2

91262M

SUPERVISOR'S USE ONLY

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Te Pāngarau me te Tauanga, Kaupae 2, 2015

91262M Te whakahāngai tikanga tuanaki hei whakaoti rapanga

2.00 i te ahiahi Rātū 10 Whiringa-ā-rangi 2015
Whiwhinga: Rima

Paetae	Kaiaka	Kairangi
Te whakahāngai tikanga tuanaki hei whakaoti rapanga.	Te whakahāngai tikanga tuanaki mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai tikanga tuanaki mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tirohia mēnā kei a koe te Rau Rauemi L2–MATHF.

Whakaaturia ngā mahinga KATOA.

Mēnā ka hiahia whārangi atu anō koe mō ō tuhinga, whakamahia ngā whārangi wātea kei muri o tēnei pukapuka, ka āta tohu ai i te tau tūmahi.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–21 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

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TŪMAHI TUATAHI

(a) Ka tohua he pānga f mā te $f(x) = x^4 + 2x^2 - 5$

Tātaihia te rōnaki o te kauwhata o te pānga kei te pūwāhi $x = -1$.

(b) $f(x) = 8 + 3x + x^2 - \frac{x^3}{3}$

He aha ngā uara o x e noho ai a f hei pānga heke?

Parahautia tō tuhinga.

Me mātua whakaatu koe kei te whakamahia ngā tikanga tuanaki.

(c) Tātaihia te pāpātanga o te huri o te rōrahi o tētahi mataono rite e ai ki tōna roa, i te wā tonu ko te roa o ia taitapa o taua mataono rite he 5 cm.

QUESTION ONE

(a) A function f is given by $f(x) = x^4 + 2x^2 - 5$

Find the gradient of the graph of the function at the point where $x = -1$.

(b) $f(x) = 8 + 3x + x^2 - \frac{x^3}{3}$

For what values of x is f a decreasing function?

Justify your answer.

You must show the use of calculus.

(c) Calculate the rate at which the volume of a cube is changing with respect to its length, at the instant when the length of each edge of the cube is 5 cm.

(d) Ka hipa tētahi tereina i tētahi pou tohu ki te tere o te 40 m s^{-1}.
Ka taea te whakaterenga, a m s^{-2}, o te tereina, t hēkona i muri i te hipanga i te pou tohu, te whakatauira mā te pānga

$$a(t) = (16 - 2t)$$

(i) He aha te tere nui rawa i taea e te tereina i muri i tana hipanga i te pou tohu?

(ii) E hia te tawhiti i muri i te hipanga i te pou tohu e haere ana te tereina i mua i tana tūnga?

MĀ TE KAIMĀKA ANAKE

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(d) A train passes a signal at a velocity of 40 m s^{-1}.
The train's acceleration, *a* m s^{-2}, *t* seconds after it passes the signal, can be modelled by the function

$$a(t) = (16 - 2t)$$

(i) What is the greatest speed attained by the train after it passes the signal?

(ii) How far past the signal does the train travel before it stops?

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUARUA

(a) Ka tohua te rōnaki o te pānga f mā te $f'(x) = 4x - 3$

E takoto ana te pūwāhi (4,6) ki te kauwhata o te pānga.

Tātaihia te whārite mō te pānga f.

(b) Ka tohua he pānga g mā te $g(\mathrm{x}) = x^2 - 3x + 18$.

(i) Tātaihia te whārite o te pātapa i te pūwāhi o te kauwhata o g ko te 0 te rōnaki.

(ii) E ai ki te kauwhata, āta whakaahuatia te pūwāhi e tūtaki ai tēnei pātapa i te pānga.

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QUESTION TWO

(a) The gradient of function f is given by $f'(x) = 4x - 3$

The point (4,6) lies on the graph of the function.

Find the equation of the function f.

(b) A function g is given by $g(x) = x^2 - 3x + 18$.

(i) Find the equation of the tangent at the point on the graph of g where the gradient is 0.

(ii) In relation to the graph, fully describe the point where this tangent meets the function.

