See back cover for an English translation of this cover

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91579M

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

Tuanaki, Kaupae 3, 2014

91579M Te whakahāngai i ngā tikanga pāwhaitua hei whakaoti rapanga

9.30 i te ata Rātū 18 Whiringa-ā-rangi 2014
Whiwhinga: Ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pāwhaitua hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pāwhaitua mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pāwhaitua mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mehemea e ōrite ana te Tau Ākonga ā-Motu (NSN) kei tō pepa whakauru ki te tau kei runga ake nei.

Whakautua e koe ngā pātai KATOA kei roto i te pukapuka nei.

Whakaaturia ngā mahinga KATOA.

Me mātua riro mai i a koe te pukaiti o ngā Tikanga Tātai me ngā Papatau L3–CALCMF.

Ki te hiahia koe ki ētahi atu wāhi hei tuhituhi whakautu, whakamahia te (ngā) whārangi kei muri i te pukapuka nei, ka āta tohu ai i ngā tau pātai.

Tirohia mehemea kei roto nei ngā whārangi 2–27 e raupapa tika ana, ā, kāore hoki he whārangi wātea.

HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

MĀ TE KAIMĀKA ANAKE

PĀTAI TUATAHI

(a) Whiriwhiria $\int\left(\frac{2}{x}-\frac{3}{x^2}\right)\mathrm{d}x$.

(b) Whiriwhiria te horahanga e taiapatia ana e te kauwhata o $y = 3\sec^2 x$, te tuaka-x, me ngā rārangi $x = \frac{\pi}{6}$ me $x = \frac{\pi}{4}$.

Whakaaturia te otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i tēnei rapanga.

(c) Ko te tere o tētahi ahanoa ko te $v(t) = 5(4 - 3e^{-0.2t})$

ina ko t te wā ā-hākona i muri mai i te whakahaeretanga o te wā,

ā, ka mutu ko v te tere i te m s^{-1}.

He aha te tawhiti i neke ai te ahanoa i ngā hākona 10 tuatahi o te whakahaeretanga o te wā?

Whakaaturia te otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i tēnei rapanga.

QUESTION ONE

(a) Find $\int \left(\frac{2}{x} - \frac{3}{x^2} \right) \mathrm{d}x$.

(b) Find the area enclosed between the graph of $y = 3\sec^2 x$, the x-axis, and the lines $x = \frac{\pi}{6}$ and $x = \frac{\pi}{4}$.

Give the result of any integration needed to solve this problem.

(c) The velocity of an object is given by $v(t) = 5(4 - 3e^{-0.2t})$

where t is the time in seconds since the timing started

and v is the velocity in m s^{-1}.

What distance did the object move in the first 10 seconds of its timed motion?

Give the result of any integration needed to solve this problem.

(d) He 2500 rita wai e puritia ana e tētahi taika. He puare iti kua puta i tana papa, ā, ka puta haere te wai ki tētahi pāpātanga hāngai ki te pūtake pūrua o te rōrahi o te wai e toe ana i te taika wai i tētahi wā.

E 2 rā mai i te tīmatanga o te putanga mai o te wai, e 475 rita wai kua puta i te taika.

E hia te roa ka pau katoa te wai i te taika?

Whakaaturia te otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i tēnei rapanga.

MĀ TE KAIMĀKA ANAKE

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(d) A tank holds 2500 litres of water. The tank develops a small hole in its base, and water leaks out at a rate proportional to the square root of the volume of water remaining in the tank at any instant.

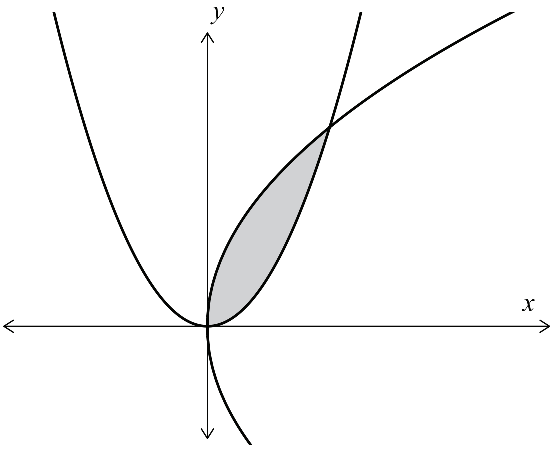
2 days after the leak started, 475 litres of water have leaked out of the tank.

