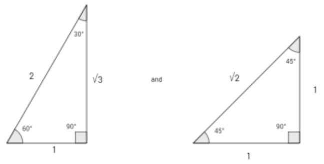
Special angles



Using these triangles and

$\sin = \frac{O}{H}$ and $\cos = \frac{A}{H}$ and $\tan = \frac{O}{A}$

I can determine that

	sin	cos	tan
30^0, $\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45^0, $\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60^0, $\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

because $\cot\theta = \frac{1}{\tan\theta}$

cot 30^0 = $\sqrt{3}$, cot 45^0 = 1, cot 60^0 = $\frac{1}{\sqrt{3}}$ 1

because $\sec\theta = \frac{1}{\cos\theta}$

sec 30^0 = $\frac{2}{\sqrt{3}}$, sec 45^0 = $\sqrt{2}$, sec 60^0 = 2

because cosec $\theta = \frac{1}{\sin\theta}$

cosec 30^0= 2, cosec 45^0 = $\sqrt{2}$, cosec 60^0 = $\frac{2}{\sqrt{3}}$

Compound angles

$15°, \frac{\pi}{12}$

$$\sin(60-45) = \sin 60\cos 45 - \cos 60\sin 45 = (\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}) - (\frac{1}{2}\times\frac{1}{\sqrt{2}}) = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$ 1

$\sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$ therefore cosec$15° = \frac{2\sqrt{2}}{\sqrt{3}-1}$

$$\cos(60-45) = \cos 60\cos 45 + \sin 60\sin 45 = (\frac{1}{2}\times\frac{1}{\sqrt{2}}) + (\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}) = \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$$

$\cos 15° = \frac{1+\sqrt{3}}{2\sqrt{2}}$ therefore sec$15° = \frac{2\sqrt{2}}{1+\sqrt{3}}$

$$\tan(60-45) = \frac{\sqrt{3}-1}{1+(\sqrt{3}\times 1)} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$\tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1}$ therefore cot$15° = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

Double angles

$120°, \frac{2\pi}{3}$

$\sin 2A = 2\sin A\cos A$ 1

$\sin 2(60°) = 2\sin 60\cos 60 = 2(\frac{\sqrt{3}}{2}\times\frac{1}{2}) = 2(\frac{\sqrt{3}}{2\sqrt{2}}) = \frac{2\sqrt{3}}{4\sqrt{2}}$

$\sin 120° = \frac{2\sqrt{3}}{4\sqrt{2}}$ and $\operatorname{cosec}120° = \frac{4\sqrt{2}}{2\sqrt{3}}$

$\cos 2A = 2\cos^2 A - 1$

$A = 60°$

$\cos 2(60°) = 2(\frac{1}{2})^2 - 1 = \frac{1}{2} - 1 = -0.5$

cos 120^0 = -0.5 and sec 120^0 = $\frac{1}{-0.5}$

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

$A = 60$

cot 120^0 = $\frac{-2}{2\frac{\sqrt{3}}{2}}$

$\tan 2(60°) = \tan 120° = \frac{2\frac{\sqrt{3}}{1}}{1-(\frac{\sqrt{3}}{1})^2} = \frac{\frac{2\sqrt{3}}{2}}{1-3} = \frac{\frac{2\sqrt{3}}{2}}{-2}$ 2

General solutions

$\sin x = \frac{1}{\sqrt{2}}$	$x = \sin^{-1}\frac{1}{\sqrt{2}}$	x=45^0	x=n180^0 + $(-1)^n 45^0$
$\cos x = \frac{1}{\sqrt{2}}$	$x = \cos^{-1}\frac{1}{\sqrt{2}}$	x=45^0	x=2(180^0)n +/- 45^0
$\tan x = 1$	$x = \tan^{-1}1$	x=45^0	x=n180^0+45^0

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Therefore

$\operatorname{cosec} x = n\frac{1}{180} + (-1)^n\frac{1}{45}$

$\sec x = n(\frac{1}{360}) \pm \frac{1}{45°}$ 4

$\cot x = n\frac{1}{180} + \frac{1}{45}$