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SUPERVISOR'S USE ONLY

Level 2 Mathematics and Statistics, 2015

91267 Apply probability methods in solving problems

2.00 p.m. Tuesday 10 November 2015
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability methods in solving problems.	Apply probability methods, using relational thinking, in solving problems.	Apply probability methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Resource Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Not Achieved

TOTAL

7

ASSESSOR'S USE ONLY

QUESTION ONE

- (a) The waiting time for a patient attending a medical centre before seeing a doctor is approximately normally distributed, with a mean of 34 minutes and a standard deviation of 8 minutes.
- (i) Find the probability that a patient will wait between 34 and 40 minutes.

$$p = \underline{0.2733} =$$

Correct answer. GC used.

- (ii) After how many minutes will 90% of patients have begun being seen by a doctor?

~~$p = 23.74$ minutes~~
~~After 23 - 24 minutes~~

After ~~24 minutes~~, 38 - 39 minutes

Incorrect answer.

- (iii) It is decided that waiting times must be changed so that at least 95% of patients will be seen by a doctor within 40 minutes.

Because of the administration required, the mean time cannot change, but it is known that for each doctor added to the duty teams, the standard deviation will reduce by 0.4 minutes.

How many doctors must be added to meet the new requirement?

About ~~24~~ 33 more doctors must be added to meet the new requirement. //

Incorrect answer.

- (b) At reception, patients are assessed on the urgency of their condition. This is done within two minutes of arrival.

It is thought that the waiting time before an assessment is done is approximately normally distributed with a mean of 60 seconds and standard deviation of 20 seconds.

- (i) What proportion of patients would be assessed at reception within 90 seconds of arrival?

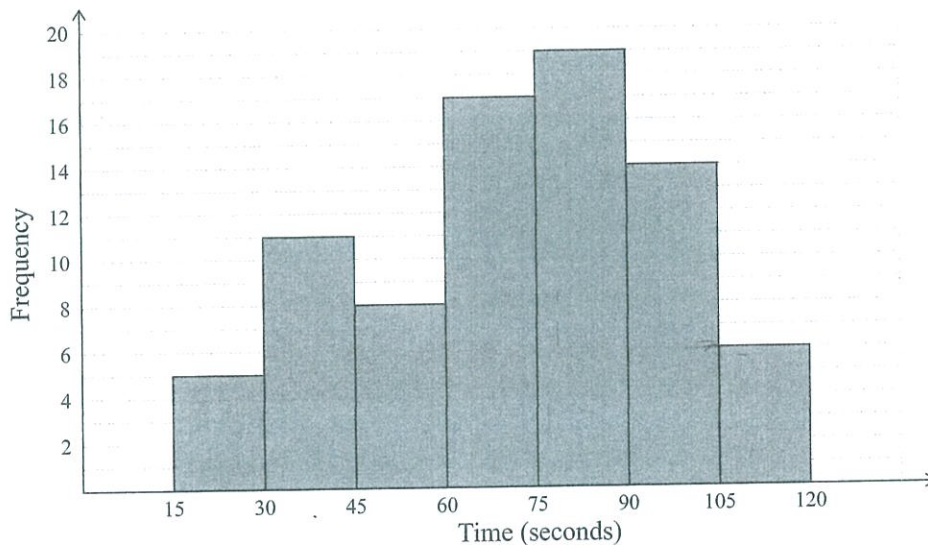
$$0.9316$$

$$= 93.16\%$$

Correct answer. GC used.

- (ii) A survey is carried out on 80 patients who arrive at reception. Patients are selected at random on a particular day. The results are shown in the frequency histogram below.

Assessment time at reception



What proportion of patients in the survey were assessed at reception within 90 seconds of arrival?

