91578M

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

3

SUPERVISOR'S USE ONLY

Tuanaki, Kaupae 3, 2016

91578M Te whakahāngai i ngā tikanga pārōnaki hei whakaoti rapanga

9.30 i te ata Rāapa 23 Whiringa-ā-rangi 2016
Whiwhinga: Ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pārōnaki hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pārōnaki mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pārōnaki mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tuhia ō mahinga KATOA.

Tirohia mēnā kei a koe te Pukapuka Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Mēnā ka hiahia whārangi atu anō koe mō ō tuhinga, whakamahia te (ngā) whārangi wātea kei muri o tēnei pukapuka, ka āta tohu ai i te tau tūmahi.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–27 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATAHI

(a) Kimihia te pārōnaki o $y = 1 + x - \frac{1}{x} + \frac{1}{x^2}$.

(b) E tukuna ana te teitei o te tai i tētahi ākau i tēnei rā mā te pānga

$$h(t) = 0.8\sin\left(\frac{4\pi}{25}t + \frac{\pi}{2}\right)$$

ina ko h te teitei ā-mita o te wai, e ai ki te pae moana toharite,
ā, ko t te wā ā-haora i muri i te waenganui pō.

c2kiwi.blogspot.co.nz/2011/01/christchurch-wedding-stroll-on-beach.html

He aha te pāpātanga o te huri o te teitei o te tai i taua ākau i te 9.00 i te ata i tēnei rā?

ASSESSOR'S USE ONLY

QUESTION ONE

(a) Differentiate $y = 1 + x - \frac{1}{x} + \frac{1}{x^2}$.

(b) The height of the tide at a particular beach today is given by the function

$$h(t) = 0.8\sin\left(\frac{4\pi}{25}t + \frac{\pi}{2}\right)$$

where h is the height of water, in metres, relative to the mean sea level and t is the time in hours after midnight.

c2kiwi.blogspot.co.nz/2011/01/christchurch-wedding-stroll-on-beach.html

At what rate was the height of the tide changing at that beach at 9.00 a.m. today?

MĀ TE KAIMĀKA ANAKE

(c) E tautuhia ana tētahi ānau mā ngā whārite tawhā

$x = 2\cos 2t$ me $y = \tan^2 t$.

Whiriwhirihia te rōnaki o te pātapa ki te ānau i te pūwāhi ina ko $t = \frac{\pi}{4}$.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

ASSESSOR'S USE ONLY

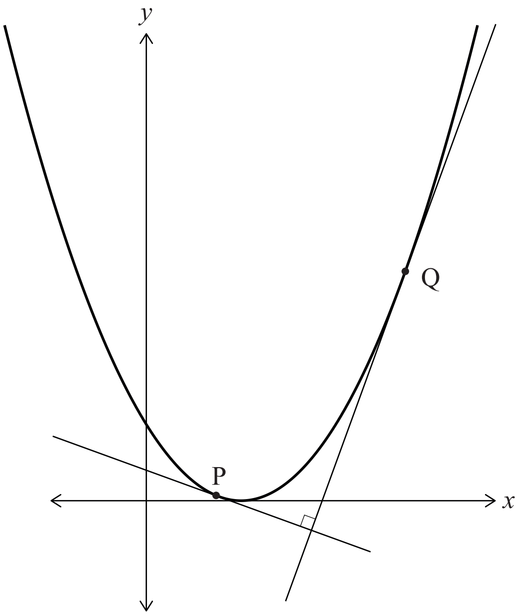
(c) A curve is defined by the parametric equations

$x = 2\cos 2t$ and $y = \tan^2 t$.

Find the gradient of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) He rārangi hāngai ngā pātapa ki te ānau $y = \frac{1}{4}(x-2)^2$ i ngā pūwāhi P me Q.



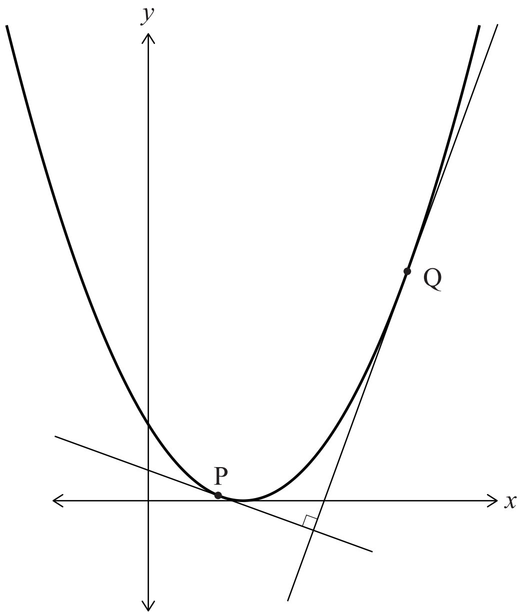
Ko te Q te pūwāhi $(6, 4)$.

