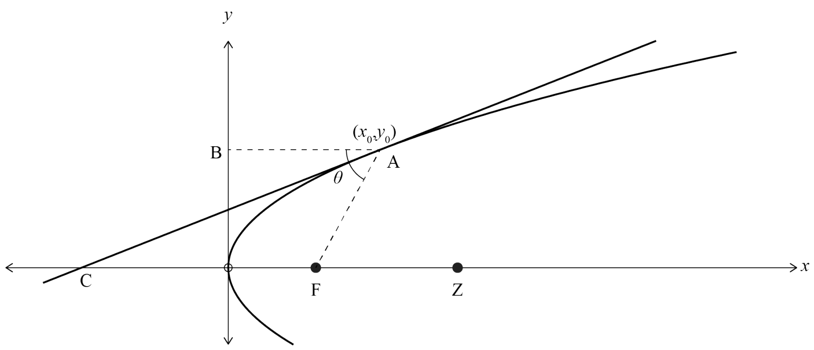
Assessment Schedule – 2017

Scholarship Calculus (93202)

Evidence Statement

Q	Solution
ONE(a)	$(x^2 + y)(x^2 - y) = 71$ But has factors ±1 and ±71. Since x and y are integers, possible solutions are: A: $x^2 + y = 1$ and $x^2 - y = 71$ B: $x^2 + y = 71$ and $x^2 - y = 1$ C: $x^2 + y = -1$ and $x^2 - y = -71$ D: $x^2 + y = -71$ and $x^2 - y = -1$ Consider A and B. Adding in each case gives $2x^2 = 72 \Rightarrow x = \pm 6$. Doing the same with C and D gives $2x^2 = -72$ which has no real solution. Solutions from A and B are the only solutions. Using $x = \pm 6$ in the original equation gives $y = \pm 35$. The four solutions are (6,35), (–6,35), (6,–35), (–6,–35).
(b)	$(x^2 - bx)(p+1) = (p-1)(ax+c)$ $px^2 - bpx + x^2 - bx = apx + pc - ax - c$ $(p+1)x^2 + (a - ap - bp - b)x + c(1-p) = 0$ **Eqn A** Let roots of **equation A** be α and β. Since $\alpha + \beta = 0$, $\dfrac{-(a - ap - bp - b)}{(p+1)} = 0$, i.e. $a - ap - bp - b = 0$ **Eqn B** $\alpha\beta < 0$ since roots are of opposite sign i.e. $\dfrac{c(1-p)}{p+1} < 0$ **From Eqn B**, $-p(a+b) = b - a$ or $p = \dfrac{a-b}{a+b}$ So $\dfrac{c(1-p)}{(p+1)} = \dfrac{c\left(1 - \dfrac{a-b}{a+b}\right)}{\left(\dfrac{a-b}{a+b} + 1\right)} < 0$ and then $\dfrac{c(a+b-(a-b))}{a-b+a+b} < 0$ $\Leftrightarrow \dfrac{2cb}{2a} < 0$ $\Leftrightarrow \dfrac{bc}{a} < 0$

(c)



As shown in the diagram, choose point Z on the x-axis on the positive side of F.

∠ZFA = ∠FAB alternate angles, CZ ∥ BA.

Coordinates of A are $(x_0, y_0) = \left(x_0, 2\sqrt{ax_0}\right)$

Length FA $= \sqrt{(a-x_0)^2 + \left(0-2\sqrt{ax_0}\right)^2}$

$= a + x_0$

Gradient of AC: $2y\frac{\mathrm{d}y}{\mathrm{d}x} = 4a$ so $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2a}{y}$

at $\left(x_0, 2\sqrt{ax_0}\right)$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{\frac{a}{x_0}}$

Equation of AC: $y - 2\sqrt{ax_0} = \sqrt{\frac{a}{x_0}}(x - x_0)$

To find x-coordinate of C, let $y = 0$

$-2\sqrt{ax_0} = \sqrt{\frac{a}{x_0}}(x - x_0)$

$\Rightarrow -2x_0 = x - x_0 \Rightarrow -x_0 = x$

Length $\mathrm{FC} = \sqrt{(a - -x_0)^2 + (0-0)^2} = a + x_0 = \mathrm{FA}$

Hence ΔAFC is isosceles.

