National Certificate of Educational Achievement
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Exemplar for Internal Achievement Standard

Mathematics and Statistics Level 1

This exemplar supports assessment against:

Achievement Standard 91032

Apply right-angled triangles in solving measurement problems

An annotated exemplar is an extract of student evidence, with a commentary, to explain key aspects of the standard. It assists teachers to make assessment judgements at the grade boundaries.

New Zealand Qualifications Authority

To support internal assessment

	Grade Boundary: Low Excellence
1.	For Excellence, the student needs to apply right-angled triangles, using extended abstract thinking, in solving measurement problems. This involves one or more of: devising a strategy to investigate or solve a problem, identifying relevant concepts in context, developing a chain of logical reasoning, or proof, forming a generalisation and also using correct mathematical statements, or communicating mathematical insight. This student's evidence is a response to the TKI assessment resource 'Demolition RATs'. The student has devised a strategy to determine the perimeter of the new cafeteria (1), investigated whether the interior angles of the new cafeteria meet the Board requirement (2) and found the perimeter of both options for the enlarged cafeteria (3). Correct mathematical statements have been used throughout the response. For a more secure Excellence, the student would need to develop the discussion about the enlarged cafeteria and clearly communicate the minimum perimeter.

Student 1: Low Excellence

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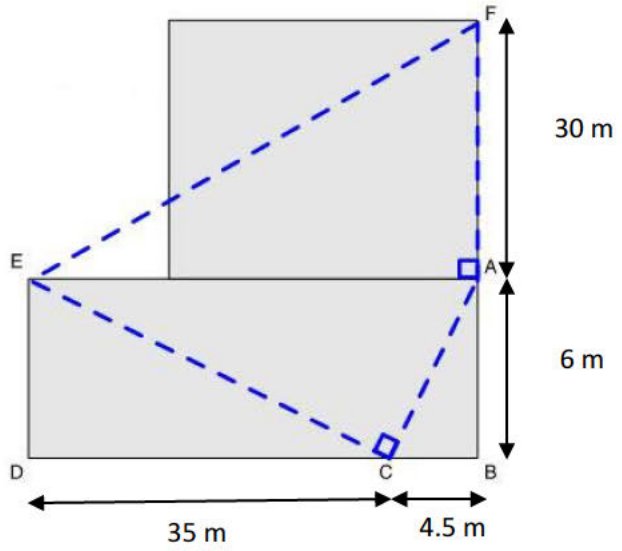


$AC^2 = 6^2 + 4.5^2$
$AC^2 = 36 + 20.25$
$AC^2 = 56.25$
AC = 7.5 m

$EC^2 = 35^2 + 6^2$
$EC^2 = 1225 + 36$
$EC^2 = 1261$
EC = 35.5 m to 1 dp

$EF^2 = 39.5^2 + 30^2$
$EF^2 = 1560.25 + 900$
$EF^2 = = 2460.25$
EF = 49.6 m to 1 dp

Perimeter of new building (4 sides)
A to F = 30 m
F to E = 49.6 m
E to C = 35.5 m
C to A = 7.5 m
Perimeter of new building is 30 m + 49.6 m + 35.5 m + 7.5 m = 122.6 m (1)

Angle EFA

$$\tan EFA = \frac{39.5}{30}$$

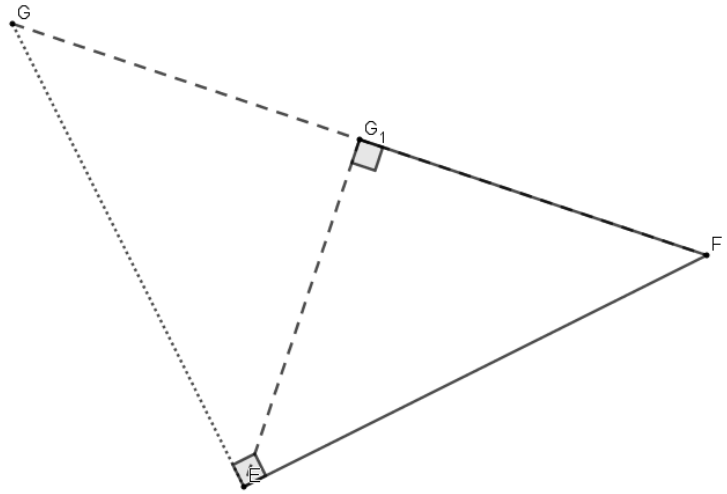
$$EFA = \tan^{-1}\frac{39.5}{30}$$

EFA = 52.8° The angle is less than 60° so does not meet the interior angle requirement.

