For the Taupo Club I need to find the shortest route from Hamilton to Taupo.

The route Hamilton – TeAwamutu – Tokoroa – Taupo = 29 + 70 + 66 = 165km
The route Hamilton – Cambridge – Tirau – Tokoroa – Taupo = 23 + 32 + 32 + 66 = 153km
The route Hamilton – Cambirdge – Te Awamutu – Tokoroa – Taupo = 23 + 24 + 70 + 66 = 183km

So this route is the shortest Hamilton – Cambridge – Tirau – Tokoroa – Taupo = 153km

For the Hamilton Club I need the minimum spanning tree based on cost. To find this I remove the most expensive edges one by one until I only have a tree, which has no loops, with all towns connected.

I removed the edges that cost 4560, 3930, 1920, 1830, 1120, 1020, 800 and 725 in that order.

The minimum cost is the total of the remaining edges which is $6,230,000. The tree is the edges not crossed off on the diagram.

For the Tirau Club I need the spanning tree with the maximum scenic value. I removed the lowest scenic value roads in this order to get the tree in the diagram, with all the towns connected.

2, 3, 4, 5, 5 and 6 (I had a choice for the 6)

The maximum scenic value is 67.
For the Tokoroa club I need to see if the network is traversable. A network can only have even nodes and it is only traversable if there are 0 or 2 odd nodes. I can work out the order of each node by counting how many roads go into them.

Hamilton 4 Matatamata 3 Pirongia 2 Cambridge 4 TeAwamutu 6 Tirau 4 TeKuiti 3 Tokoroa 4 Taupo 2

Because Matamata and TeKuiti are odd and the rest even the network is not traversable starting and finishing at the same place.

There is no network that can satisfy all three clubs because the networks for the Hamilton, Taupo and Tirau club are all different. Either of the trees for Hamilton and Tirau will still allow people to go from Hamilton to Taupo, although it will be a longer route. I don’t think that is important because cyclists like long trips and the exercise will do them good.

I think the best thing to do is to choose one of the other two networks I have found. Which one depends on what the priorities of the Government are. If they want to attract tourists to the region that it would be best to select the maximum scenic route, but that is going to cost more as it is not the minimum cost network. I think this is still the best idea as the investment will be returned by the number of people attracted into the region. If the cost is too much they could consider removing the Tekuiti to Taupo road and replacing it with the Tokoroa to Taupo road.