91577M

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

3

SUPERVISOR'S USE ONLY

Tuanaki, Kaupae 3, 2015

91577M Te whakahāngai i te taurangi o ngā tau matatini hei whakaoti rapanga

2.00 i te ahiahi Rāapa 25 Whiringa-ā-rangi 2015
Whiwhinga: Rima

Paetae	Kaiaka	Kairangi
Te whakahāngai i te taurangi o ngā tau matatini hei whakaoti rapanga.	Te whakahāngai i te taurangi o ngā tau matatini mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai i te taurangi o ngā tau matatini mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tuhia ō mahinga KATOA.

Tirohia mēnā kei a koe te pukapuka Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Mēnā ka hiahia whārangi atu anō koe mō ō tuhinga, whakamahia ngā whārangi wātea kei muri o tēnei pukapuka, ka āta tohu ai i te tau tūmahi.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–21 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

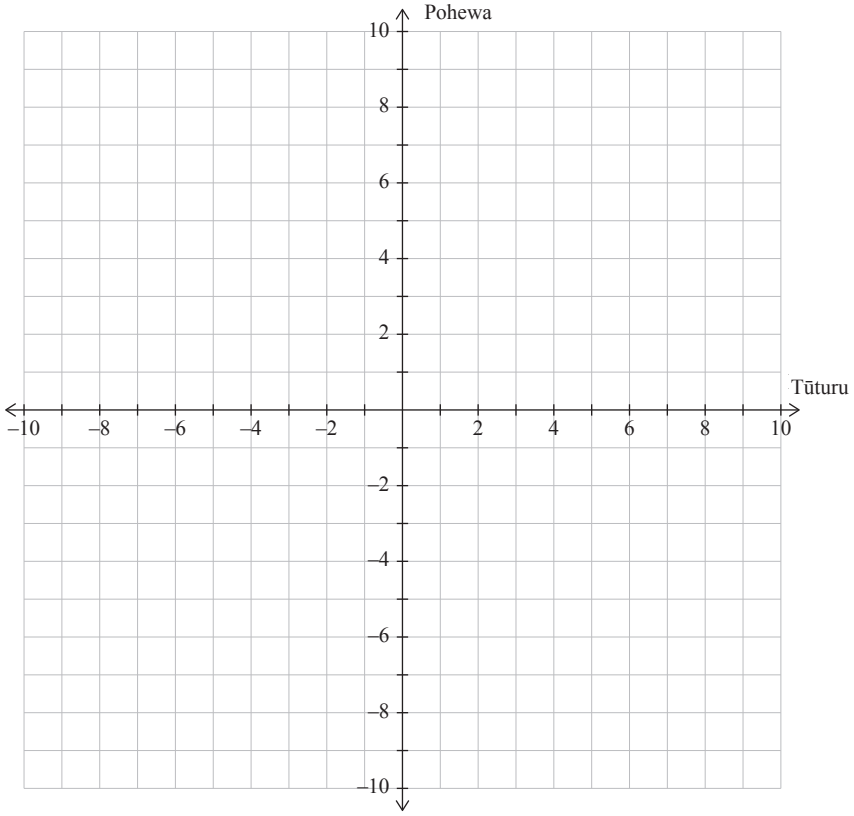
TŪMAHI TUATAHI

MĀ TE KAIMĀKA ANAKE

(a) Whakaotihia te whārite $x^2 - 8x + 4 = 0$.

Tuhia tō whakautu ki te āhua $a \pm b\sqrt{c}$, ina ko a, b me c he tau tōpū, ā, ko $b \neq 1$.

(b) Mēnā $u = 1 + \sqrt{3}\text{i}$, āta whakaaturia a u^3 ki te hoahoa Argand i raro.