He aha te taunga-x o te pūwāhi P?

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

MĀ TE KAIMĀKA ANAKE

(d) The tangents to the curve $y = \frac{1}{4}(x-2)^2$ at points P and Q are perpendicular.



Q is the point $(6, 4)$.

What is the x-coordinate of point P?

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) E tautuhia ana tētahi ānau mā te pānga $f(x) = e^{-(x-k)^2}$.

Kimihia, e ai ki a k, te (ngā) taunga-x mō $f''(x) = 0$

Me whakamahi rawa te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

MĀ TE KAIMĀKA ANAKE

ASSESSOR'S USE ONLY

(e) A curve is defined by the function $f(x) = e^{-(x-k)^2}$.

Find, in terms of k, the x-coordinate(s) for which $f''(x) = 0$.

You must use calculus and show any derivatives that you need to find when solving this problem.

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUARUA

(a) Kimihia te pārōnaki o $f(x) = x\ln(3x - 1)$.

(b) Whiriwhiria te rōnaki o te pātapa ki te pānga $y = \sqrt{2x-1}$ i te pūwāhi $(5, 3)$.

Me whakamahi rawa te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

QUESTION TWO

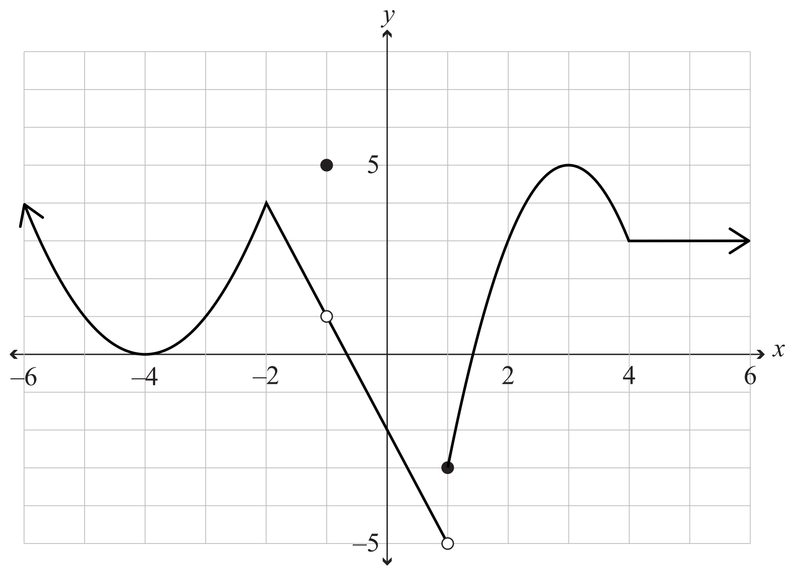
ASSESSOR'S USE ONLY

(a) Differentiate $f(x) = x\ln(3x - 1)$.

(b) Find the gradient of the tangent to the function $y = \sqrt{2x-1}$ at the point $(5, 3)$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) E tohu ana te kauwhata i raro nei i te pānga $y = f(x)$.



Mō te pānga $y = f(x)$ i runga ake:

(i) Kimihia te (ngā) uara mō x e ū ana ki ēnei whakaritenga e whai ake:

