National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Exemplar for Internal Achievement Standard

Mathematics and Statistics Level 1

This exemplar supports assessment against:

Achievement Standard 91038

Investigate a situation involving elements of chance

An annotated exemplar is an extract of student evidence, with a commentary, to explain key aspects of the standard. It assists teachers to make assessment judgements at the grade boundaries.

New Zealand Qualifications Authority

To support internal assessment

	Grade Boundary: Low Excellence
1.	For Excellence, the student needs to investigate, showing statistical insight, a situation involving elements of chance. This involves integrating contextual information and knowledge with an understanding of applications of probability and may involve considering the possible effects of other related variables or factors. This student's evidence is a response to the TKI assessment resource 'Free Throw'. The student has posed a question to explore a situation involving elements of chance (1), planned an experiment to explore the situation (2), gathered data by performing the experiment (3), selected and used appropriate displays including experimental probability distributions (4), identified and communicated patterns in the data (5) and communicated findings in a conclusion (6). They have also integrated contextual information and knowledge with an understanding of applications of probability in the investigation (7) and considered the possible effect of a related factor on the probability (8). For a more secure Excellence, the student would need to strengthen the integration of contextual information and knowledge with an understanding of applications of probability. For example, they could give a reason(s) why they would expect these results and give a reason(s) for the final comment about the result not being like a real game.

Student 1: Low Excellence

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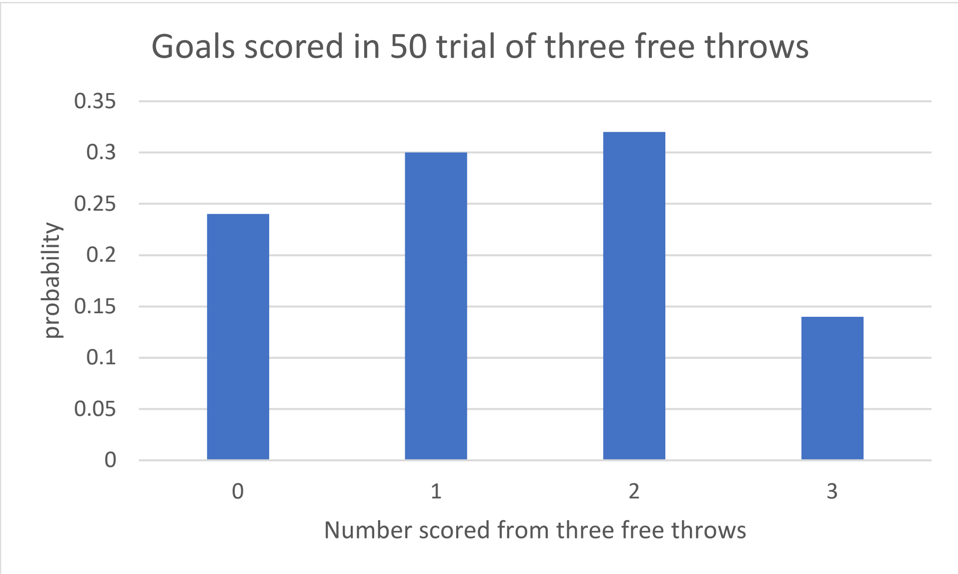
I want to investigate the chances of getting each possible outcomes (0,1,2,3 goals) when 1
taking three free shots. I would expect to get one or two goals because I play a bit and so I
think I would get quite few in. I will take a break of 30 secs between sets of three and two
minutes between each lot of 10. I will also ensure that I stand in the same place on the free 2
throw line every time. To do this I will mark with masking tape where my feet are so as each
time my feet are in the position. This is to try to keep the shooting conditions the same. I will
also try and ensure that each time I let the basketball go that the movement is the same I
think it will be as I have practice throwing free throws a lot and so my muscles have
something called muscle memory so my PE teacher tells me. 50 trials should be good
number, it is enough to get a reasonable set of results but not too many so that I get too tired 3
or change the I way I throw too much.

