

See back cover for an English translation of this cover

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91577M



915775



NEW ZEALAND QUALIFICATIONS AUTHORITY  
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD  
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

SUPERVISOR'S USE ONLY

Tohua tēnei pouaka mēnā  
KĀORE koe i tuhituhi i  
roto i tēnei pukapuka

# Tuanaki, Kaupae 3, 2021

## 91577M Te whakahāngai i te taurangi o ngā tau matatini hei whakaoti rapanga

Ngā whiwhinga: Rima

Paetae	Kaiaka	Kairangi
Te whakahāngai i te taurangi o ngā tau matatini hei whakaoti rapanga.	Te whakahāngai i te taurangi o ngā tau matatini mā te whakaaro tuhonohono hei whakaoti rapanga.	Te whakahāngai i te taurangi o ngā tau matatini mā te whakaaro waitara hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

**Me whakamātau koe i ngā tūmahi KATOAA kei roto i tēnei pukapuka.**

Tuhia ō mahinga KATOAA.

Tirohia mēnā kei a koe te Pukapuka o ngā Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Ki te hiahia koe ki ētahi atu wāhi hei tuhituhi whakautu, whakamahia te wāhi wātea kei muri i te pukapuka nei.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–19 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi ki roto i tētahi wāhi kairuku whakahāngai (X). Ka tapahia pea tēnei wāhi ina mākahia te pukapuka.

**ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.**

## TŪMAHI TUATAHI

- (a) Ina ko  $w = d + 5i$ ,  $\bar{a}$ ,  $z = 3 - 4i$ , whiriwhiria te uara o  $d$  mēnā  $wz = 38 - 9i$ .

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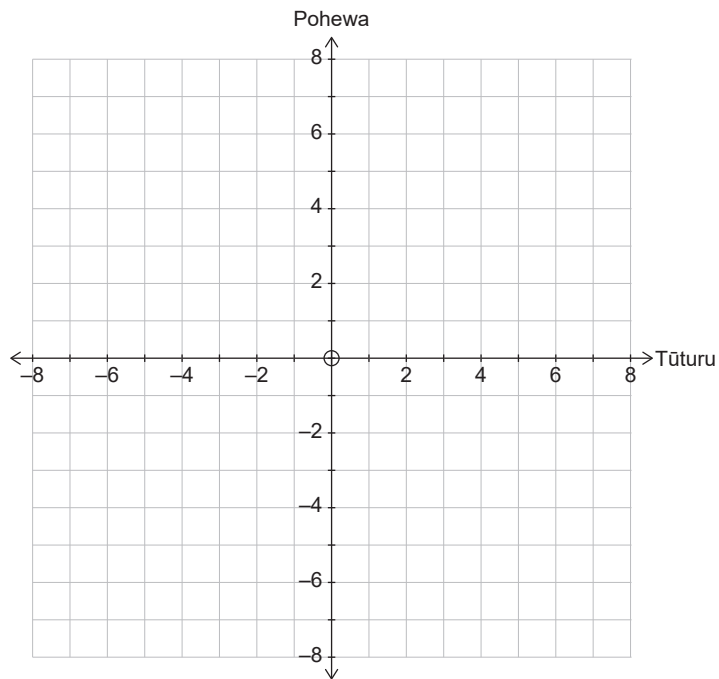


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- (b) Mēnā ko  $z = 2 + 3i$ , whakaaturia  $\frac{26}{z}$  ki te hoahoa Argand i raro nei.



**QUESTION ONE**

- (a) Given that  $w = d + 5i$  and  $z = 3 - 4i$ , find the value of  $d$  if  $wz = 38 - 9i$ .

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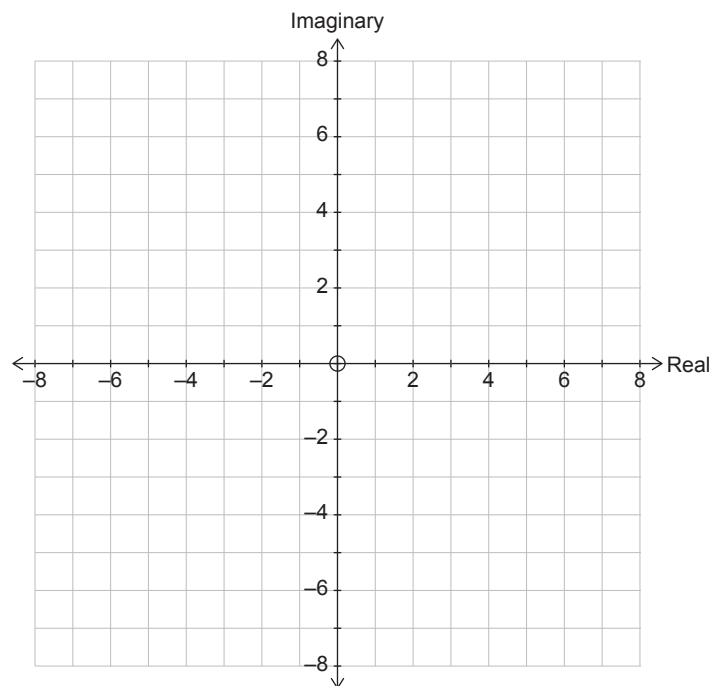


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- (b) If  $z = 2 + 3i$ , show  $\frac{26}{z}$  on the Argand diagram below.