How long will it take the tank to empty completely?

Give the result of any integration needed to solve this problem.

MĀ TE KAIMĀKA ANAKE

(e) E whakaatu ana te hoahoa i raro i ngā kauwhata o ngā ānau $y^2 = px$ me $y = px^2$, ina ko $p > 1$.



Whakaaturia ko te horahanga i waenga i ngā ānau e rua ko $\frac{1}{3}$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

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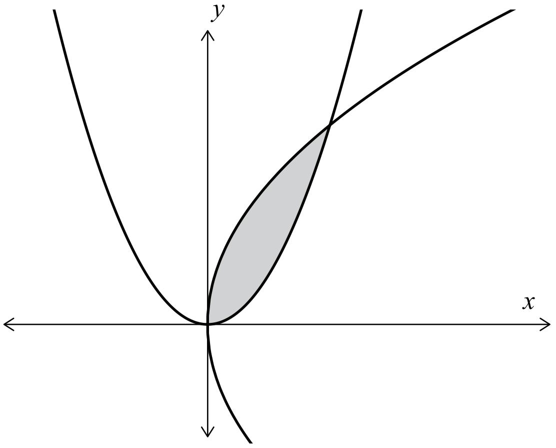
(e) The diagram below shows the graphs of the curves $y^2 = px$ and $y = px^2$, where $p > 1$.



Show that the area between the two curves is $\frac{1}{3}$.

You must use calculus and give the results of any integration needed to solve this problem.

PĀTAI TUARUA

MĀ TE KAIMĀKA ANAKE

(a) Whiriwhiria $\int (\sec x \tan x - \sin 2x)\,\mathrm{d}x$.

(b) Whakaotihia te whārite pārōnaki $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 6x^2 - 6x$, ina ko $x = 2$, kāti ko $y = 10$ me $\frac{\mathrm{d}y}{\mathrm{d}x} = 8$.

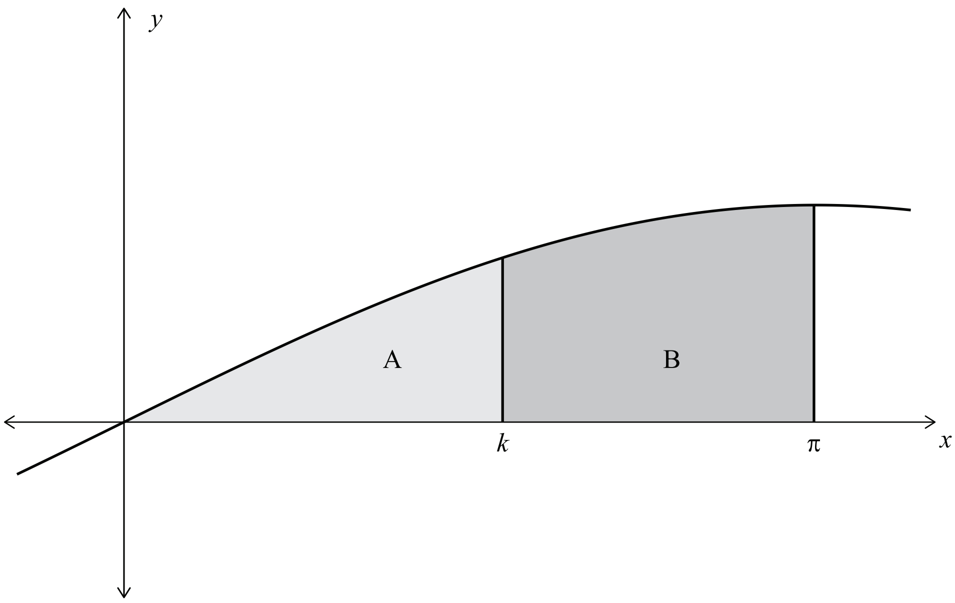
QUESTION TWO

ASSESSOR'S USE ONLY

(a) Find $\int (\sec x \tan x - \sin 2x)\,\mathrm{d}x$.