Since ∠ZFA is the external angle of triangle FAC,

$\angle\mathrm{FAC} + \angle\mathrm{FCA} = \angle\mathrm{ZFA} = \theta$, i.e. $\angle\mathrm{FCA} = \frac{1}{2}\theta$

OR alternate angles FCA and CAB – no need for point Z.

Alternate method

Using $(at^2, 2at)$ for point and gradient

$\tan(ACF) = \frac{1}{t}$

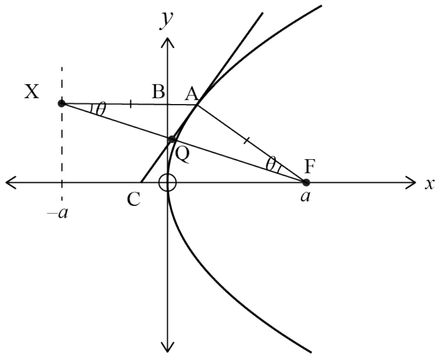
$\tan\vartheta = \frac{2t}{t^2 - 1}$

Double angle formula to show one is double the other.

(c)

Other methods possible:

METHOD ONE



$y^2 = 4ax \quad \frac{dy}{dx} = \frac{2a}{y}$

AX = AF dist from A to directrix
= dist from A to focus

$\therefore \angle AXF = \angle AFX$ base ∠'s isos Δ

Slope of AC = $\frac{2a}{y_0}$ (grad tangent)

Slope of XF = $\frac{-y_0}{2a}$ (rise/run)

$\frac{2a}{y_0} \times \frac{-y_0}{2a} = -1 \Rightarrow \angle AQF = \angle AQX = 90°$

$\therefore$ ΔAXQ and ΔAFQ are similar, and hence the tangent bisects ∠BAF

METHOD TWO

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Angle BAC $\qquad \tan(\text{BAC}) = \left|\frac{0 - \frac{2a}{y_0}}{1+0}\right| = \left|\frac{2a}{y_0}\right|$

Angle CAF $\qquad \tan(\text{CAF}) = \left|\frac{\frac{2a}{y_0} - \frac{y_0}{x_0 - a}}{1 + \frac{2a}{y_0}\left(\frac{y_0}{x_0 - a}\right)}\right|$

$$= \left|\frac{\frac{2a(x_0 - a) - {y_0}^2}{y_0(x_0 - a)}}{\frac{x_0 - a + 2a}{(x_0 - a)}}\right|$$

$$= \left|\frac{2a(x_0 - a) - {y_0}^2}{y_0(x_0 + a)}\right|$$

${y_0}^2 = 4ax_0 \Rightarrow \qquad = \left|\frac{2a(x_0 - a) - 4ax_0}{y_0(x_0 + a)}\right|$

