For the Taupo Club I need to find the shortest route from Hamilton to Taupo.

I used the tree method do this, starting in Hamilton. I draw lines to each of the towns I can go to at each step and record the total distance from Hamilton. If I have two ways or more to get a town I choose the shortest one and don’t go any further with the diagram except from the shortest distance.

The tree shows me the shortest distance from Hamilton to Taupo is 153km, travelling from Hamilton to Cambridge, to Tirau, to Tokoroa and then to Taupo.

For the Hamilton Club I need the spanning tree with the minimum cost. To find this I used the edge deletion method, removing the highest edge at each turn provided I don’t disconnect a town from the network. As there are 9 towns in the network I know I will need 8 edges, so I must delete 8 of the edges on the network as there are 16 altogether.

I deleted the edges that cost: 4560, 3930, 1920, 1830, 1120, 1020, 800, 725 in that order.

This leaves me with this minimum spanning tree which has the minimum cost of $6 230 000.

The tree has these edges:
- Hamilton to Cambridge
- Cambridge to Matamata
- Matamata to Tirau
- Cambridge to Te Awamutu
- Te Awamutu to Pirongia
- Te Awamutu to Te Kuiti
- Te Awamutu to Tokoroa
- Tokoroa to Taupo

For the Tirau Club I need the spanning tree with the maximum scenic value. To find this I used the edge deletion method, removing the lowest edge at each turn provided I don’t disconnect a town from the network. I have to delete 8 edges again.
I deleted the edges with scenic values:
2, 3, 4, 5 then I have choice of two edges worth 6, either Hamilton to Matamata or Cambridge to Te Awamutu. I am going to delete the second one.
The leaves me with a maximum scenic value of 67

The edges in the spanning tree are:
- Hamilton to Pirongia
- Hamilton to Matamata
- Cambridge to Matamata
- Matamata to Tauru
- Te Kuiti to Tokoroa
- Te Kuiti to Taupo

For the Tokoroa club I need to see if the network is traversable.

The order of the nodes are:

<table>
<thead>
<tr>
<th>Places</th>
<th>Ham</th>
<th>Mat</th>
<th>Pir</th>
<th>Cam</th>
<th>TeA</th>
<th>Tauru</th>
<th>TeK</th>
<th>Tok</th>
<th>Tau</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

Because Matamata and TeKuiti are odd and the rest even the network is not traversable starting and finishing at Tokoroa. But it could be traversed and finished at the 2 odd nodes, so starting at Matamata and finishing at TeKuiti or vice versa.

This diagram shows the shortest route from Hamilton to Taupo (in Red), the minimum cost tree (in Blue) and the maximum scenic value tree (in Green). Because these three don’t overlap there is no one route that will satisfy all three clubs.

To satisfy as best I can all three clubs I am going to first select the edges which are in more than one colour as they are common to two clubs. This gives me 5 of the edges in the compromise network (HamCam, CamMat, MatTir, TokTau and TeATEK) and I need another three. I am going to choose edges which have the highest scenic value to complete my network because one of the main reasons that tourists visit New Zealand and the Waikato is for the scenery.

I have no choice but to add in HamTeA and HamPir but I have a choice of adding TeKTok or TeKTau. These have scenic ratings of 9 and 4 so I will use the first one to maximise the scenic value.

My chosen compromise is in orange on the diagram.