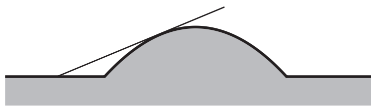
(c) He ahu tō tētahi papa papareti he h mita te teitei i te pūwāhi o te tawhiti huapae, mai i tētahi pūwāhi pūmau P, he x mita.

Ka taea te ahu te whakatauira mā te:

$$h = -0.5x^2 + 3x - 1.5$$

(i) He aha te teitei mōrahi o te ahu?

(ii) Ko tētahi papa rōnaki ki te taha o te ahu he pātapa ki te ahu.



Ka taea te ahu te whakatauira mā te pānga

$$h = 0.5x - c$$

Whakamahia te tuanaki hei whiriwhiri i te tawhiti poutū i raro i te ahu e tūtaki ai te papa rōnaki i te ahu.

Kaua e aro atu ki te mātotoru o te papa rōnaki.

ASSESSOR'S USE ONLY

(c) A skateboard park has a mound that is h metres high at the point where the horizontal distance, from a fixed point P, is x metres.

The mound can be modelled by

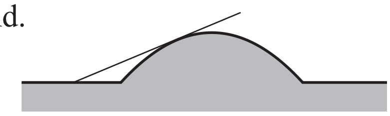
$$h = -0.5x^2 + 3x - 1.5$$

(i) What is the maximum height of the mound?

(ii) A ramp up the side of the mound is a tangent to the mound.

The ramp can be modelled by the function

$$h = 0.5x - c$$



Use calculus to find the vertical distance below the top of the mound where the ramp will meet the mound.

Ignore the thickness of the ramp.

MĀ TE KAIMĀKA ANAKE

(iii) Ko te teitei h mita o tētahi ara papareti ki tētahi tawhiti huapae r mita mai i tētahi atu pūwāhi Q, ka taea te whakatauira mā te pānga

$$h = \frac{r^3}{3} - 2r^2 + 3r \qquad (0.15 < r < 3.5)$$

He ture mō te teitei e here ana kia kaua tētahi wāhanga o te ara papareti e eke atu i te 3 m te teitei i runga ake i te papa.

Āta whakaahuatia tēnei ānau tae atu ki ngā pūwāhi huringa, me te kī anō mēnā e ū ana te ara papareti ki taua ture, kāore rānei.

Me whakaatu ngā tikanga tuanaki hei whakatutuki i tēnei tūmahi.

(iii) The height *h* metres of a skateboard path at a horizontal distance *r* metres from another point Q, can be modelled by the function

$$h = \frac{r^3}{3} - 2r^2 + 3r \qquad (0.15 < r < 3.5)$$

There is a height regulation that requires no part of the skateboard path to be more than 3 m above the ground.

Fully describe this curve including its turning points, and state whether or not the skateboard path complies with the height regulation.

You must show calculus in answering this question.

ASSESSOR'S USE ONLY

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATORU

(a) Ko te tere v m s^{-1} o tētahi ahanoa e t hēkona i muri i tana hipanga i tētahi pūwāhi pūmau ka taea te whakatauira mā te pānga

$$v(t) = 4t^3 - t^2 + 2t$$

Whiriwhiria te whārite o te whakaterenga o te ahanoa.

(b) Whiriwhiria te whārite o te pātapa ki te ānau $f(x) = x^3 - 2x^2 + x$ i te pūwāhi (2,2) i runga i te ānau.

(c) I roto i tētahi takiwā e karapoti ana i tētahi papa wakarererangi pāmu, he tikanga whakatiki teitei mō ngā pahū ahi o te 50 m.

Ko te teitei h mita i runga ake i te papa e taea e te pahū ahi i te t hēkona i muri i te pahūnga, ka taea te whakatauira mā te pānga

$$h = 20t - 5t^2$$

Ka wāhia e te pahū ahi te tepenga 50 m?

Whakamahia ngā tikanga tuanaki hei parahau i tō tuhinga.