(b) Solve the differential equation $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 6x^2 - 6x$, given that when $x = 2$, $y = 10$, and $\frac{\mathrm{d}y}{\mathrm{d}x} = 8$.

(c) E whakaatu ana te kauwhata i raro i te pānga $y = \sin\left(\frac{x}{2}\right)$ me ngā rārangi $x = k$ me $x = \pi$.

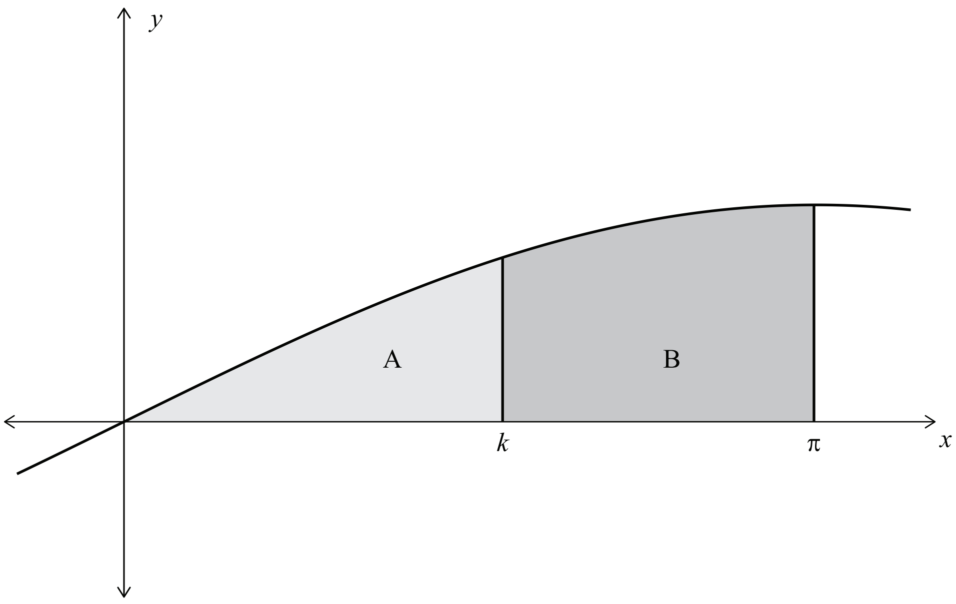


Kimihia te uara o k kia noho ōrite ngā horahanga A me B.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

ASSESSOR'S USE ONLY

(c) The graph below shows the function $y = \sin\left(\frac{x}{2}\right)$ and the lines $x = k$ and $x = \pi$.



Find the value of k so that the areas A and B are equal.

You must use calculus and give the results of any integration needed to solve this problem.

MĀ TE KAIMĀKA ANAKE

(d) Mēnā ko $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3\sqrt{x}}{2y}$ me $y = 5$ ina ko $x = 4$, kimihia te uara o y ina ko $x = 9$.

ASSESSOR'S USE ONLY

(d) If $\frac{dy}{dx} = \frac{3\sqrt{x}}{2y}$ and $y = 5$ when $x = 4$, find the value of y when $x = 9$.

MĀ TE KAIMĀKA ANAKE

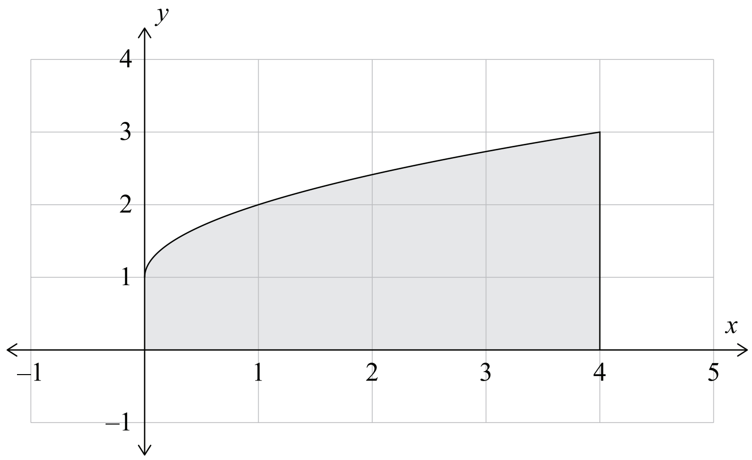
(e) E kīia ana ko te papatipu waenga pū o tētahi ahanoa ko te pū āhuahanga. Mō tētahi ahanoa rahirahi, ko te pū āhuahanga kei $(\bar{x},\bar{y})$ ina ko

$$\bar{x} = \frac{1}{A}\int_a^b x f(x)\,\mathrm{d}x \quad \text{me} \quad \bar{y} = \frac{1}{A}\int_a^b \frac{[f(x)]^2}{2}\,\mathrm{d}x$$