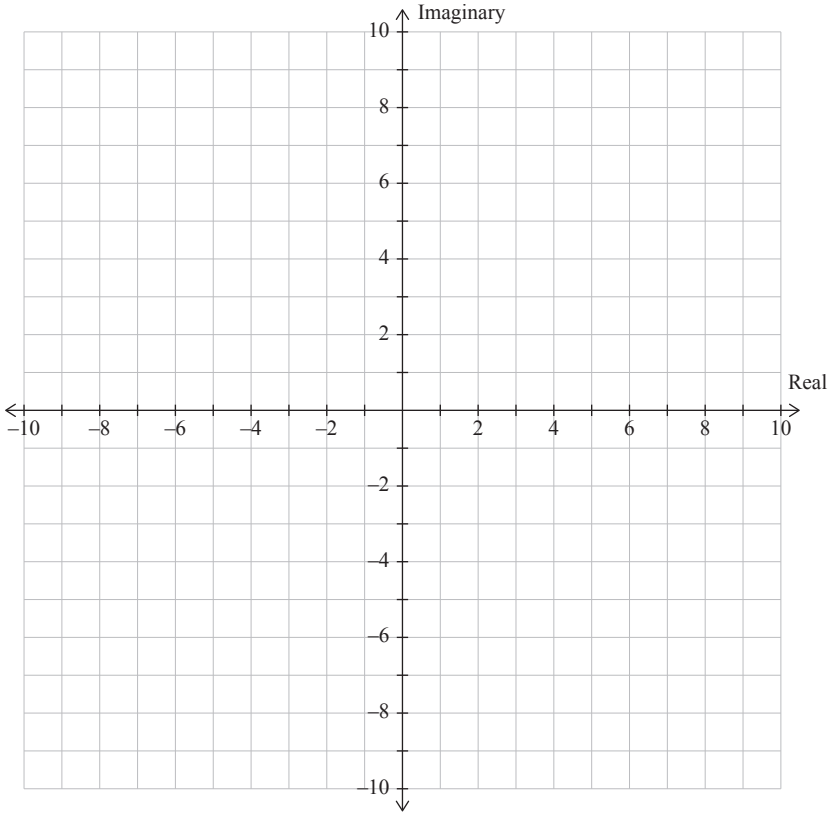
QUESTION ONE

ASSESSOR'S USE ONLY

(a) Solve the equation $x^2 - 8x + 4 = 0$.

Write your answer in the form $a \pm b\sqrt{c}$, where a, b, and c are integers and $b \neq 1$.

(b) If $u = 1 + \sqrt{3}\mathrm{i}$, clearly show u^3 on the Argand diagram below.



(c) Ko v te tau matatini $3 - 7i$
Ko w te tau matatini $-4 + 6i$.

Kimihia ngā tau tūturu p me q kia puta ko $pv + qw = 6.5 - 11i$.

MĀ TE KAIMĀKA ANAKE

(d) Hāponotia ko ngā pūtake o te whārite $3x^2 + (2c + 1)x - (c + 3) = 0$ he tūturu i ngā wā katoa mō ngā uara katoa o c, ina ko c he tūturu.

ASSESSOR'S USE ONLY

(c) v is the complex number $3 - 7i$
w is the complex number $-4 + 6i$.

Find the real numbers p and q such that $pv + qw = 6.5 - 11i$.

(d) Prove that the roots of the equation $3x^2 + (2c + 1)x - (c + 3) = 0$ are always real for all values of c, where c is real.

(e) Mēnā kei $x^2 + bx + c$ me $x^2 + dx + e$ he tauwehe pātahi o $(x - p)$,

hāponotia ko $\frac{e-c}{b-d} = p$, ina ko *b, c, d, e,* me *p* he tūturu katoa.

MĀ TE KAIMĀKA ANAKE

(e) If $x^2 + bx + c$ and $x^2 + dx + e$ have a common factor of $(x - p)$,

prove that $\frac{e-c}{b-d} = p$, where $b, c, d, e,$ and p are all real.

ASSESSOR'S USE ONLY

TŪMAHI TUARUA

MĀ TE KAIMĀKA ANAKE

(a) He aha te toenga ina ko $2x^3 + x^2 - 5x + 7$ ka whakawehea mā $x + 3$?

(b) Ka taea te tau matatini $\frac{2+3\mathrm{i}}{5+\mathrm{i}}$ te kī mā te āhua $k(1 + \mathrm{i})$, ina ko k he tau tūturu.

Kimihia te uara o k.

QUESTION TWO

ASSESSOR'S USE ONLY

(a) What is the remainder when $2x^3 + x^2 - 5x + 7$ is divided by $x + 3$?