QUESTION THREE

ASSESSOR'S USE ONLY

(a) The velocity v m s^{-1} of an object t seconds after it passes a fixed point can be modelled by the function

$$v(t) = 4t^3 - t^2 + 2t$$

Find the equation for the acceleration of the object.

(b) Find the equation of the tangent to the curve $f(x) = x^3 - 2x^2 + x$ at the point (2,2) on the curve.

(c) In an area surrounding a farming airstrip there is a height restriction for fireworks of 50 m.

The height h metres above the ground reached by a firework t seconds after it is fired, can be modelled by the function

$$h = 20t - 5t^2$$

Will the firework break the 50 m limit?

Use calculus methods to justify your answer.

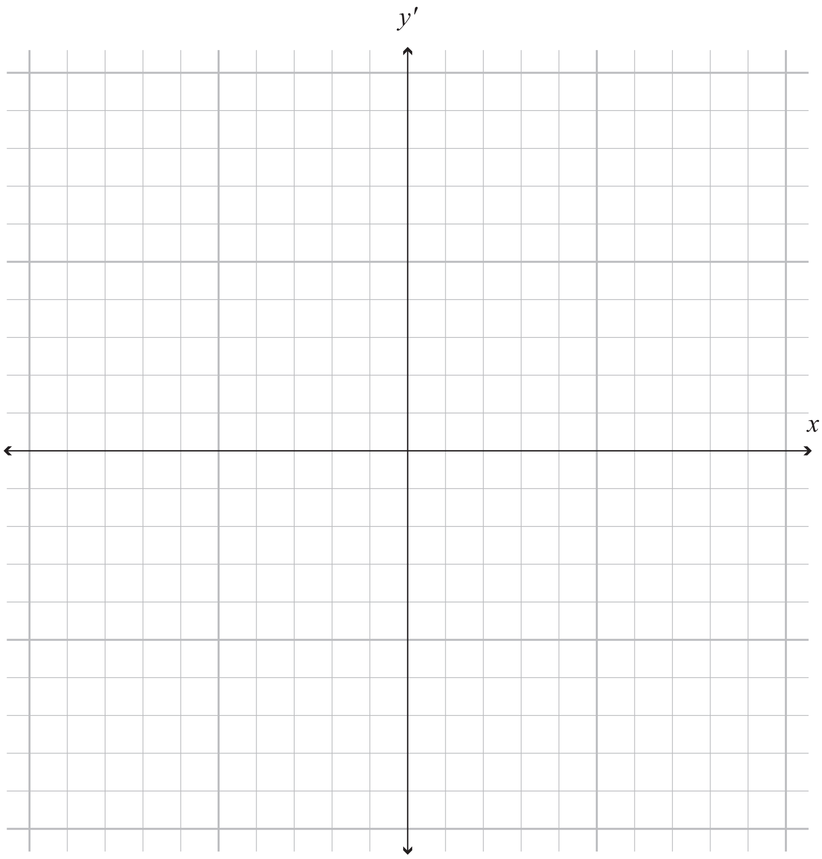
MĀ TE KAIMĀKA ANAKE

(d) Mō tētahi pānga $y = -ax^2 + bx + c$,

he tau tōrunga a a, b, me c, ā, ko $b = 2a$.

Ki te tukutuku o raro, tuhia te pānga rōnaki.

Whakaaturia te uara o ngā haukotinga katoa. Me homai te haukotinga-y' e ai ki a b.