$$= \left|\frac{-2a(a + x_0)}{y_0(x_0 + a)}\right| = \frac{2a}{y_0}$$

$\angle \text{BAC} = \angle \text{CAF}$

Q	Solution
TWO (a)(i)	Since $\angle SPD = \theta$, $\angle SPQ = (\pi - 2\theta)$ In ΔAPQ and ΔBRQ, $\angle A = \angle B = \frac{\pi}{2}$, since ABCD is a rectangle. $\angle AQP = \angle BQR$ so $\angle APQ = \angle QRB = \theta$ (Sum of angles of Δ) The opposite angles of SPQR are equal so SPQR is a parallelogram with perimeter 2(PQ + PS). But $PQ = \frac{x}{\cos\theta}$ and $PS = \frac{3-x}{\cos\theta}$ Perimeter = $2\left(\frac{x}{\cos\theta}+\frac{3-x}{\cos\theta}\right)=\frac{6}{\cos\theta}$, i.e. does not depend on x.
(a)(ii)	Using Cosine Rule $PR^2 = PQ^2 + QR^2 - 2PQ.QR.\cos(\angle PQR)$ $\angle AQP = \frac{\pi}{2} - \theta = \angle BQR \Rightarrow \angle PQR = 2\theta$ $PR^2 = \left(\frac{2}{\cos\frac{\pi}{3}}\right)^2 + \left(\frac{3-2}{\cos\frac{\pi}{3}}\right)^2 - 2\frac{2}{\cos\frac{\pi}{3}}\cdot\frac{3-2}{\cos\frac{\pi}{3}}\cdot\cos\left(2\frac{\pi}{3}\right)$ $PR^2 = 16 + 4 + 8 = 28$ i.e. $PR = 2\sqrt{7}$ units
(b)	From (1) $x + y = 1 + z$ $(x + y)^2 = 1 + 2z + z^2$ (A) From (2) $x^2 + 2xy + y^2 = 5 + z^2$ (B) $(B) - (A)$ gives: $0 = 4 - 2z \Rightarrow z = 2$ This gives $x + y = 3$ and $(x + y)^3 = 27$ Also $x^3 + 3x^2y + 3xy^2 + y^3 = x^3 + y^3 + 3xy(x + y) = x^3 + y^3 + 9xy$ So $x^3 + y^3 + 9xy = 27$ or $x^3 + y^3 = 27 - 9xy$ (C) From (3) and using $z = 2$, we have $x^3 + y^3 = 51 - 3xy$ (D) $(D) - (C)$ gives: $0 = 24 + 6xy$ $xy = -4$ In summary, we have $x + y = 3$, $z = 2$, and $xy = -4$ The solutions are: (1) $x = 4$, $y = -1$, and $z = 2$ (2) $x = -1$, $y = 4$, and $z = 2$

Q	Solution
THREE (a)	$\ln y = \ln x^{(x^x)} = x^x \ln x$ $\ln(\ln y) = \ln(x^x \ln x) = \ln x^x + \ln(\ln x)$ $\frac{1}{\ln y}\frac{1}{y}\frac{dy}{dx} = \ln x + x\frac{1}{x} + \frac{1}{\ln x}\frac{1}{x}$ And substituting: when $x = 2, y = 16$ $\frac{1}{\ln 16}\frac{1}{16}\frac{dy}{dx} = \ln 2 + 1 + \frac{1}{\ln 2} \times \frac{1}{2}$ $\frac{dy}{dx} = 64\ln 2\left(\ln 2 + 1 + \frac{1}{2\ln 2}\right) = 107.1$
(b)(i)	$\frac{d}{dx}\left(e^x \sin x\right) = e^x\left(\sin x + \cos x\right)$ $= \sqrt{2}e^x\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right)$ $= 2^{\frac{1}{2}}e^x \sin\left(x + \frac{\pi}{4}\right)$
(b)(ii)	$\frac{d}{dx}\left(2^{\frac{1}{2}}e^x \sin\left(x + \frac{\pi}{4}\right)\right)$ $= 2^{\frac{1}{2}}e^x\left(\sin\left(x + \frac{\pi}{4}\right) + \cos\left(x + \frac{\pi}{4}\right)\right)$ $= 2^{\frac{1}{2}}e^x\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\left(x + \frac{\pi}{4}\right) + \frac{1}{\sqrt{2}}\cos\left(x + \frac{\pi}{4}\right)\right)$ $= 2^{2\times\frac{1}{2}}e^x\left(\sin\left(x + 2\frac{\pi}{4}\right)\right)$
(b)(iii)	Similarly $\frac{d^3y}{dx^3} = 2^{3\times\frac{1}{2}}e^x\left(\sin\left(x + 3\frac{\pi}{4}\right)\right)$ and so $\frac{d^ny}{dx^n} = 2^{n\times\frac{1}{2}}e^x\left(\sin\left(x + n\frac{\pi}{4}\right)\right)$ n is even $\Rightarrow \left(\frac{d^ny}{dx^n}\right)_{x=0} = 2^{\frac{n}{2}}\left(\sin\left(n\frac{\pi}{4}\right)\right)$ which evaluates to: $2^{\frac{n}{2}}$ if $n = 2, 10, 18 \ldots$ or $\{n = 8k + 2, k = \{0, 1, 2 \ldots\}\}$ 0 if $n = 4, 8, 12 \ldots$ or $\{n = 4k, k = \{1, 2, 3 \ldots\}\}$ $-2^{\frac{n}{2}}$ if $n = 6, 14, 22 \ldots$ or $\{n = 8k + 6, k = \{0, 1, 2 \ldots\}\}$ n is odd: $2^{\frac{n}{2}} \times \frac{1}{\sqrt{2}} = 2^{\frac{n-1}{2}}$ if $n = 1, 3, 9, 11 \ldots$ or $\{n = 2 + 8k \pm 1,\ k = \{0, 1, 2 \ldots\}\}$ $-2^{\frac{n}{2}} \times \frac{1}{\sqrt{2}} = -2^{\frac{n-1}{2}}$ if $n = 5, 7, 13, 15 \ldots$ or $\{n = 6 + 8k \pm 1,\ k = \{0, 1, 2 \ldots\}\}$