Angle FEC
FEA = 90° - 52.8°
FEA = 37.2°

$$\sin AEC = \frac{7.5}{39.5}$$

AEC = 10.9°
FEC = 37.2 + 10.9 = 48.1° This angle is also less than 60° (2)



Point G and G_1 are there for two reasons. Reason one because the two sides needed to be even due to the board wanting an additional room to be an isosceles triangle. Reason two: the board also wanted the room to be right-angled and to find the right angle I could use FE and make the right angle and put G or find the middle of the E-F line and from that point I could put G_1.

$x^2 + x^2 = 49.6^2$
$2x^2 = 2460.16$
x = 35.1 m to 1 dp
$GF^2 = 49.6^2 + 49.6^2$
$GF^2 = 4920.32$
GF =70.1 m to 1 dp
A to F = 30 m
F to G = 35.1 m
G to E = 35.1 m
E to C = 35.5 m
C to A = 7.5 m
30 + 35.1 + 35.1 + 35.5 + 7.5 = 143.2 m

3

A to F = 30 m
F to G = 70.1 m
G to E 49.6 m
E to C = 35.5 m
C to A = 7.5 m
30 + 70.1 + 49.6 + 35.5 + 7.5 = 192.7 m

	Grade Boundary: High Merit
2.	For Merit, the student needs to apply right-angled triangles, using relational thinking, in solving measurement problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts and representations, demonstrating understanding of concepts, forming and using a model and also relating findings to a context, or communicating thinking using appropriate mathematical statements. This student's evidence is a response to the TKI assessment resource 'What's the Angle?'. The student has selected and carried out a logical sequence of steps to correctly determine most of the dimensions of the sail (1) and the minimum dimensions of the van without including the extra 2.5 m in the length of the pole (2). Appropriate mathematical statements have been used throughout the response. To reach Excellence, the student would need to determine all dimensions of the sail, relate the findings to the context by describing its shape, consider the extra 2.5 m in the length of the pole and identify the minimum dimensions of the van.

Student 2: High Merit

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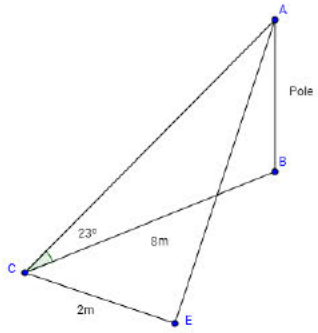
1. $\tan 23 = \frac{AB}{8}$

AB = tan 23 x 8

AB = 3.4m (1dp)

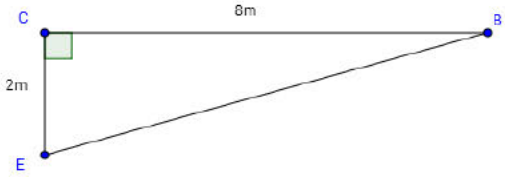


2. $AC^2 = CB^2 + AB^2$

$AC^2 = 8^2 + 3.4^2$

$AC = \sqrt{64 + 11.56}$

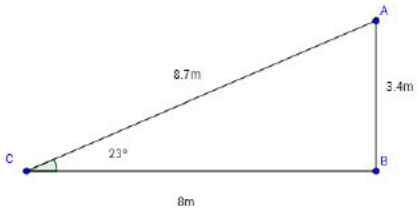
AC = 8.7m (1dp)



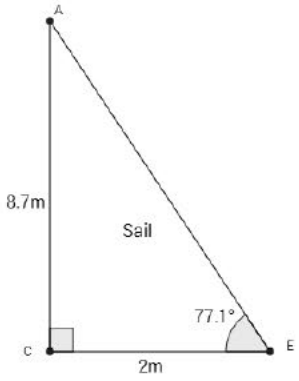
3. $\tan E = \frac{8.7}{2}$

$E = \tan^{-1}\left(\frac{8.7}{2}\right)$

$E = 77.1^0$ (1dp)



The Sail



1

$\sin 13 = \frac{x}{AB}$

x = sin13 x 3.4 = 0.76m

$\cos 13 = \frac{AD}{AB}$

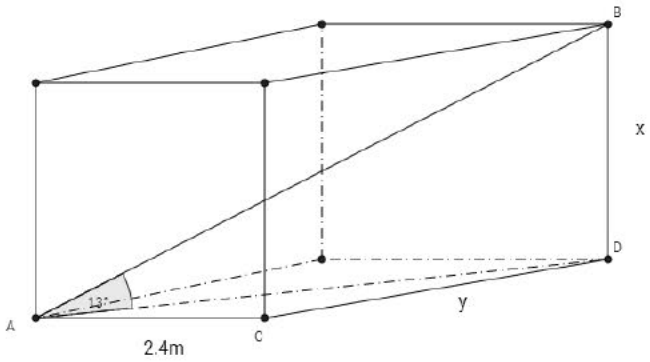
$\cos 13 = \frac{AD}{3.4}$

AD = cos13 x 3.4 = 3.31m (2dp)