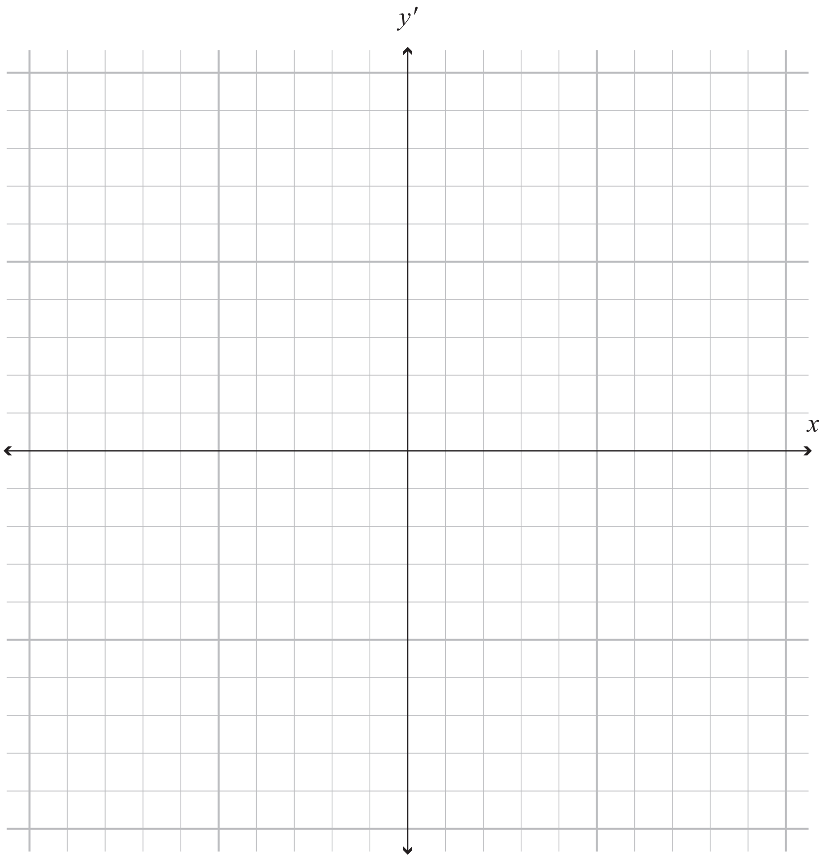
Ki te hiahia koe ki te tuhi anō i tēnei kauwhata, whakamahia te tukutuku i te whārangi 18.

(d) For a function $y = -ax^2 + bx + c$,

a, b, and c are positive numbers and $b = 2a$.

On the grid below, sketch the gradient function.

Show the value of all intercepts. The y'-intercept should be given in terms of b.



If you need to redraw this graph, use the grid on page 19.

ASSESSOR'S USE ONLY

(e) Ko y te uara o x i muri i te tango i te 3 me te whakarearua i te otinga, ā, kei waenga a x i a -0.5 me 3.

Whiriwhiria ngā uara mōrahi me te mōkito o te hua o x^2y.

Parahautia tō tuhinga.

(e) y is the value of x after 3 has been subtracted and then the answer doubled, and x is between -0.5 and 3.

Find the maximum and minimum values of the product of x^2y.

Justify your answer.