(c)	Let $y = \sinh^{-1} x \Rightarrow \sinh y = x$ $x = \frac{1}{2}\left(e^{y} - e^{-y}\right) \Rightarrow$ $1 = \frac{1}{2}\left(e^{y}\frac{dy}{dx} + e^{-y}\frac{dy}{dx}\right)$ $\frac{dy}{dx}\left(\frac{1}{2}\left(e^{y} - e^{-y}\right)\right) = 1 \Rightarrow$ $\frac{dy}{dx}\cosh y = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{\cosh y}$ From the definition: $\sinh^2 y - \cosh^2 y$ $= \left(\frac{1}{2}\left(e^{y} - e^{-y}\right)\right)^2 - \left(\frac{1}{2}\left(e^{y} - e^{-y}\right)\right)^2 = -1$ $\frac{dy}{dx} = \frac{1}{\cosh y} = \frac{1}{\sqrt{(\sinh y)^2 + 1}} = \frac{1}{\sqrt{x^2 + 1}}$ METHOD ONE $y = \sinh x \quad = \frac{1}{2}\left(e^{x} - e^{-x}\right)$ $2y \quad = e^{x} - e^{-x}$ $e^{x} \cdot 2y \quad = e^{2x} - 1$ $\left(e^{x}\right)^2 - e^{x} \cdot 2y - 1 = 0$ $e^{x} \quad = \frac{2y \pm \sqrt{4y^2 + 4}}{2}$ $e^{x} \quad = y \pm \sqrt{y^2 + 1}$ $x \quad = \ln\left(y + \sqrt{y^2 + 1}\right)$ as $y - \sqrt{y^2 + 1} < 0$ $\therefore y = \sinh^{-1}(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$ $\frac{d}{dx}\left(\sinh^{-1}(x)\right) \quad = \frac{1}{x + \sqrt{x^2 + 1}} \cdot \left[1 + \frac{2x}{2\sqrt{x^2 + 1}}\right]$ $= \frac{1}{x + \sqrt{x^2 + 1}} \cdot \left[\frac{\sqrt{x^2 + 1} + x}{\sqrt{x^2 + 1}}\right]$ $= \frac{1}{\sqrt{x^2 + 1}}$