Number of goals	first 10	second 10	third 10	fourth 10	fifth 10	total
0	5 (0.5)	2 (0.2)	1 (0.1)	1 (0.1)	3 (0.3)	12 (0.24)
1	2 (0.2)	3(0.3)	4 (0.4)	2 (0.2)	4 (0.4)	15 (0.3)
2	2 (0.2)	4 (0.4)	4 (0.4)	4 (0.4)	2 (0.2)	16 (0.32)
3	1 (0.1)	1 (0.1)	1 (0.1)	3 (0.3)	1 (0.1)	7 (0.14)



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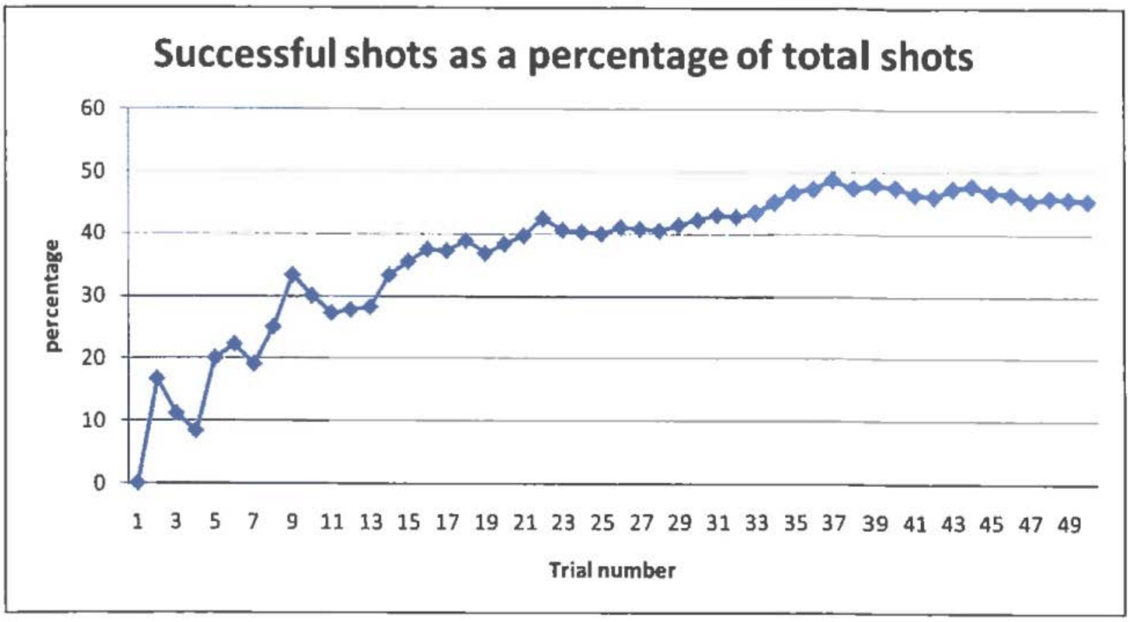
The graph shows that the chance of getting one or two goals are about the same (0.30 and
0.32) and are greater than getting zero (0.24) or three with three being the least (0.14). I
expected this because I practice a lot for my games at my free throws because when I watch
the Australian basketball league they are constantly showing the players career success rate
for free throws and when the game is very close in the 4th quarter the team trailing always 5
try's and foul the person with the lowest percentage so they can get the ball back to get a
field goal and either close the gap or even take the lead. So about 40% of the time throwing
free throws successfully can be the difference between winning and losing a game. However,
these results would only apply to me because different people have different ability. Also the 7
table shows that as the investigation went on things changed. I improved but then went

downhill at the end. For example, with the first 10 sets my probability of no goals was 0.5 but in fourth before I went downhill, it was 0.1. I think was because when I practice my free throws I only practice shooting two free throws. Also in a game situation you spend a lot of time running up and down the court so you are not concentrating all the time on making the free throws, whereas for this experiment I was concentrating a lot more than I usually do. (7) On the other hand my probability of getting 3 goals from the first 10 set was 0.1 but in the fourth set it 0.3. While this pattern might seem strange it could be like a real game, you improve with more shoots but then might get tired. (8)



Most magazines talk about shooting percentages which is total goals as a percentage of total shots and so I did this. (6) My percentage varied a lot early on but that is because the number of shots is small. My overall percentage is about 45% but this is increased to about trial 37 and then could be dropping, the same as in my table. I can conclude my overall percentage is about 45% but the experiment is not really like real games.

