

L3-CALCF



Mana Tohu Mātauranga o Aotearoa  
New Zealand Qualifications Authority

## Level 3 Calculus 2023

### FORMULAE AND TABLES BOOKLET for 91577, 91578 and 91579

Refer to this booklet to answer the questions in your Question and Answer Booklets.

Check that this booklet has pages 2–4 in the correct order and that none of these pages is blank.

**YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.**

## MATHEMATICS – USEFUL FORMULAE

### ALGEBRA

#### Quadratics

$$\text{If } ax^2 + bx + c = 0$$

$$\text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

#### Logarithms

$$y = \log_b x \Leftrightarrow x = b^y$$

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

$$\log_b(x^n) = n \log_b x$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

#### Complex numbers

$$z = x + iy$$

$$= r \operatorname{cis} \theta$$

$$= r(\cos \theta + i \sin \theta)$$

$$\bar{z} = x - iy$$

$$= r \operatorname{cis}(-\theta)$$

$$= r(\cos \theta - i \sin \theta)$$

$$r = |z| = \sqrt{z\bar{z}} = \sqrt{(x^2 + y^2)}$$

$$\theta = \arg z$$

$$\text{where } \cos \theta = \frac{x}{r}$$

$$\text{and } \sin \theta = \frac{y}{r}$$

#### De Moivre's Theorem

If  $n$  is any integer, then

$$(r \operatorname{cis} \theta)^n = r^n \operatorname{cis} (n\theta)$$

### COORDINATE GEOMETRY

#### Straight Line

$$\text{Equation } y - y_1 = m(x - x_1)$$

### CALCULUS

#### Differentiation

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
$\ln x$	$\frac{1}{x}$
$e^{ax}$	$ae^{ax}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$

#### Integration

$f(x)$	$\int f(x) dx$
$x^n$	$\frac{x^{n+1}}{n+1} + c$ ( $n \neq -1$ )
$\frac{1}{x}$	$\ln x  + c$
$\frac{f'(x)}{f(x)}$	$\ln f(x)  + c$

#### Parametric Function

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt} \left( \frac{dy}{dx} \right) \cdot \frac{dt}{dx}$$

**Product Rule**

$$(f \cdot g)' = g \cdot f' + f \cdot g' \text{ or if } y = uv \text{ then } \frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

**Quotient Rule**

$$\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{g^2} \text{ or if } y = \frac{u}{v} \text{ then } \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

**Composite Function or Chain Rule**

$$(f(g))' = f'(g) \cdot g'$$

$$\text{or if } y = f(u) \text{ and } u = g(x) \text{ then } \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

**NUMERICAL METHODS****Trapezium Rule**

$$\int_a^b f(x) \, dx \approx \frac{1}{2}h[y_0 + y_n + 2(y_1 + y_2 + \dots + y_{n-1})]$$

$$\text{where } h = \frac{b-a}{n} \text{ and } y_r = f(x_r)$$

**Simpson's Rule**

$$\int_a^b f(x) \, dx \approx \frac{1}{3}h[y_0 + y_n + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})]$$

$$\text{where } h = \frac{b-a}{n}, y_r = f(x_r) \text{ and } n \text{ is even.}$$

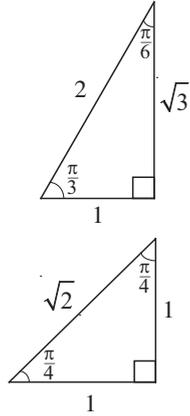
**TRIGONOMETRY**

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

**Sine Rule**

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

**Cosine Rule**

$$c^2 = a^2 + b^2 - 2ab \cos C$$

**Identities**

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$$

**General Solutions**

$$\text{If } \sin \theta = \sin \alpha \text{ then } \theta = n\pi + (-1)^n \alpha$$

$$\text{If } \cos \theta = \cos \alpha \text{ then } \theta = 2n\pi \pm \alpha$$

$$\text{If } \tan \theta = \tan \alpha \text{ then } \theta = n\pi + \alpha$$

where  $n$  is any integer

**Compound Angles**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

**Double Angles**

$$\sin 2A = 2 \sin A \cos A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A \end{aligned}$$

**Products**

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

**Sums**

$$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$$

**MEASUREMENT****Triangle**

$$\text{Area} = \frac{1}{2} ab \sin C$$

**Trapezium**

$$\text{Area} = \frac{1}{2} (a+b)h$$

**Sector**

$$\text{Area} = \frac{1}{2} r^2 \theta$$

$$\text{Arc length} = r\theta$$

**Cylinder**

$$\text{Volume} = \pi r^2 h$$

$$\text{Curved surface area} = 2\pi r h$$

**Cone**

$$\text{Volume} = \frac{1}{3} \pi r^2 h$$

$$\text{Curved surface area} = \pi r l \text{ where } l = \text{slant height}$$

**Sphere**

$$\text{Volume} = \frac{4}{3} \pi r^3$$

$$\text{Surface area} = 4\pi r^2$$