Q	**Solution**
FOUR (a)	$\tan 3x = \tan(x+2x) = \dfrac{\tan x + \tan 2x}{1 - \tan x \tan 2x}$ and $\tan 3x - \tan x \ \tan 2x \ \tan 3x = \tan x + \tan 2x$ $\tan x \ \tan 2x \ \tan 3x = -\tan x - \tan 2x + \tan 3x$ $\int \tan x \ \tan 2x \ \tan 3x \, \mathrm{d}x = \int (-\tan x - \tan 2x + \tan 3x)\, \mathrm{d}x$ And since $\int \tan x \, dx = \int \dfrac{\sin x}{\cos x} \mathrm{d}x$, by substitution with $u = \cos x$ $\int \dfrac{\sin x}{u} \dfrac{du}{-\sin x} = -\ln\lvert\cos x\rvert + c$ $\int \tan x \ \tan 2x \ \tan 3x \, \mathrm{d}x = \ln\lvert\cos x\rvert + \frac{1}{2}\ln\lvert\cos 2x\rvert - \frac{1}{3}\ln\lvert\cos 3x\rvert + K$
(b)	The portions of the curve defined by $0 \le \theta \le \pi$ and $\pi \le \theta \le 2\pi$ are symmetric. $S = 2\int_0^{\pi} \sqrt{\left\{ r^2 + \left(\dfrac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2 \right\}} \, \mathrm{d}\theta$ $r^2 + \left(\dfrac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2 = a^2(1-\cos\theta)^2 + (a\sin\theta)^2$ $= a^2 - 2a^2\cos\theta + a^2\cos^2\theta + a^2\sin^2\theta$ $= 2a^2(1-\cos\theta)$ $= 2a^2\left(1 - \left(\cos^2\dfrac{\theta}{2} - \sin^2\dfrac{\theta}{2}\right)\right)$ $= 2a^2\left(2\sin^2\dfrac{\theta}{2}\right)$ $S = 2\int_0^{\pi} \sqrt{2a^2\left(2\sin^2\dfrac{\theta}{2}\right)} \, \mathrm{d}\theta$ $= 4a\int_0^{\pi} \sin\dfrac{\theta}{2} \, \mathrm{d}\theta$ $= 4a\left[-2\cos\dfrac{\theta}{2}\right]_0^{\pi} = -8a\left[\cos\dfrac{\pi}{2} - \cos 0\right] = 8a$

	METHOD ONE $1-\cos\theta = \frac{r}{a}$ $\sin\theta\,\mathrm{d}\theta = \frac{1}{a}\mathrm{d}r$ $\mathrm{d}\theta = \frac{\mathrm{d}r}{a\sin\theta}$ $\sin\theta = \sqrt{1-\cos^2\theta}$ $= \sqrt{1-\left(1-\frac{r}{a}\right)^2}$ $= \sqrt{\frac{2ar-r^2}{a^2}}$ $so\ I = 2\times\int_0^{2a}\sqrt{2}\cdot a\cdot\sqrt{\frac{r}{a}}\cdot\frac{1}{a}\cdot\frac{a}{\sqrt{2ar-r^2}}\mathrm{d}r$ $= 2\sqrt{2a}\cdot\int_0^{2a}\sqrt{\frac{r}{2ar-r^2}}\,\mathrm{d}r$ $= 2\sqrt{2a}\cdot\int_0^{2a}(2a-r)^{-\frac{1}{2}}\mathrm{d}r$ $= 2\sqrt{2a}\left[-2(2a-r)^{\frac{1}{2}}\right]_0^{2a} = 2\sqrt{2a}\left(0+2\sqrt{2a}\right) = 8$ METHOD TWO Note: $1-\cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$ $I = \sqrt{2}a\int_0^{2\pi}\sqrt{2\sin^2\frac{\theta}{2}}\mathrm{d}\theta$ $= 4a\int_0^{\pi}\sin\frac{\theta}{2}\mathrm{d}\theta$ $= 4a\left[-2\cos\frac{\theta}{2}\right]_0^{\pi} = 4a\left(0-(-2)\right)$ $= 8a$

(c)	Firstly by chain rule, $m\frac{\mathrm{d}v}{\mathrm{d}t} = m\frac{\mathrm{d}v}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t} = mv\frac{\mathrm{d}v}{\mathrm{d}x}$ $mv\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{mgR^2}{(x+R)^2}$ $m\int v\,\mathrm{d}v = -\int \frac{mgR^2}{(x+R)^2}dx$ $\frac{1}{2}v^2 = gR^2(x+R)^{-1} + C$ The max height h is reached when $v = 0$ i.e. $C = \frac{-gR^2}{h+R}$ Exact soln: $\frac{1}{2}v^2 = \frac{gR^2}{x+R} - \frac{gR^2}{h+R}$ $\frac{1}{2}v^2 = \frac{gR^2(h+R-x-R)}{(x+R)(h+R)} = \frac{gR^2(h-x)}{(x+R)(h+R)}$ $v = \sqrt{\frac{2gR^2(h-x)}{(x+R)(h+R)}}$ At v_0, $x = 0$ $v_0 = \sqrt{\frac{2gRh}{h+R}}$