1. Kāore e motukore te f:

2. Kāore e tāea te kimi pārōnaki mō f:

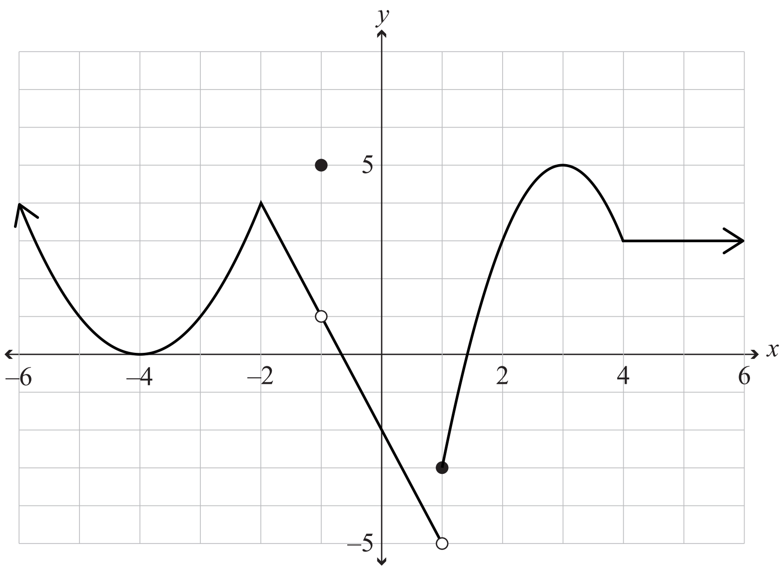
3. $f'(x) = 0$:

4. $f''(x) < 0$:

(ii) He aha te uara o $\lim_{x \to -1} f(x)$?
Āta kōrero mai mēnā kāore rawa he uara mō te tepe.

ASSESSOR'S USE ONLY

(c) The graph below shows the function $y = f(x)$.



For the function $y = f(x)$ above:

(i) Find the value(s) of x that meet the following conditions:

1. f is not continuous:

2. f is not differentiable:

3. $f'(x) = 0$:

4. $f''(x) < 0$:

(ii) What is the value of $\lim_{x \to -1} f(x)$?

State clearly if the value of the limit does not exist.

MĀ TE KAIMĀKA ANAKE

(d) Kei te whakakīngia tētahi poihau porohita nui haumāmā ki te pāpātanga aumou o te $4800\ cm^3\ s^{-1}$.

He aha te pāpātanga e nui haere ana te pūtoro o te poihau ina eke te rōrahi o te poihau ki te $288\,000\pi\ cm^3$?

Me whakamahi rawa te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

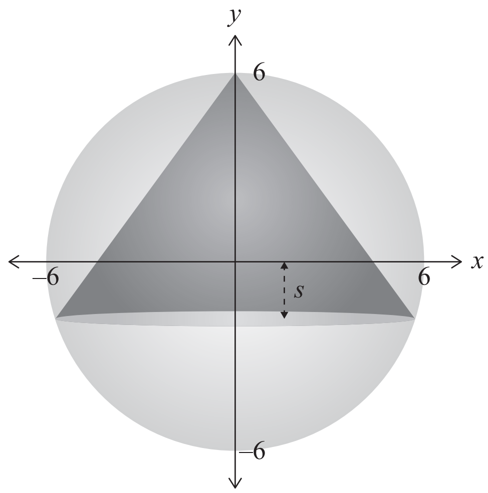
ASSESSOR'S USE ONLY

(d) A large spherical helium balloon is being inflated at a constant rate of $4800 \text{ cm}^3 \text{ s}^{-1}$.

At what rate is the radius of the balloon increasing when the volume of the balloon is $288\,000\pi \text{ cm}^3$?

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) Kua whakairohia tētahi koeke he h te teitei me te pūtoro r ki roto i tētahi poi me te pūtoro o te 6 cm, e ai ki te pikitia.



Ko te kaupapa o te koeke he s cm i raro i te tuaka-x.

Tātaihia te uara o s e whakanui rawahia ai te rōrahi o te koeke.

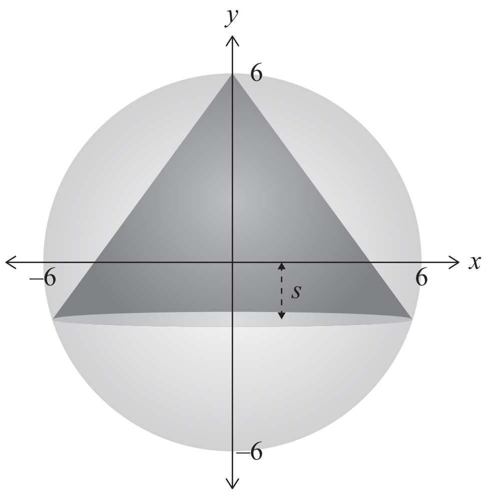
Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

Kāore he tikanga kia hāponotia e koe he mōrahi te rōrahi i tātaihia.