$$19 + 17 + 8 + 11 + 5 = 60$$

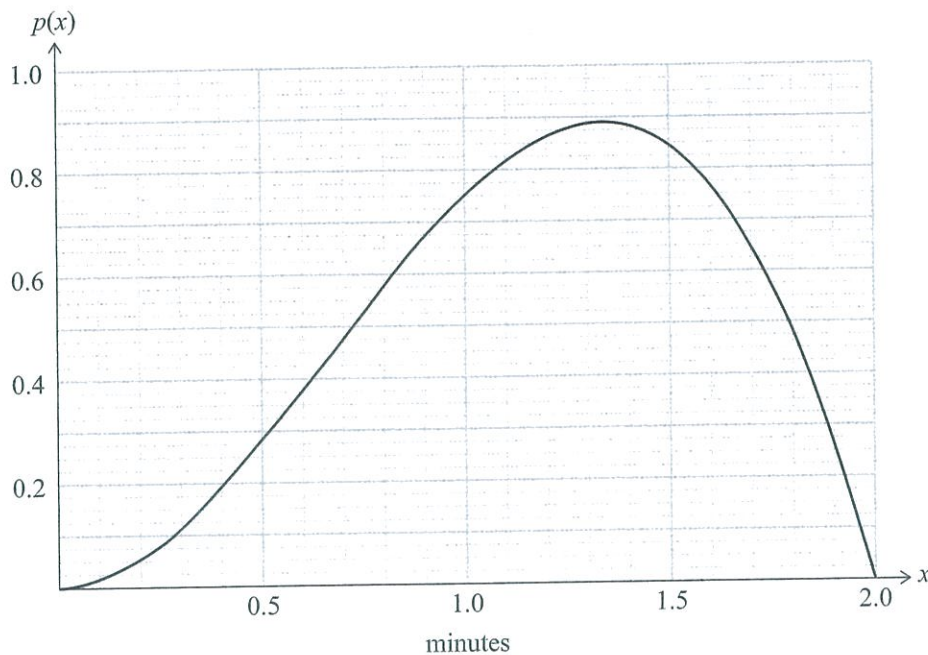
Correct answer.

$$60 + 14 = 74$$

60/80 patients were assessed within 90s

$$60/80 = 0.75 \quad \underline{\underline{75\%}}$$

- (iii) A statistician states that the assessment times are not normally distributed, but are more likely to approximate the distribution $p(x)$ below.



The associated probabilities (with minutes converted to seconds) are given in the following table:

Assessment Time (seconds)	0 –	15 –	30 –	45 –	60 –	75 –	90 –	105 – 120
Probability	0.01	0.05	0.10	0.16	0.21	0.22	0.17	0.08

Compare the frequency histogram for the survey of 80 patients with the distribution curve $p(x)$.

You should comment on the comparative shape, centre, and spread of the two distributions.

It is important to give numerical values to support your statements where possible.

You can see that both graphs are quite similar. The histogram is slightly skewed to the left and the distribution curve is also slightly skewed to the left.

0.75 → patients assessed before 90s

0.25 → patients assessed after 90s

75% of patients assessed before 90s

25% of patients assessed after 90s

A comparative answer on shape but nothing else of relevance.

There is more space for your answer on the following page.



A4

QUESTION TWO

A study is conducted of 1500 randomly selected candidates for an international examination to investigate whether Year 12 candidates were as successful as those from Year 13.

The results are summarised in the table below:

	Year 12	Year 13	Total
Passed	347	853	1200
Failed	33	267	300
Total	380	1120	1500

- (a) (i) What proportion of candidates in the study passed the examination?

$$1200 / 1500$$

$$= 0.8$$

Correct answer.

- (ii) What proportion of candidates who failed the examination were from Year 12?

$$33 / 380$$

$$= 0.0868$$

Incorrect answer. Division by 380 rather than 300 – a common error.

- (iii) There were about 52 500 candidates from Year 12 and Year 13 who attempted the examination.

Using the results of this study, how many candidates would be expected to be from Year 13, and pass the examination?

$$853 / 1120 = 0.7616 = 76.16\%$$

$$52\,500 \times 0.7616 = 39\,984$$

39 984 candidates would be expected to be from year 13, and pass the examination.

Incorrect answer.

1120 used instead of 1500 – a common error.

- (iv) It is claimed that Year 13 candidates are four times more likely to fail the examination than Year 12 candidates.

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State whether or not you agree with this claim, showing full calculations to support your view.

From the study \rightarrow Yr 12 & 13

Yr 13 Failed $\rightarrow 267 / 300 = 0.89$

Yr 12 Failed $\rightarrow 33 / 300 = 0.11$

~~$267 / 33 = 8.09$~~ (rounded to 1st)

~~I agree with this claim.
Reason being is that the year 13s are
8 times more likely to fail than the
year 12s because of the calculations
shown above.
 $8 > 4$ therefore this claim is correct
and I agree with this claim.~~

I agree with this claim because
 $0.89 / 0.11 = 8.0909$ therefore the year
13s aren't ⁴ more times more likely to fail
but are 8.0909 times more likely to
fail. $8.0909 > 4$ hence this claim
is justified.