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<tr><th>Q</th><th>Solution</th></tr>
<tr><td>FIVE (a)(i)</td><td>

$\cos 5\theta = Re(\cos\theta + i\sin\theta)^5$

$(\cos\theta + i\sin\theta)^5 = \cos^5\theta + \binom{5}{1}(\cos^4\theta)(i\sin\theta) + \binom{5}{2}(\cos^3\theta)(i\sin\theta)^2$

$+\binom{5}{3}(\cos^2\theta)(i\sin\theta)^3 + \binom{5}{4}(\cos\theta)(i\sin\theta)^4 + (i\sin\theta)^5$

$= \cos^5\theta + 5i(\cos^4\theta)\sin\theta - 10(\cos^3\theta)\sin^2\theta - 10i\cos^2\theta\sin^3\theta + 5\cos\theta\sin^4\theta + i\sin^5\theta$

Taking the real part only:

$\cos 5\theta = \cos^5\theta - 10(\cos^3\theta)\sin^2\theta + 5\cos\theta\sin^4\theta$

$= \cos^5\theta - 10(\cos^3\theta)(1-\cos^2\theta) + 5\cos\theta(1-\cos^2\theta)^2$

$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

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Or using trig identities

$\cos(5\theta) = \cos(4\theta + \theta)$

$= \cos 4\theta\cos\theta - \sin 4\theta\sin\theta$

$= (2\cos^2 2\theta - 1)\cos\theta - 2\sin 2\theta\cos 2\theta\sin\theta$

$= \left(2(2\cos^2\theta - 1)^2 - 1\right)\cos\theta - 4\sin^2\theta\cos\theta\cos 2\theta$

$= (8\cos^4\theta - 8\cos^2\theta + 1)\cos\theta - 4(1-\cos^2\theta)\cos\theta\cos 2\theta$

$= 8\cos^5\theta - 8\cos^3\theta + \cos\theta + 4(\cos^3\theta - \cos\theta)(2\cos^2\theta - 1)$

$= 8\cos^5\theta - 8\cos^3\theta + \cos\theta + 8\cos^5\theta - 12\cos^3\theta + 4\cos\theta$

$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

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<tr><td>(a)(ii)</td><td>

$\cos 5\theta = \cos 4\theta$ has solns $5\theta = 2k\pi \mp 4\theta$, i.e. $\theta = 2k\pi$ or $9\theta = 2n\pi$, k and n integers. Also $\cos 5\theta - \cos 4\theta = 0$ will give a degree five polynomial which can be factored to a quadratic and a cubic. It is the cubic that will yield the roots required.

$\cos 4\theta = Re(\text{cis}\theta)^4$

$= \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$

$= \cos^4\theta - 6\cos^2\theta(1-\cos^2\theta) + (1-\cos^2\theta)^2$

$= 8\cos^4\theta - 8\cos^2\theta + 1$

so: $\cos 5\theta - \cos 4\theta = 16\cos^5\theta - 8\cos^4\theta - 20\cos^3\theta + 8\cos^2\theta + 5\cos\theta - 1 = 0$

Using $\theta = 2k\pi$ or $9\theta = 2n\pi$, k and n integers the roots are:

$\cos 0$, $\cos\frac{2\pi}{9}$, $\cos\frac{4\pi}{9}$, $\cos\frac{6\pi}{9}$ and $\cos\frac{8\pi}{9}$

But $\cos 0 = 1$ and $\cos\frac{6\pi}{9} = \cos\frac{2\pi}{3} = -\frac{1}{2}$

$\cos\theta - 1$ and $2\cos\theta + 1$ are factors: using equating coefficients

$(2\cos^2\theta - \cos\theta - 1)(A\cos^3\theta + B\cos^2\theta + C\cos\theta + D)$

$= 16\cos^5\theta - 8\cos^4\theta - 20\cos^3\theta + 8\cos^2\theta + 5\cos\theta - 1$