MĀ TE KAIMĀKA ANAKE

(e) A cone of height h and radius r is inscribed, as shown, inside a sphere of radius 6 cm.

ASSESSOR'S USE ONLY



The base of the cone is s cm below the x-axis.

Find the value of s which maximises the volume of the cone.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not need to prove that the volume you have found is a maximum.

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATORU

(a) Kimihia te pārōnaki o $f(x)=\sqrt[4]{3x+2}$.

(b) Kimihia te uara-x e whakarara ana tētahi pātapa ki te ānau $y = 6x - e^{3x}$ ki te tuaka-x.

Me whakamahi rawa te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

QUESTION THREE

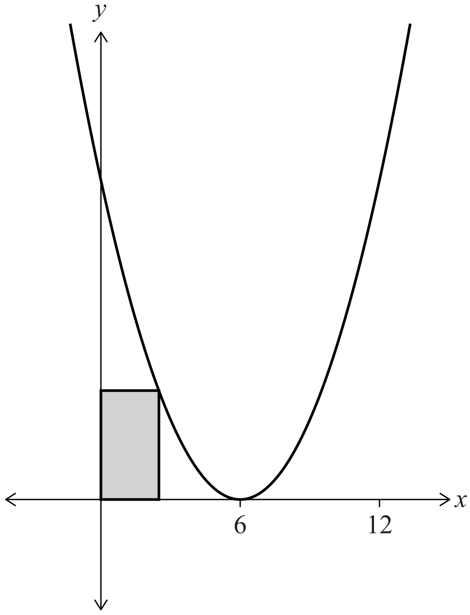
ASSESSOR'S USE ONLY

(a) Differentiate $f(x) = \sqrt[4]{3x+2}$.

(b) Find the x-value at which a tangent to the curve $y = 6x - e^{3x}$ is parallel to the x-axis.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) Kotahi te akitu o tētahi tapawhā hāngai i (0,0) me te akitu kōaro o te ānau $y = (x - 6)^2$, ina ko $0 < x < 6$, e ai ki te kauwhata i raro.



Kimihia te horahanga mōrahi rawa ka taea o te tapawhā hāngai.

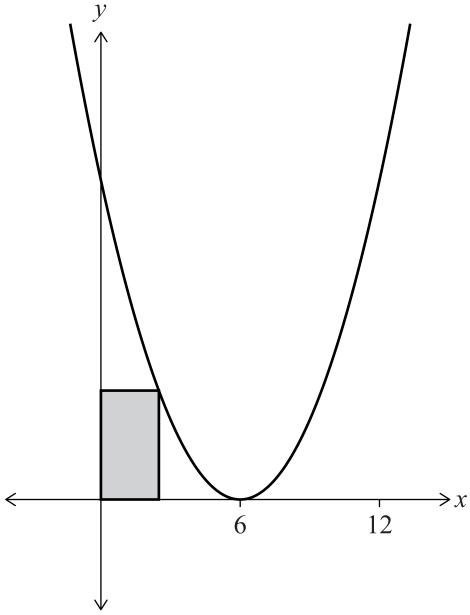
Me whakamahi rawa te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

Kāore he tikanga kia hāponotia e koe he mōrahi te horahanga i tātaihia.

MĀ TE KAIMĀKA ANAKE

ASSESSOR'S USE ONLY

(c) A rectangle has one vertex at (0,0) and the opposite vertex on the curve $y = (x - 6)^2$, where $0 < x < 6$, as shown on the graph below.



Find the maximum possible area of the rectangle.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not need to prove that the area you have found is a maximum.

