

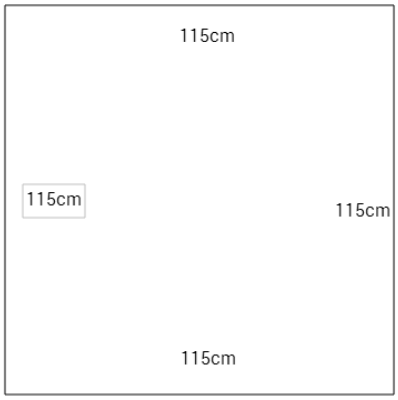
3



115x4 = 460

Length of fence = 4.6m

2



π x 10^2 = 314 314 ÷ 2 = 157
157 x 55 = 8635 cm^3

20+55 = 75 75 ÷ 2 = 37.5
37.5 x 20 = 750 750x55 = 41250cm^3

20 x 10 = 200 200 x 55 = 11000
11000 x 2 = 22000cm^3

4

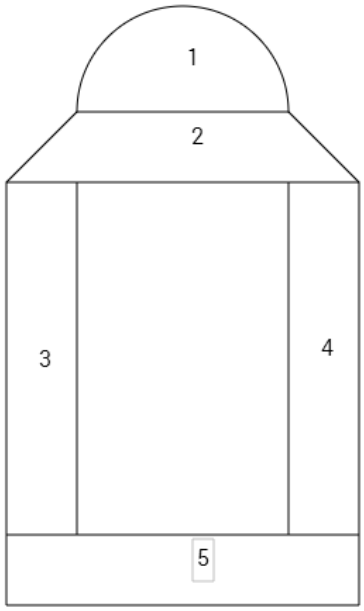
10 x 55 = 550 550 x 55 = 30250cm^3

8635 + 41250 = 49885 + 22000 = 71885
+ 3025 = 74910cm^3

Total volume = 74910cm^3

5

1



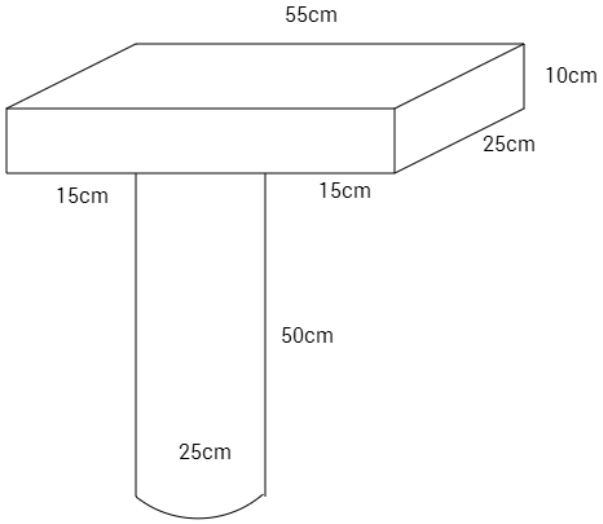
	Grade Boundary: High Not Achieved
6.	For Achieved, the student needs to apply measurement in solving problems. This involves selecting and using a range of methods, demonstrating knowledge of measurement concepts and terms and also communicating solutions which usually require only one or two steps. This student's evidence is a response to the TKI assessment resource 'Garden Sculpture'. This student has used units correctly throughout their work and found the length of the fence (1). The calculation for the cylinder is incorrect (2). Calculating the volume of a cuboid is at too low a level of the curriculum to provide evidence of selecting and using a method (3). To reach Achieved, the student would need to provide evidence of selecting and using another method in the solving of a problem (to demonstrate more range), and provide calculations that are at the appropriate curriculum level. For example, the student could do this by finding the correct volumes of the cylinder and the total sculpture.

Mathematics and Statistics – Garden Sculpture

My sculpture will consist of a rectangular prism on top of a cylinder.

Volume of cube =
Length x with x height =
10 x 25 x 55 = 13750cm^3

3



Volume of cylinder =
π x radius^2 x height =
π x 25^2 x 50 = 98174cm^3

2

Total volume =
98174 + 13750 =
111924cm^3

The base of the sculpture is a circle with a diameter of 25cm. To find the length of the fence and the area of the paving we must first add the paving onto the base, then find the circumference of the whole circle.

25 + 60 = 85cm (diameter of whole area = 85cm)

circumference = π x d = π x 85 = 267cm

1

The length of the surrounding fence is 267cm.