Incorrect divisors in calculating
the absolute risks.

- (b) The same study also considered the number of subjects the candidates were taking in their normal academic courses. It found that of the same sample of 1500 candidates, 682 were taking six subjects, while the rest were taking five subjects. Of the candidates who were taking five subjects, 192 failed the examination.

The table from page 7 is repeated here to help you answer the questions that follow.

	Year 12	Year 13	Total
Passed	347	853	1200
Failed	33	267	300
Total	380	1120	1500

	Six Subjects	Five Subjects	Total
Passed	341	626	967 533
Failed	341	192	533 967
Total	682	818	1500

- (i) What proportion of candidates in the study took six subjects and passed?

$$\frac{341}{967}$$

$$= 0.3526$$

Incorrect answer arising
from faulty table.

- (ii) On the evidence of this study, would you recommend that candidates take six subjects? Support your answer with numerical calculations that consider the absolute and relative risks. You may also wish to comment on the sensibility of drawing any conclusions on this evidence.

No, I would recommend that candidates should take 5 subjects because according to my calculations:

Ppl who ~~take~~^{took} 6 subjects and passed were $341/967$

Ppl who took 5 subjects and passed were $626/967$.

$$341/967 = 0.3526 = 0.35$$

$$626/967 = 0.6473 = 0.64$$

Therefore if you took 6 subjects ~~you~~^{you'd} you would have a $(0.35/0.64 = 0.54)$ 54% chance of failing. whereas if you took 5 subjects then you would have a ~~46%~~ 46% chance of failing.

$$\text{Failed with 6subs} \rightarrow 341/533 = 0.64$$

$$\text{Failed with 5subs} \rightarrow 192/533 = 0.36$$

~~Therefore~~ Therefore ~~if~~^{for} the people who took 6 subjects ~~had~~^{are} 1.78 ~~the~~ times more likely to fail than the people who take 5 subjects.

$$0.64/0.36 = 1.78$$

Incorrect absolute risks and relative risk found.

QUESTION THREE

ASSESSOR'S
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- (a) When calves are born into a pedigree beef herd, decisions are made after they are one month old, and again when they are three months old, as to whether they will be kept in the herd or sold.

55% of calves born are male. At age one month, 70% of male and 20% of female calves are sold. Of the remainder, at age three months, 80% of males and 35% of females are also sold.

- (i) Find the probability that a randomly chosen calf born into the herd will be male and sold at age one month.

No response.

- (ii) Find the probability that a randomly chosen calf born into the herd will be female and sold at age three months.

No response.

- (iii) What percentage of calves will eventually be kept in the pedigree herd?

No response.

- (iv) In a particular year 550 calves were born.

How many male calves can be expected to be kept in the pedigree herd?

No response.

- (v) The ratio of male to female calves being kept in the herd after three months is about one male to every seven females. This is to be changed to one male to every ten females.

If the number of male calves remains the same, what proportion of females would have been sold?

No response.

- (b) New Zealand fantails are birds which are either pied or black.

Pied fantail

Black fantail

Cherryl Mariner, www.nzbirdsonline.org.nz/species/new-zealand-fantail

They interbreed, and pairs with successful nests are found in the following proportions:

Pair	Two pied fantails	One pied and one black fantail	Two black fantails
Proportion	0.75	0.2	0.05

Successful nests have between one and four eggs. The proportions of eggs are given in the table below.

Pair	Two pied fantails	One pied and one black fantail	Two black fantails
One egg	0.15	0.2	0.3
Two eggs	0.3	0.35	0.5
Three eggs	0.4	0.35	0.15
Four eggs	0.15	0.1	0.05

- (i) What proportion of pairs with two pied fantails will have a successful nest with more than one egg?

$$0.3 + 0.4 + 0.15 = 0.85$$

$$\frac{0.75}{0.85}$$

$$= 0.8823$$

Incorrect answer.

- (ii) A researcher claims that only one out of every 50 nests found with three eggs is likely to be from a pair of two black fantails.

Use calculations to show that the researcher's claim is justified.

$\frac{1}{50}$ nests found with 3 eggs is likely to be from a pair of two black fantails.

$$\frac{1}{50} = 0.02$$

$$\text{Three eggs \& Two black fantails} = 0.15$$

$$\frac{0.02}{0.15} = 0.1\bar{3} = 0.13$$

Incorrect answer.