$y^2 = AD^2 - AC^2$

$y = \sqrt{(10.96 - 5.76)}$

y = 2.28m



2

	Grade Boundary: Low Merit
3.	For Merit, the student needs to apply right-angled triangles, using relational thinking, in solving measurement problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts and representations, demonstrating understanding of concepts, forming and using a model and also relating findings to a context, or communicating thinking using appropriate mathematical statements. This student's evidence is a response to the TKI assessment resource 'What's the Angle?'. The student has selected and carried out a logical sequence of steps in calculating the lengths of AB (1) and AC (2) correctly, but there is a transfer error in calculating the angle in the sail (3). The student has demonstrated relational thinking by considering the correct length of the pole in finding minimum dimensions for the van (4). For a more secure Merit, the student would need to calculate the correct dimensions of the sail and relate the findings to the context by clearly describing the shape of the sail.

Student 3: Low Merit

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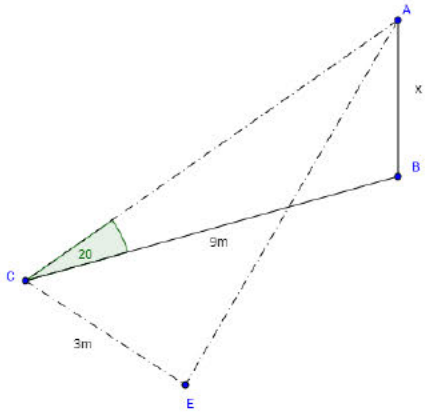
Calculate Sail dimensions

$$\tan 20 = \frac{x}{9}$$

x = tan20 = 9tan 20 = 3.28

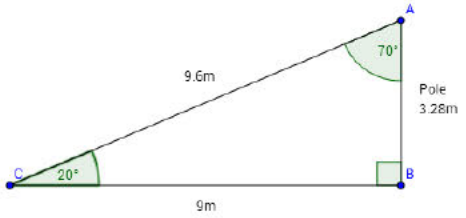
x = 3.28 m (pole height)



1

$$AC = \sqrt{(9^2 + 3.28^2)}$$

AC = 9.58 m (Sail side)



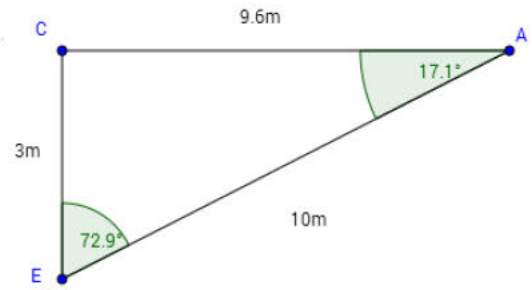
2

$$\tan x = \frac{9.78}{3} = \angle CEA$$

$$x = (\frac{9.78}{3}) \tan^{-1}$$

$$x = 72.9°$$

$\angle CEA = 72.9°$ (Sail angle)



3

Length of pole

3.28 + 2.5 = 5.78 m

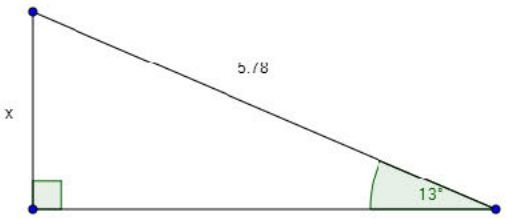
$$\frac{x}{5.78} = \sin 13$$

x = 5.78sin13 = 1.3 m

$y = \sqrt{(5.78^2 - 1.3^2)}$ = 5.63 m

$\sqrt{(5.63^2 - 2.4^2)}$ = 5.1 m (1dp

Smallest van is 5.1 m long



4

	Grade Boundary: High Achieved
4.	For Achieved, the student needs to apply right-angled triangles in solving measurement problems. This involves selecting and using a range of methods, demonstrating knowledge of measurement and geometric concepts and terms, and communicating solutions which would usually require only one or two steps. This student's evidence is a response to the TKI assessment resource 'What's the Angle?'. The student has measured at a level of precision appropriate to the task, calculated the height of the pole (1), the length of one side of the sail (2) and an angle in the sail (3). To reach Merit, the student would need to identify the shape and size of the sail and correctly link the length of the pole to the dimensions of the trailer.

