3

91579

915790

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

SUPERVISOR'S USE ONLY

Level 3 Calculus, 2014

91579 Apply integration methods in solving problems

9.30 am Tuesday 18 November 2014
Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–15 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

ASSESSOR'S USE ONLY

QUESTION ONE

ASSESSOR'S USE ONLY

(a) Find $\int \left(\frac{2}{x} - \frac{3}{x^2} \right) \mathrm{d}x$.

(b) Find the area enclosed between the graph of $y = 3\sec^2 x$, the x-axis, and the lines $x = \frac{\pi}{6}$ and $x = \frac{\pi}{4}$.

Give the result of any integration needed to solve this problem.

(c) The velocity of an object is given by $v(t) = 5(4 - 3e^{-0.2t})$

where t is the time in seconds since the timing started

and v is the velocity in m s^{-1}.

What distance did the object move in the first 10 seconds of its timed motion?

Give the result of any integration needed to solve this problem.

ASSESSOR'S USE ONLY

(d) A tank holds 2500 litres of water. The tank develops a small hole in its base, and water leaks out at a rate proportional to the square root of the volume of water remaining in the tank at any instant.

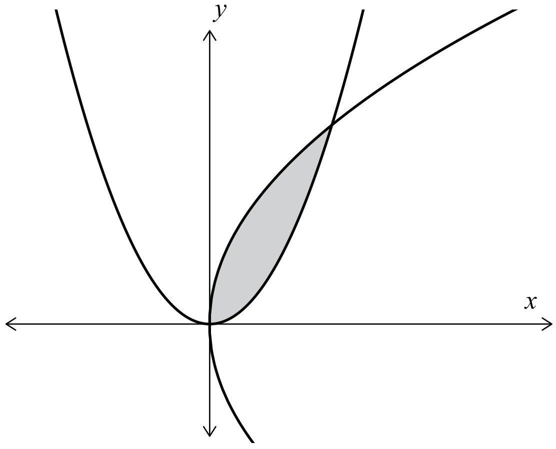
2 days after the leak started, 475 litres of water have leaked out of the tank.

How long will it take the tank to empty completely?

Give the result of any integration needed to solve this problem.

(e) The diagram below shows the graphs of the curves $y^2 = px$ and $y = px^2$, where $p > 1$.

ASSESSOR'S USE ONLY



Show that the area between the two curves is $\frac{1}{3}$.

You must use calculus and give the results of any integration needed to solve this problem.

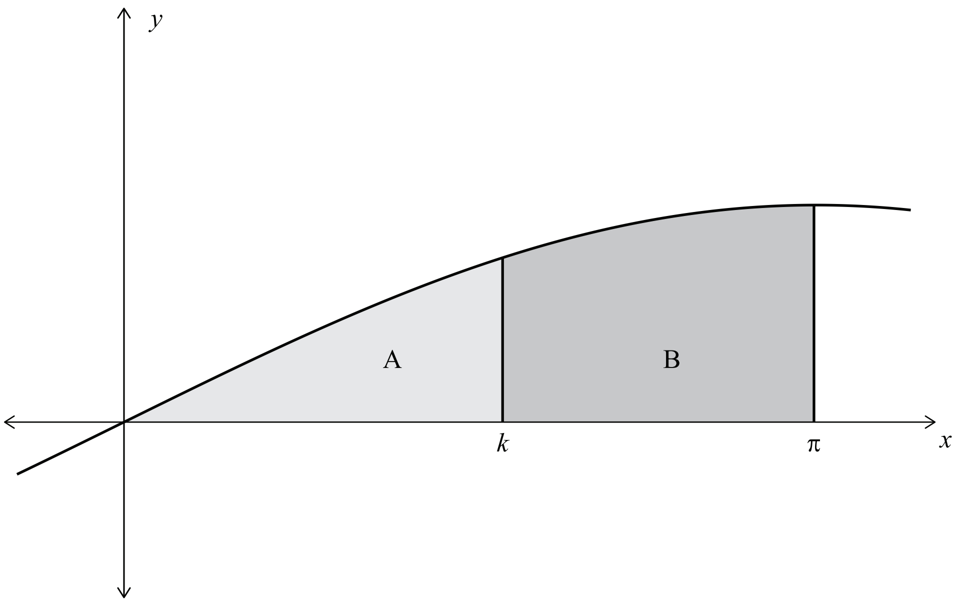
QUESTION TWO

ASSESSOR'S USE ONLY

(a) Find $\int(\sec x \tan x - \sin 2x)\,\mathrm{d}x$.