- Whiriwhiria ngā uara o  $a$  me  $b$ .

- (c) The polynomial  $f(x) = x^3 + 3x^2 + ax + b$  has the same remainder when divided by  $(x - 2)$  as it does when divided by  $(x + 1)$ .

The polynomial  $f(x)$  also has  $(x + 2)$  as a factor.

Find the values of  $a$  and  $b$ .

- (d) Show that if  $z = 1 + 3i$ , then  $\arg\left(\frac{z-1}{z-2i}\right) = \frac{\pi}{4}$ .



- (e) Given that the real part of  $\frac{z-2i}{z-4}$  is zero and  $z \neq 4$ , prove that the locus of points described by  $z$  is given by the Cartesian equation  $(x-2)^2 + (y-1)^2 = 5$ .

## TŪMAHI TUARUA

- (a) Ina ko  $u = 2i$ ,  $\bar{a}$ ,  $w = 2\text{cis}\left(\frac{2\pi}{3}\right)$ , whiriwhiria  $z = \frac{u}{w}$ .

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- (b) Whakaotihia te whārite  $x^2 - 12qx + 20q^2 = 0$  mō  $x$  e ai ki  $q$ , me te whakapuaki i ngā otinga ki te āhua māmā rawa atu.

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- (c) Hāponotia ko  $\frac{a+bi}{b-ai}$  he pohewa, ina ko  $a$  me  $b$  he uara pūmau tūturu.

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**QUESTION TWO**

- (a) Given that  $u = 2i$  and  $w = 2\text{cis}\left(\frac{2\pi}{3}\right)$ , find  $z = \frac{u}{w}$ .

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- (b) Solve the equation  $x^2 - 12qx + 20q^2 = 0$  for  $x$  in terms of  $q$ , expressing any solutions in their simplest form.

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- (c) Prove that  $\frac{a+bi}{b-ai}$  is purely imaginary, where  $a$  and  $b$  are real constants.

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- (d) Solve the equation  $z^3 = k^6 + k^6 i$ , where  $k$  is a real constant.

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- (e) If  $z$  is a complex number and  $|z + 16| = 4|z + 1|$ , find the value of  $|z|$ .

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## TŪMAHI TUATORU

- (a) Kei te tau hiato  $u = 5 + mi$  he  $|u| = 6$ .

Ina ko  $0 < \arg(u) < \frac{\pi}{2}$ , whiriwhiria te uara pū o te tau tūturu  $m$ .

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- (b) Tuhia  $\frac{18}{4 - 2\sqrt{3}}$  ki te āhua  $a + b\sqrt{3}$ , ina ko  $a$  me  $b$  he tau tōpū.

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**QUESTION THREE**

- (a) The complex number  $u = 5 + mi$  has  $|u| = 6$ .

Given that  $0 < \arg(u) < \frac{\pi}{2}$ , find the exact value of real number  $m$ .

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- (b) Write  $\frac{18}{4 - 2\sqrt{3}}$  in the form  $a + b\sqrt{3}$ , where  $a$  and  $b$  are integers.

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- (c) Ko tētahi otinga o  $4z^3 - 19z^2 + 128z + A = 0$  ko  $z = 2 + 5i$ .

Mēnā he tau tūturu a  $A$ , whiriwhiria te uara o  $A$  me ērā atu otinga e rua o te whārite.

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- (d) Whakaotihia te whārite e whai ake mō  $x$  e ai ki  $m$ .

$$6\sqrt{2x} - 5 = 6\sqrt{2x + m}$$

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- (c) One solution of  $4z^3 - 19z^2 + 128z + A = 0$  is  $z = 2 + 5i$ .

If  $A$  is real, find the value of  $A$  and the other two solutions of the equation.

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- (d) Solve the following equation for  $x$  in terms of  $m$ .

$$6\sqrt{2x} - 5 = 6\sqrt{2x + m}$$

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Question Three  
continues on page 17.







**He whārangi anō ki te hiahiatia.  
Tuhia te (ngā) tau tūmahi mēnā e tika ana.**

TAU TŪMAHI

**Extra space if required.**  
**Write the question number(s) if applicable.**

QUESTION  
NUMBER

*English translation of the wording on the front cover*

## Level 3 Calculus 2021

### 91577M Apply the algebra of complex numbers in solving problems

Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply the algebra of complex numbers in solving problems.	Apply the algebra of complex numbers, using relational thinking, in solving problems.	Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.


**You should attempt ALL the questions in this booklet.**

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–19 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (). This area may be cut off when the booklet is marked.

**YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.**

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