	Grade Boundary: High Merit
2.	For Merit, the student needs to investigate, with justification, a situation involving elements of chance. This involves linking aspects of the investigation to the situation and making supporting statements which refer to evidence such as summary statistics, probabilities, trends or features of visual displays. The student has posed a question to explore a situation involving elements of chance (1), planned an experiment to explore the situation (2), selected and used appropriate displays including experimental probability distributions (3), identified and communicated patterns in the data (4), compared discrete theoretical distributions and experimental distributions (5) and communicated findings in a conclusion (6). This student response also included evidence of gathering data by performing the experiment at an appropriate level for the award of Merit. The student has linked aspects of the investigation, made comments about the distribution and provided supporting comments in context for the answer to the question (7). To reach Excellence, this student would need to provide more detail and greater contextual understanding of the theoretical probability in their discussion. For example, the student could have drawn a side by side comparison graph and commented in detail about the experimental and theoretical probability. The student should also explain why not including a total of 7 as a win would have influenced the probability of winning.

Student 2: High Merit
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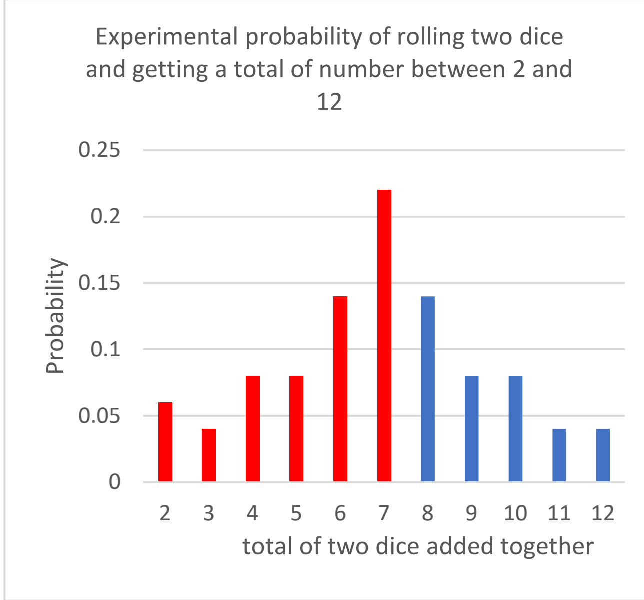
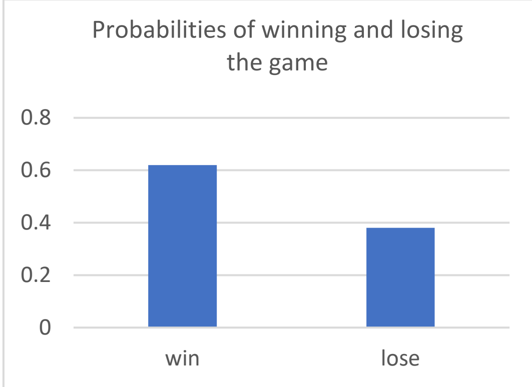
I want to investigate the probability of winning if winning means getting a total of 2-7 from two dice when they are thrown together, and the numbers shown are added? 1

I will use 2 dice to carry out my experiment to find out the probability of my problem which is “me winning a game of dice, where a total of 2-7 are found when throwing 2 dice. I will play my probability game rolling 2 dice 50 times and each time I roll the two die I will then add the two dice together and that will be my sum. After I have calculated my sum I will then mark it in tally chart next to which option it sums up to. After my 50 rolls I will use my tally marks to calculate the outcomes of my frequency’s. After my frequency’s calculations I will use my frequency’s to find out the probabilities percentages that will be out of 100. I will transfer my 2
data I collected drawing appropriate displays and analyse my results, explaining my data. The possible outcomes for the total sums in this experiment are rolling totals of 2, 3, 4, 5, 6, 7, 8, 9,10,11 and 12 when throwing 2 dice. This means that there will be a win if there is a total of 2,3,4,5,6 or a 7 when throwing 2 dice.





The red bars show the probability of getting the total that would give me a win. The blue bars show the probabilities when I would lose. I can see from this that a total of 7 has the highest probability of being rolled (22%) whereas totals of 3, 11 and 12 have the lowest probability of 4
being rolled (4%). I also noticed that both the totals of 6 and 8 have the same probability of being rolled (14%) and when I look at the distribution of the graph I see that is fairly symmetrical around the total the two dice being 7.

If I add up all the probabilities of rolling 2 dice and getting a total of between 2 and 7 I found 3
that I had .62 or 62% chance of winning the game. There is a 0.38 or 38% chance of losing 7
the game.