(b) The complex number $\dfrac{2+3\mathrm{i}}{5+\mathrm{i}}$ can be expressed in the form $k(1 + \mathrm{i})$, where k is a real number.

Find the value of k.

MĀ TE KAIMĀKA ANAKE

(c) Kimihia ngā tau tūturu A, B me C kia puta ko $\dfrac{1}{x^2(x-1)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{(x-1)}$

(d) Tuhia te tau matatini $\left(\dfrac{4\mathrm{i}^7 - \mathrm{i}}{1+2\mathrm{i}}\right)^2$ ki te āhua $a + b\mathrm{i}$, ina ko a me b he tau tūturu.

ASSESSOR'S USE ONLY

(c) Find real numbers A, B and C such that $\dfrac{1}{x^2(x-1)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{(x-1)}$

(d) Write the complex number $\left(\dfrac{4\mathrm{i}^7 - \mathrm{i}}{1+2\mathrm{i}}\right)^2$ in the form $a + b\mathrm{i}$, where a and b are real numbers.

(e) Kimihia te whārite Cartesian o te huanui e whakaahua ana mā $\arg\left(\frac{z-2}{z+5}\right)=\frac{\pi}{4}$

MĀ TE KAIMĀKA ANAKE

(e) Find the Cartesian equation of the locus described by $\arg\left(\frac{z-2}{z+5}\right)=\frac{\pi}{4}$

ASSESSOR'S USE ONLY

TŪMAHI TUATORU

MĀ TE KAIMĀKA ANAKE

(a) Mēnā ko $z = 4 + 2i$ me $w = -1 + 3i$, kimihia a $\arg(zw)$.

(b) Mō tēhea, ēhea uara tūturu rānei o k he pūtake ōrite tō te whārite $kx^2 + \frac{x}{k} + 2 = 0$?

(c) Ko tētahi otinga kotahi o te whārite $3w^3 + Aw^2 - 3w + 10 = 0$ ko $w = -2$.

Mēnā ko A he tau tūturu, kimihia te uara o te A me ērā atu otinga e rua o te whārite.

QUESTION THREE

ASSESSOR'S USE ONLY

(a) If $z = 4 + 2i$ and $w = -1 + 3i$, find $\arg(zw)$.

(b) For what real value(s) of k does the equation $kx^2 + \frac{x}{k} + 2 = 0$ have equal roots?

(c) One solution of the equation $3w^3 + Aw^2 - 3w + 10 = 0$ is $w = -2$.

If A is a real number, find the value of A and the other two solutions of the equation.

(d) Whakaotihia te whārite $z^3 = k + \sqrt{3}\,k\mathrm{i}$, ina he tūturu, he tōrunga hoki a *k*.
Tuhia ō otinga ki te āhua pākoki e pā ana ki *k*.

MĀ TE KAIMĀKA ANAKE

Ka haere tonu te Tūmahi Tuatoru i te whārangi 18.

(d) Solve the equation $z^3 = k + \sqrt{3}\, k\mathrm{i}$, where k is real and positive.
Write your solutions in polar form in terms of k.

ASSESSOR'S USE ONLY

Question Three continues on page 19.

(e) (i) Kimihia ia pūtake o te whārite $z^5 - 1 = 0$.

MĀ TE KAIMĀKA ANAKE

(ii) Waiho ko p te pūtake i te wāhanga (i) me te tohenga tōrunga iti rawa.

Me whakaatu ko ngā pūtake i te wāhanga (i) ka taea te tuhi hei $1, p, p^2, p^3, p^4$.

(e) (i) Find each of the roots of the equation $z^5 - 1 = 0$.

ASSESSOR'S USE ONLY

(ii) Let p be the root in part (i) with the smallest positive argument.

Show that the roots in part (i) can be written as $1, p, p^2, p^3, p^4$.

He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

English translation of the wording on the front cover

Level 3 Calculus, 2015

91577M Apply the algebra of complex numbers in solving problems

2.00 p.m. Wednesday 25 November 2015
Credits: Five

91577M

Achievement	Achievement with Merit	Achievement with Excellence
Apply the algebra of complex numbers in solving problems.	Apply the algebra of complex numbers, using relational thinking, in solving problems.	Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–21 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.