Student 4: High Achieved

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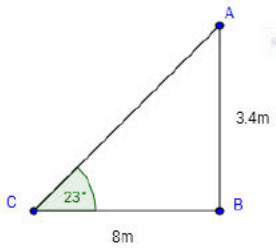
$T\frac{O}{A}C\frac{A}{H}S\frac{O}{H}$

Height of pole

AB = tan23x8 = 3.4m



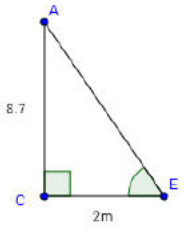
1

Dimensions of sail

$AC=\sqrt{(8^2+3.4^2)}$

$=\sqrt{75.56}$

$=8.7m$

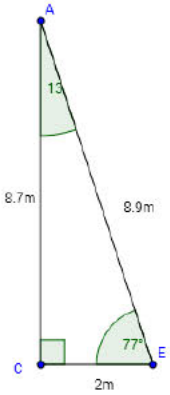


2

Triangle ACE

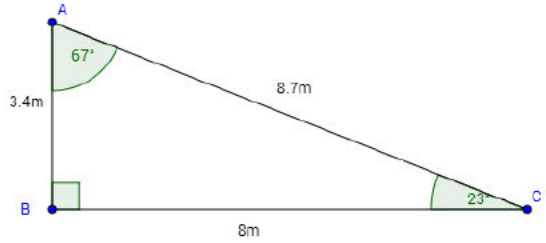
$\angle CEA=\tan^{-1}(\frac{8.7}{2})$

$=77.1°$



3

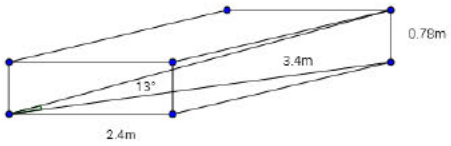
Triangle ABC



$\frac{x}{3.4}=\tan 13°$

$x=3.4\times\tan 13$

$x=0.78m$



	Grade Boundary: Low Achieved
5.	For Achieved, the student needs to apply right-angled triangles in solving measurement problems. This involves selecting and using a range of methods, demonstrating knowledge of measurement and geometric concepts and terms, and communicating solutions which usually require only one or two steps. This student's evidence is a response to the TKI assessment resource 'What's the Angle?'. The student has measured at a level of precision appropriate to the task, and calculated the height of the pole (1) and the length of one side of the sail (2). For a more secure Achieved, the student would need to more clearly communicate what the second and third calculations represent.

Student 5: Low Achieved

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a)

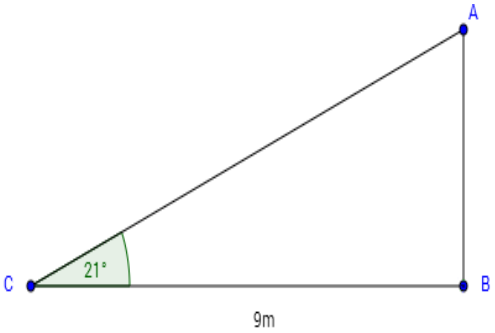
$$\tan 21 = \frac{x}{9}$$

$$\tan 21 \times 9 = x$$

$$x = 3.45$$

$$= 3.45$$

The height of the pole is 3.45m



1

b)

$$a^2 + b^2 = c^2$$

$$9^2 + 3.45^2 = c^2$$

$$c^2 = 92.903$$

$$c = 9.6m$$

$$= 9.6m$$

2

c)

$$\tan^{-1}\left(\frac{9}{2.1}\right)$$

$$= 76.9$$

$$= 76.9$$

	Grade Boundary: High Not Achieved
6.	For Achieved, the student needs to apply right-angled triangles in solving measurement problems. This involves selecting and using a range of methods, demonstrating knowledge of measurement and geometric concepts and terms, and communicating solutions which usually require only one or two steps. This student's evidence is a response to the TKI assessment resource 'What's the Angle?'. The student has taken some measurements at a level of precision appropriate to the task and found the length of one edge of the sail (1). The student has not calculated the length of AB correctly. To reach Achieved, the student would need to provide evidence of selecting and using another method correctly in solving the problem.

Student 6: High Not Achieved

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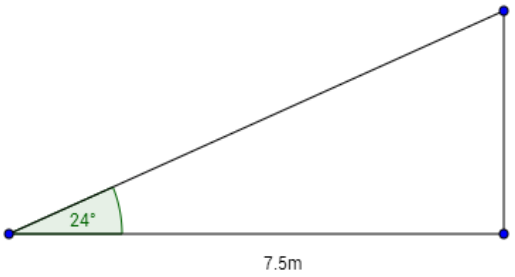
$$AB = \sin 24 = \frac{O}{H}$$

$$= \frac{AB}{7.5}$$

$$7.5 \sin 24 = AB$$

$$AB = 3.05m(2dp)$$



$$AC = \cos 24 = \frac{A}{H}$$

$$\frac{A}{H} = \frac{7.5}{H}$$

$$\frac{7.5}{\cos 24} = AC$$

$$AC = 8.21m(2dp)$$

1