I wonder if the theoretical probability follows the same general pattern. I will need to calculate these.

- So the theoretical probability of rolling a total of 2 which means rolling 2 ones only so is $= \frac{1}{6} \times \frac{1}{6}$ = 0.0277
- The theoretical probability of rolling a total of 3 which means rolling a 2 and 1 or a 1 and 2
- $= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36} + \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$ so therefore the theoretical probability of rolling a total of 3
- $= \frac{1}{36} + \frac{1}{36}$ = 0.0555
- The theoretical probability of rolling a total of 4 is rolling a 2 on each dice, a 1 and 3 or a 3 and 1 so therefore I would get = $\frac{1}{36} + \frac{1}{36} + \frac{1}{36}$ = 0.0833
- The theoretical probability of rolling a total of 5 is rolling a 2 and 3 on each dice, a 3 and 2, a 1 and 4 and a 4 and 1 = 0.111
- The theoretical probability of rolling a total of 6 is rolling a 3 on each dice a 4 and 2, a 2 and 4, a 5 and 1 and a 1 and 5 = 0.1388
- The theoretical probability of rolling a total of 7 is rolling a 4 and 3 on each dice, a 3 and 4, a 2 and 5, a 5 and 2, a 6 and 1 and a 1 and 6 = 0.1666
- The theoretical probability of rolling a total of 8 is rolling a 4 on each dice, a 2 and 6, a 6 and 2, a 5 and 3 and a 3 and 5 = 0.1388
- The theoretical probability of rolling a total of 9 is rolling a 5 and 4on each dice, a 4 and 5, a 6 and 3 and a 3 and 6 = 0.111
- The theoretical probability of rolling a total of 10 is rolling a 5 on each dice and a 6 and 4 and a 4 and 6 = 0.0833
- The theoretical probability of rolling a total of 11 is rolling a 5 and 6 aon each dice and a 6 and 5 = 0.05555
- The theoretical probability of rolling a 12 is rolling a 6 on each dice = 0.0277

It is interesting to see that the theoretical follows reasonably closely to my experimental probability. For example, both graphs peak at rolling a total of 7. My experimental probability does not exactly match the theoretical for example for rolling a total of 2 my experimental 5
probability was 6% whereas the theoretical probability was 2.7% likewise for rolling total of 7 my experimental probability was 22% whereas the theoretical was only 16.6%. A possible reason why this might have occurred was that when I was rolled the dice I may not have 7
rolled them the same each time as I was getting tired and was under time pressure to complete.

From my trial investigation I can see it was pretty obvious and predictable given the likelihood to rolling each number and this is shown in graph the less number of possible outcomes the lower the times the number was rolled. The highest value of the outcomes in both my experiment and theoretical was 7. This is because 7 has the most possible combinations of being rolled (6). I am surprised with 3 only having a experimental probability of 4%

From the data and given what I have found out about the frequency of the possible number of different combinations to get certain numbers I can say that it would be likely for me win more times with a total of 2-7 as my experimental probability gave me a result of 62% chance 6
and this is supported when I compared to the theoretical probability which was 58.3%. If 7 was not included as a total then I think I would win as many games as someone who had number 8-12 based off my theoretical and experimental results. However as 7 is included in my numbers I believe from experiment I will win more games than my opponent

	Grade Boundary: Low Merit
3.	For Merit, the student needs to investigate, with justification, a situation involving elements of chance. This involves linking aspects of the investigation to the situation and making supporting statements which refer to evidence such as summary statistics, probabilities, trends or features of visual displays. This student's evidence is a response to the TKI assessment resource 'Free Throw'. The student has posed a question to explore a situation involving elements of chance (1), planned an experiment to explore the situation (2), gathered data by performing the experiment (3), selected and used appropriate displays including experimental probability distributions (4), identified and communicated patterns in the data (5) and communicated findings in a conclusion (6). The student has linked aspects of the investigation to the situation by calculating the average number of free throws that were successful out of three. The student has also made some supporting comments about the distribution (7). For a more secure Merit, the student would need a more detailed discussion about the average and the comparison for 0 and 3 goals. They would also need to discuss the distribution more thoroughly by referring to additional evidence, and more clearly link the context back to the question and the conclusion.