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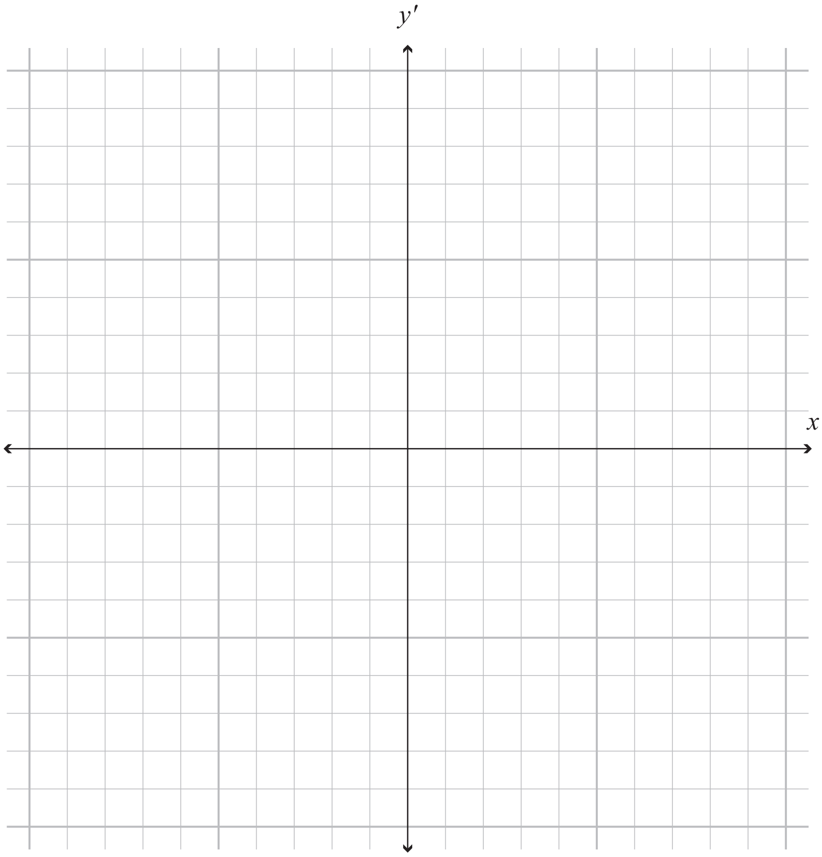
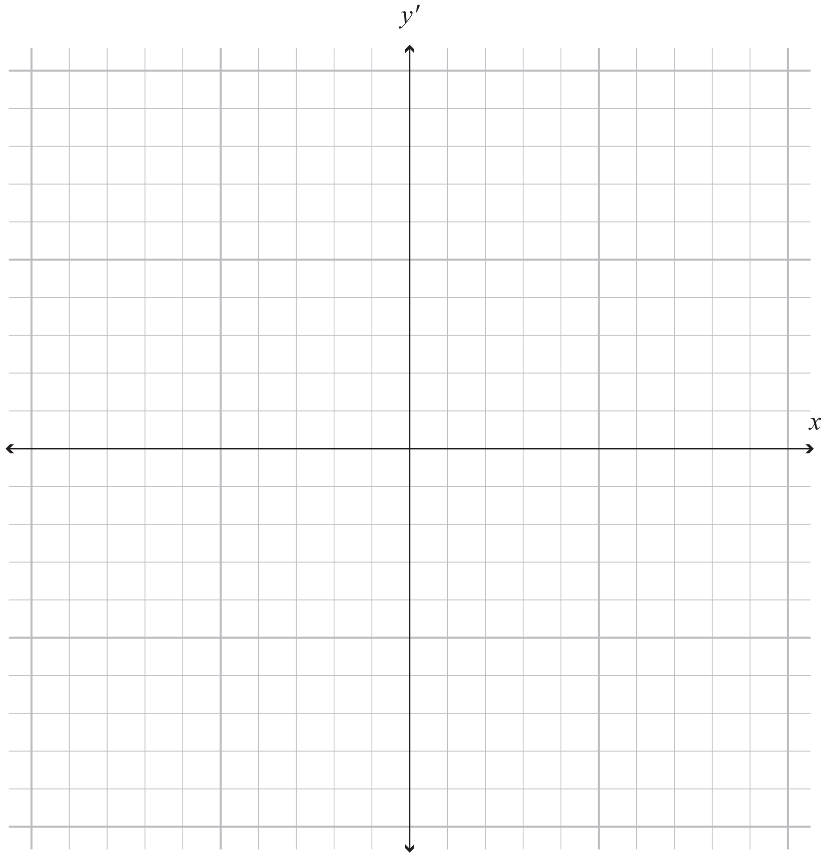
Ki te hiahia koe ki te tuhi anō i tō kauwhata mai i te Tūmahi Tuatoru (d), tuhia ki te tukutuku o raro. Kia mārama te tohu ko tēhea te kauwhata ka hiahia koe kia mākahia.

MĀ TE KAIMĀKA ANAKE



If you need to redraw your graph from Question Three (d), draw it on the grid below. Make sure it is clear which answer you want marked.



He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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English translation of the wording on the front cover

Level 2 Mathematics and Statistics, 2015

91262 Apply calculus methods in solving problems

2.00 p.m. Tuesday 10 November 2015
Credits: Five

91262M

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Resource Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–21 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.