A = horahanga o te ahanoa
ko a te tepe pāpaku me b te tepe teitei o x.

Ko te āhua e kaurukutia ana i roto te hoahoa i raro kei te rohea e tētahi wāhanga o te ānau $y = \sqrt{x} + 1$ me ngā rārangi $x = 0$, $x = 4$, me $y = 0$.

Kimihia ngā taunga $(\bar{x},\bar{y})$ o te pū āhuahanga o te āhua.



Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

He wāhi anō mō tō whakautu ki tēnei pātai kei te whārangi 16.

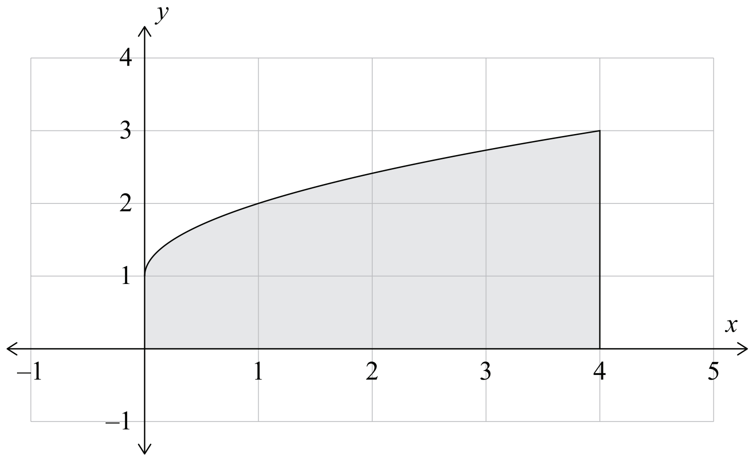
(e) The centre of mass of an object is called the centroid. For a uniformly thin object, the centroid is at $\left(\bar{x},\bar{y}\right)$ where

$$\bar{x}=\frac{1}{A}\int_a^b x f\left(x\right)\mathrm{d}x \text{ and } \bar{y}=\frac{1}{A}\int_a^b \frac{\left[f\left(x\right)\right]^2}{2}\mathrm{d}x$$

A = area of object
a and b are the lower and upper limits of x respectively.

The shape shown shaded in the diagram below is bounded by part of the curve $y=\sqrt{x}+1$ and the lines $x = 0$, $x = 4$, and $y = 0$.

Find the coordinates $\left(\bar{x},\bar{y}\right)$ of the centroid of the shape.



You must use calculus and give the results of any integration needed to solve this problem.

There is more space for your answer to this question on page 17.

ASSESSOR'S USE ONLY

MĀ TE KAIMĀKA ANAKE

ASSESSOR'S USE ONLY

PĀTAI TUATORU

MĀ TE KAIMĀKA ANAKE

(a) Whakamahia ngā uara i raro ki te kimi i tētahi āwhiwhitanga ki $\int_2^5 f(x)\,\mathrm{d}x$, mā te whakamahi i te Ture a Simpson.

x	2	2.5	3	3.5	4	4.5	5
$f(x)$	0.8	1.12	2.02	2.17	2.28	1.56	1.2

(b) Whiriwhiria $\int\left(\sqrt[3]{x}+6\mathrm{e}^{3x-5}\right)\mathrm{d}x$.

QUESTION THREE

ASSESSOR'S USE ONLY

(a) Use the values given below to find an approximation to $\int_2^5 f(x)\,\mathrm{d}x$, using Simpson's Rule.

x	2	2.5	3	3.5	4	4.5	5
$f(x)$	0.8	1.12	2.02	2.17	2.28	1.56	1.2

(b) Find $\int\left(\sqrt[3]{x}+6\mathrm{e}^{3x-5}\right)\mathrm{d}x$.