Student 3: Low Merit

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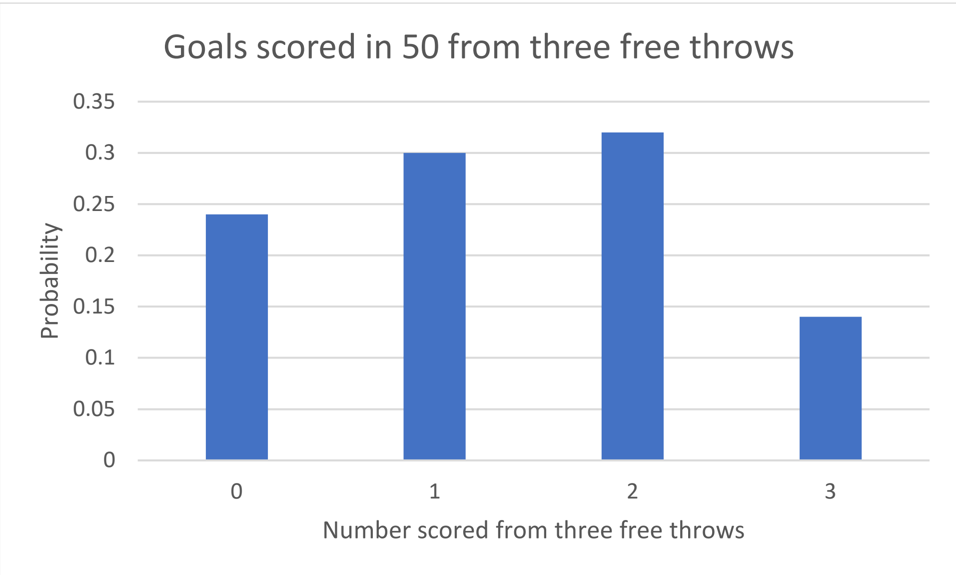
I wonder how the chances of each of the possible number of goals (0,1,2,3) compare. To do 1
this I will complete 50 sets of 3 throws and record the shots that go through the hoop. I will
make sure that each time I throw the basketball from the free throw line that after I have
picked up the ball either from the ball going through the hop and net or rebounding off the 2
back board that I reset myself by standing in the free throw line and bouncing the ball the few
times which takes approximately 20 seconds like I do in my games. Once I have completed
each set of three throws I will then give myself a minute before I start again to ensure that
each set is consistent. I think that the chance of getting two should be highest and then one
followed by three and then none at all. I should keep conditions the same by taking a break
between each set of throws.

Number of goals	total	Probability	
0	12	0.24	3
1	15	0.30	
2	16	0.32	
3	7	0.14	



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I scored a total of 68 goals in 50 trials. So my average was 1.36 goals for any set of 3 free
throws. My probabilities show that the chance of getting one goal (0.3) and two goals (0.32) 5
are near enough to the same. I was surprised that the chance of getting no goals was (0.24). 7
The probability of three (0.14) is less than zero (0.24) and I did not expect this. Also I thought
I would get more two's (0.32) than one's (0.30) as when I practice my free throwing I always 6
take two free throws. The order is more like both 1 and 2 the same, then zero and finally
three.

	Grade Boundary: High Achieved
4.	For Achieved, the student needs to investigate a situation involving elements of chance. This involves using the experimental probability process. The student has posed a question to explore a situation involving elements of chance (1), planned an experiment to explore the situation (2), gathered data by performing the experiment (3), selected and used appropriate displays including experimental probability distributions (4), identified and communicated patterns in the data (5) and communicated findings in a conclusion (6). To reach Merit, the student would need to further justify comments made. For example, they could explain with supporting evidence from the experiment what they mean when they say that their findings are consistent with both their experimental and theoretical probability results.

Student 4: High Achieved

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Introduction:

My investigative question is: I wonder what the probability of me getting as free ticket when rolling two dice is. The possible outcomes could be a giant popcorns worth, 50% discount on my ticket, a free ticket and not winning at all. I predict winning a giant popcorn will have the highest frequency overall.