MĀ TE KAIMĀKA ANAKE

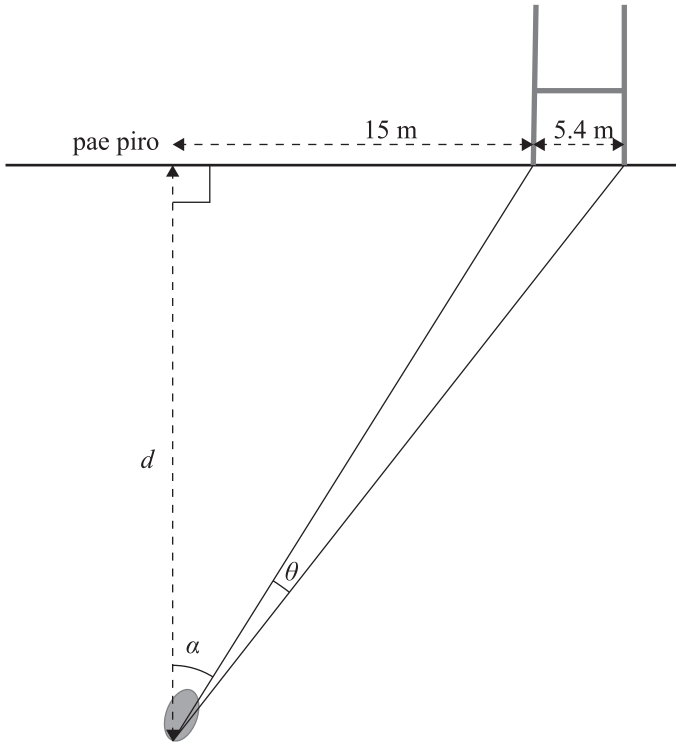
(d) Mēnā ko $y = \frac{e^x}{\sin x}$, me whakaatu ko $\frac{dy}{dx} = y(1 - \cot x)$.

ASSESSOR'S USE ONLY

(d) If $y = \frac{e^x}{\sin x}$, show that $\frac{dy}{dx} = y(1 - \cot x)$.

(e) I tētahi kēmu whutupōro, i tutuki tētahi piro i te 15 m mai i te poutūmārō mauī. Ka kīkia te whana turuki i tētahi pūwāhi i te pae e hāngai tonu ana ki te pae piro mai i te pūwāhi i tutuki te piro, e ai ki te hoahoa i raro.

Me uru te pōro ki waenga o ngā poutūmārō, he 5.4 m te wehe.



Kimihia te tawhiti d mai i te pae piro me kiki te whana turuki hei whakanui ake i te koki θ i waenga i ngā rārangi mai i te pōro ki ngā poutūmārō.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

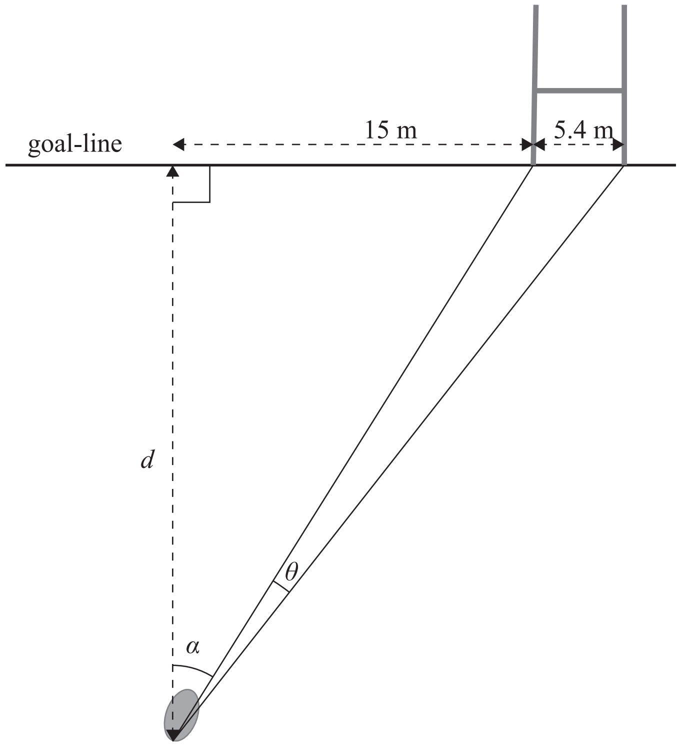
Kāore he tikanga kia hāponotia e koe he mōrahi te koki i tātaihia.

MĀ TE KAIMĀKA ANAKE

ASSESSOR'S USE ONLY

(e) In a rugby game, a try is scored 15 m from the left-hand goal-post. The conversion kick is taken at some point on the line perpendicular to the goal-line from the point where the try was scored, as shown in the diagram below.

The ball needs to pass between the goal-posts, which are 5.4 m apart.



Find the distance d from the goal-line that the conversion kick should be taken from in order to maximise the angle θ between the lines from the ball to the goal-posts.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not need to prove that the angle you have found is a maximum.

He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

English translation of the wording on the front cover

Level 3 Calculus, 2016

91578 Apply differentiation methods in solving problems

9.30 a.m. Wednesday 23 November 2016
Credits: Six

91578M

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.