(b) Solve the differential equation $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 6x^2 - 6x$, given that when $x = 2$, $y = 10$, and $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 8$.

(c) The graph below shows the function $y = \sin\left(\frac{x}{2}\right)$ and the lines $x = k$ and $x = \pi$.



Find the value of k so that the areas A and B are equal.

You must use calculus and give the results of any integration needed to solve this problem.

ASSESSOR'S USE ONLY

(d) If $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3\sqrt{x}}{2y}$ and $y = 5$ when $x = 4$, find the value of y when $x = 9$.

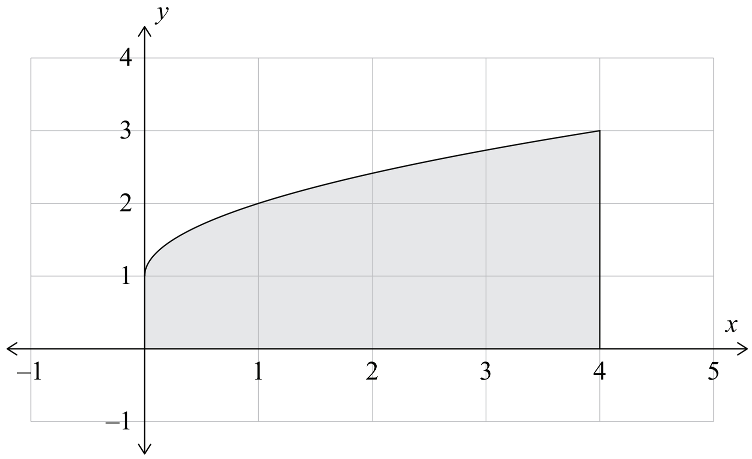
(e) The centre of mass of an object is called the centroid. For a uniformly thin object, the centroid is at $(\bar{x}, \bar{y})$ where

$$\bar{x} = \frac{1}{A}\int_a^b x f(x)\,\mathrm{d}x \quad \text{and} \quad \bar{y} = \frac{1}{A}\int_a^b \frac{\left[f(x)\right]^2}{2}\,\mathrm{d}x$$

A = area of object
a and b are the lower and upper limits of x respectively.

The shape shown shaded in the diagram below is bounded by part of the curve $y = \sqrt{x} + 1$ and the lines $x = 0$, $x = 4$, and $y = 0$.

Find the coordinates $(\bar{x}, \bar{y})$ of the centroid of the shape.



You must use calculus and give the results of any integration needed to solve this problem.

ASSESSOR'S USE ONLY

ASSESSOR'S USE ONLY

QUESTION THREE

ASSESSOR'S USE ONLY

(a) Use the values given below to find an approximation to $\int_2^5 f(x)\,\mathrm{d}x$, using Simpson's Rule.

x	2	2.5	3	3.5	4	4.5	5
$f(x)$	0.8	1.12	2.02	2.17	2.28	1.56	1.2

(b) Find $\int\left(\sqrt[3]{x}+6\mathrm{e}^{3x-5}\right)\mathrm{d}x$.

ASSESSOR'S USE ONLY

(c) An oven tray is taken from a hot oven and placed in a room that has a constant temperature of 20ºC.

The rate at which the temperature of the oven tray changes at any instant is proportional to the difference between the temperature of the oven tray and the room temperature at that instant.

(i) Write a differential equation that models this situation.

(ii) The temperature of the oven tray is originally 220ºC.

After 3 minutes its temperature is 100ºC.

Solve the differential equation in (i) to find what the temperature of the oven tray will be after a further 2 minutes.

Give the result of any integration needed to solve this problem.

Question Three continues on the following page.

ASSESSOR'S USE ONLY

(d) Find $\int_4^9 \frac{18}{x\sqrt{x}}\,\mathrm{d}x$.

Give the result of any integration needed to solve this problem.

ASSESSOR'S USE ONLY

(e) If a plant is grown at a constant temperature in a glasshouse, then the rate of growth of the plant depends on the length of the day.

The rate of growth is given by the equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = k\left(12 + 3\cos\left(\frac{2\pi t}{365}\right)\right)$$

where t is the time measured from the longest day of the year **in days**

h is the height of the plant, in cm

and k is the growth constant, which is different for each plant.

On the longest day of a particular year, a plant has a height of 84 cm.

75 days later the plant has a height of 91 cm.

What will the height of the plant be on the longest day of the next year?

Assume the length of a year is 365 days.

Give the result of any integration needed to solve this problem.

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

91579