MĀ TE KAIMĀKA ANAKE

(c) Ka tangohia he paepae umu mai i tētahi umu wera ka whakatakotoria i roto i tētahi rūma he $20^{o}C$ te pāmahana aumou.

Ko te pāpātanga e huri ai te pāmahana o te paepae umu i tētahi wā ko te pāpātanga hāngai ki te rerekētanga i waenga i te pāmahana o te paepae umu me te pāmahana takiwā i taua wā tonu.

(i) Tuhia tētahi whārite pārōnaki e whakatauira ana i tēnei āhuatanga.

(ii) Ko te pāmahana o te paepae umu i te tuatahi he $220^{o}C$.

I muri i te 3 meneti he $100^{o}C$ tana pāmahana.

Whakaotihia te whārite pārōnaki i (i) kia kitea te pāmahana o te paepae umu i muri i te 2 meneti anō.

Whakaaturia te otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

Ka haere tonu te Pātai Tuatoru i te whārangi 22.

ASSESSOR'S USE ONLY

(c) An oven tray is taken from a hot oven and placed in a room that has a constant temperature of 20°C.

The rate at which the temperature of the oven tray changes at any instant is proportional to the difference between the temperature of the oven tray and the room temperature at that instant.

(i) Write a differential equation that models this situation.

(ii) The temperature of the oven tray is originally 220°C.

After 3 minutes its temperature is 100°C.

Solve the differential equation in (i) to find what the temperature of the oven tray will be after a further 2 minutes.

Give the result of any integration needed to solve this problem.

Question Three continues on page 23.

(d) Whiriwhiria $\int_4^9 \frac{18}{x\sqrt{x}}\, \mathrm{d}x$.

Whakaaturia te otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i tēnei rapanga.

MĀ TE KAIMĀKA ANAKE

ASSESSOR'S USE ONLY

(d) Find $\displaystyle\int_4^9 \frac{18}{x\sqrt{x}}\,dx$.

Give the result of any integration needed to solve this problem.

MĀ TE KAIMĀKA ANAKE

(e) Mēnā ka whakatipuhia tētahi tipu ki tētahi pāmahana aumou i roto i tētahi koropū karaehe, ko te pāpātanga tipu o te tipu kei te roa o te rangi.

Ko te pāpātanga tipu kei te whārite

$$\frac{\mathrm{d}h}{\mathrm{d}t} = k\left(12 + 3\cos\left(\frac{2\pi t}{365}\right)\right)$$

ina ko t te wā ka inea mai i te rangi roa rawa o te tau, **ā-rangi**

ina ko h te teitei ā-henemita[1] o te tipu

ka mutu ko k te aumou tipu, ā, he rerekē mō ia tipu.

I te rangi roa rawa o tētahi tau, he 84 henemita te teitei o tētahi tipu.

I ngā rā 75 i muri he 91 henemita te teitei o te tipu.

He aha te teitei o te tipu ā te rangi roa rawa o te tau o muri mai?

Ko te tikanga e 365 ngā rangi o te tau.

Whakaaturia te otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

[1] mitarau

(e) If a plant is grown at a constant temperature in a glasshouse, then the rate of growth of the plant depends on the length of the day.

ASSESSOR'S USE ONLY

The rate of growth is given by the equation

$$\frac{dh}{dt} = k\left(12 + 3\cos\left(\frac{2\pi t}{365}\right)\right)$$

where t is the time measured from the longest day of the year **in days**

h is the height of the plant, in cm

and k is the growth constant, which is different for each plant.

On the longest day of a particular year, a plant has a height of 84 cm.

75 days later the plant has a height of 91 cm.

What will the height of the plant be on the longest day of the next year?

Assume the length of a year is 365 days.

Give the result of any integration needed to solve this problem.

He puka anō mēnā ka hiahiatia.
Tuhia te (ngā) tāu pātai mēnā e hāngai ana.

TAU PĀTAI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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English translation of the wording on the front cover

91579M

Level 3 Calculus, 2014

91579 Apply integration methods in solving problems

9.30 am Tuesday 18 November 2014
Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.