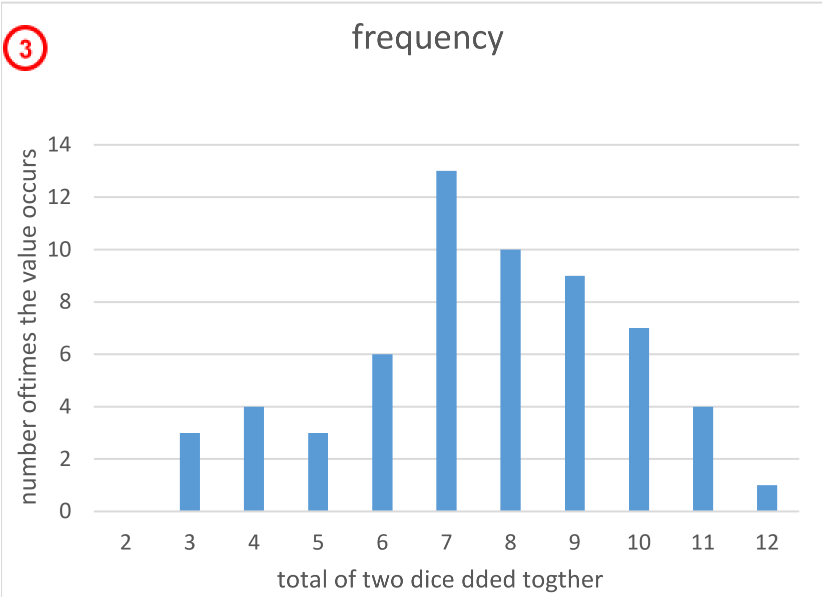
The promotion requires me to buy a ticket to the film and be given two dice to roll and to win the following prizes:

- If the total is 7 or an 11 I will win a giant popcorn.
- If the total is 3, 5 or 9 I will get a 50% discount on my ticket.
- If I throw a double, I will get a free ticket.

The possible outcomes for this experiment are rolling a 2, 3, 4, 5, 6, 7, 8, 9,10,11 and 12

On the other hand, if I do not win any of these prizes, then I will pay the full prize on my ticket.
In this investigation, the tools that I am going to use are the two coloured dice (Red & Blue) and my laptop. I will throw the two dice on the table and record the total of the two dice on the data table 60 times. If one or both dice falls onto the ground, the result will not be recorded into the data table. I will then calculate the experimental probability from the 60 trials and graph it.

Total	Tally	Frequency	Pizes
2			No Prize
3	II	2	50% discount
4	III	3	1 free ticket
5	IIII	4	50% discount
6	~~IIII~~ II	7	No Prize
7	~~IIII~~ ~~IIII~~ III	13	Giant popcorn
8	~~IIII~~ ~~IIII~~	10	No Prize
9	~~IIII~~ IIII	9	50% discount
10	~~IIII~~ II	7	I free ticket
11	IIII	4	Giant popcorn
12	I	1	1 free ticket



I can see from my table:
The probability of winning 50% discount on the ticket is 15/60 is 0.25.
The probability of winning free ticket is 11/60 is 0.1833.
The probability of winning giant popcorn is 17/60 is 0.283.
The probability of getting no prize is 17/60 is 0.2833. 4

I can see on my graph above, the sum of 7 has the highest frequency and the lowest frequency is at 12 and comparing my frequency with the others, the most frequent number is 7. I also noticed that the chance of getting no prize and winning a giant popcorn had the same probability outcome, while winning a free ticket had the lowest probability. 5

Given my experimental probably results above I wonder if the theoretical probability winning a free ticket will also end up being lowest probability.

Below I have listed the possible outcomes

This table 1 shows the possible outcomes for the total outcome of the Red and Blue dice

R,B	R,B	R,B	R,B	R,B	R,B
1,1	2,1	3,1	4,1	5,1	6,1
1,2	2,2	3,2	4,2	5,2	6,2
1,3	2,3	3,3	4,3	5,3	6,3
1,4	2,4	3,4	4,4	5,4	6,4
1,5	2,5	3,5	4,5	5,5	6,5
1,6	2,6	3,6	4,6	5,6	6,6

- Possible outcomes of giant popcorns are 8/36.
- Possible outcomes of 50% discount on tickets are 10/36.
- Possible outcomes of free tickets are 6/36.

This table 2 shows the probabilities of winning a prize.