So $D = 1$, $A = 8$,

$-8\cos^4\theta + 2B\cos^4\theta = -8\cos^4\theta \Rightarrow B = 0$

$-8\cos^3\theta + 2C\cos^3\theta = -20\cos^3\theta \Rightarrow C = -6$

The equation is: $(2\cos^2\theta - \cos\theta - 1)(8\cos^3\theta - 6\cos\theta + 1)$

The polynomial with roots $\cos\frac{2\pi}{9}$, $\cos\frac{4\pi}{9}$, and $\cos\frac{8\pi}{9}$ is $(8\cos^3\theta - 6\cos\theta + 1)$

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(b)(i)	$a_{n+1}-a_n=\frac{1}{2}(a_n+a_{n-1})-a_n$ $=\left(-\frac{1}{2}\right)[a_n-a_{n-1}]$ $=\left(-\frac{1}{2}\right)\left[\frac{1}{2}(a_{n-1}+a_{n-2})-a_{n-1}\right]$ $=\left(-\frac{1}{2}\right)^2[a_{n-1}-a_{n-2}]$ $=\left(-\frac{1}{2}\right)^3[a_{n-2}-a_{n-3}]$ Continuing .... $=\left(-\frac{1}{2}\right)^{n-1}[a_2-a_1]=\left(-\frac{1}{2}\right)^{n-1}\times 5$ (Candidate may instead exhaustively calculate terms for $n\geq 2$ and arrive at the same equation. Acceptable!) So: $a_n-a_{n-1}=5\times\left(-\frac{1}{2}\right)^{n-2}$ $a_{n-1}-a_{n-2}=5\times\left(-\frac{1}{2}\right)^{n-3}$ $a_3-a_2=5\times\left(-\frac{1}{2}\right)^1$ $a_2-a_1=5\times\left(-\frac{1}{2}\right)^0$ Adding these equations gives $a_n-a_1=5\left[1+\left(-\frac{1}{2}\right)^1+\left(-\frac{1}{2}\right)^2....+\left(-\frac{1}{2}\right)^{n-2}\right]$ $=\frac{5\left[1-\left(-\frac{1}{2}\right)^{n-1}\right]}{1-\left(-\frac{1}{2}\right)}$ and $a_n=\frac{10}{3}\left[1-\left(-\frac{1}{2}\right)^{n-1}\right]+2$ $=\frac{1}{3}\left[16-10\left(-\frac{1}{2}\right)^{n-1}\right]$
(b)(ii)	$\lim_{n\to\infty}\frac{1}{3}\left[16-10\left(-\frac{1}{2}\right)^{n-1}\right]=\frac{16}{3}$

(b)(i)	**Further solution** First, generate some terms Term: 1, 2, 3, 4, 5, 6, 7 Value: 2, 7, 4.5, 5.75, 5.125, 5.4375, etc Difference: 5, –2.5, +1.25, –0.625, +0.3125, etc $x - ½$, $x - ½$, $x - ½$, $x - ½$, $x - ½$, etc The differences alternate by multiple $-\frac{1}{2}$ So, difference D_i follow a geometric pattern With $t_1 = 5$ $r = -\frac{1}{2}$ Each D_i has value $5\left(-\frac{1}{2}\right)^{i-1}$ [t_n for geometric progression] So in main sequence: $T_2 = T_1 + D_1$, $T_3 = T_2 + D_2 = T_1 + D_1 + D_2$, $T_4 = T_3 + D_3$, $= T_1 + D_1 + D_2 + D_3$ $\therefore T_n = T_1 + \sum_{i=1}^{n-1} D_i$ $= 2 + \frac{5\left(1 - \left(-\frac{1}{2}\right)^{n-1}\right)}{1.5}$ [sum of geometric progression]
(ii)	$\lim_{n\to\infty} T_n = \lim_{n\to\infty} 2 + \frac{5\left(1 - \left(-\frac{1}{2}\right)^{n-1}\right)}{1.5}$ $= 2 + \frac{5}{1.5} = \frac{16}{3}$