Prizes	Probability of winning a prize	Percentage of winning a prize
Win Giant Popcorn Worth $5	0.22	22%
Win 50% discount on ticket	0.28	28%
Win Free Ticket	0.17	17%
No Prize	0.33	33%
Total	1.00	100%

Conclusion:
I found from this investigation that my prediction was correct. The investigation also showed that the overall probability of winning a free ticket had the lowest probability of happening. So the probability of me winning a free movie ticket is 0.1833. This is consistent with not only my experimental probability but also my theoretical probability result. 6

	Grade Boundary: Low Achieved
5.	For Achieved, the student needs to investigate a situation involving elements of chance. This involves using the experimental probability process. This student’s evidence is a response to the TKI assessment resource ‘Games of Chance’. The student has posed a question to explore a situation involving elements of chance (1), planned an experiment to explore the situation (2), gathered data by performing the experiment (3), selected and used appropriate displays including experimental probability distributions (4), identified and communicated patterns in the data (5) and communicated findings in a conclusion (6). For a more secure Achieved, the student would need to provide a more detailed answer to the question and draw the graph of the experimental distribution accurately.

Student 5: Low Achieved

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Question
How many sixes would I expect to get when five dice are rolled? 1

Plan
I am going to roll five dice together and count the number of sixes.
I am going to roll the five dice 50 times. 2
The number of sixes I can get could be all of the dice showing a six, none of them showing a six and any number in between.

Number of sixes		Number of times	Probability
0	𝍸 𝍸 𝍸 𝍸 𝍷	21	0.42
1	𝍸 𝍸 𝍸 𝍸	20	0.40
2	𝍸 𝍷	6	0.12
3	𝍷𝍷	2	0.04
4	𝍷	1	0.02
5		0	0.00

3

4

The most common number was 0. 5
The larger numbers happens less times than the smaller numbers.

I would expect to get 0 or 1 six when I toss five dice. 6

	Grade Boundary: High Not Achieved
6.	For Achieved, the student needs to investigate a situation involving elements of chance. This involves using the experimental probability process. This student's evidence is a response to the TKI assessment resource 'Games of Chance'. The student has posed a question to explore a situation involving elements of chance (1), planned an experiment to explore the situation (2), gathered data by performing the experiment (3), selected and used appropriate displays (4), identified and communicated a pattern in the data (5) and communicated findings in a conclusion (6). To reach Achieved, the student would need to give the experimental probability distribution and identify a further pattern in the data. The student would also need to clearly discuss and define the set of possible outcomes for the experiment.

Student 6: High Not Achieved

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What are the chances of getting a double with five dice? 1

I am going to roll five dice fifty times and count the number of doubles of any number. 2

3

1	5	1	1	2	3	1
2	3	3	3	4	4	2
3	4	4	6	1	3	1
4	2	4	6	6	5	1
5	3	3	2	2	1	2
6	2	4	6	5	6	1
7	1	2	5	3	4	0
8	1	3	6	1	6	2
9	2	4	6	5	3	0
10	2	3	2	2	4	1
11	4	1	2	2	4	2
12	2	1	6	6	6	1
13	5	6	5	1	2	1
14	6	3	4	1	4	1
15	4	4	4	1	6	1
16	2	1	6	2	1	2
17	6	3	5	4	5	1
18	2	5	6	1	2	1
19	3	3	6	3	2	1
20	5	3	2	3	6	1
21	6	1	3	5	2	0
22	4	5	1	4	4	1
23	5	2	4	3	3	1
24	2	5	1	6	4	0
25	6	5	5	3	2	1

26	2	1	5	2	2	1
27	2	4	5	6	2	1
28	1	6	1	4	5	1
29	3	5	5	4	5	1
30	2	1	3	6	4	0
31	6	5	1	4	6	1
32	1	6	1	6	5	2
33	3	3	4	3	1	1
34	2	5	4	5	2	2
35	4	1	4	1	4	2
36	6	4	4	2	4	1
37	2	4	6	6	5	1
38	5	2	2	5	4	2
39	2	4	5	2	6	1
40	3	2	2	4	1	1
41	2	4	4	1	2	2
42	5	1	1	6	2	1
43	3	6	5	2	5	1
44	4	6	6	5	6	1
45	3	4	1	2	6	0
46	2	3	2	2	4	1
47	5	1	2	2	6	1
48	4	6	6	2	3	1
49	3	4	4	3	6	2
50	2	6	4	4	1	1

Number of doubles	Number of times
0	6
1	33
2	11

4

Sometimes I got a triple and counted this as a double. 5

My chances of getting a double are 33/50 = 66% 6