91579M

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NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

3

SUPERVISOR'S USE ONLY

Tuanaki, Kaupae 3, 2016

91579M Te whakahāngai i ngā tikanga pāwhaitua hei whakaoti rapanga

9.30 i te ata Rāapa 23 Whiringa-ā-rangi 2016
Whiwhinga: Ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pāwhaitua hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pāwhaitua mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pāwhaitua mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tuhia ō mahinga KATOA.

Tirohia mēnā kei a koe te Pukapuka Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Mēnā ka hiahia whārangi atu anō koe mō ō tuhinga, whakamahia te (ngā) whārangi wātea kei muri o tēnei pukapuka, ka āta tohu ai i te tau tūmahi.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–21 kei roto i tēnei pukapuka, ā, kāore tētahi o aua whārangi i te takoto kau.

ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATAHI

MĀ TE KAIMĀKA ANAKE

Mō ngā wāhanga (a) me (b), whiriwhiria ia pāwhaitua.

Kaua e wareware ki te aumou o te tikanga pāwhaitua.

(a) $\int \frac{2x^4 - x^2}{x^3} \, dx$

(b) $\int \sec(3x)\tan(3x) \, dx$

(c) Mēnā ko $\frac{dy}{dx} = \frac{\cos x}{3y}$ me $y = 1$ ina ko $x = \frac{\pi}{6}$, kimihia te uara o y ina ko $x = \frac{7\pi}{6}$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i tēnei rapanga.

QUESTION ONE

ASSESSOR'S USE ONLY

For parts (a) and (b) find each integral.

Remember the constant of integration.

(a) $\displaystyle\int \frac{2x^4 - x^2}{x^3}\,\mathrm{d}x$

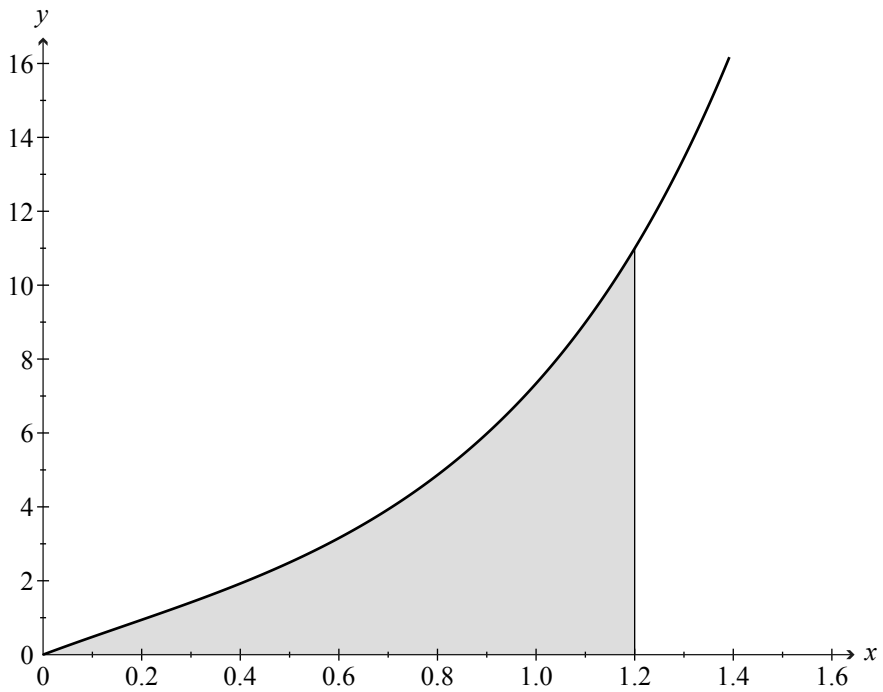
(b) $\displaystyle\int \sec(3x)\tan(3x)\,\mathrm{d}x$

(c) If $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\cos x}{3y}$ and $y = 1$ when $x = \dfrac{\pi}{6}$, find the value of y when $x = \dfrac{7\pi}{6}$.

You must use calculus and give the results of any integration needed to solve this problem.

MĀ TE KAIMĀKA ANAKE

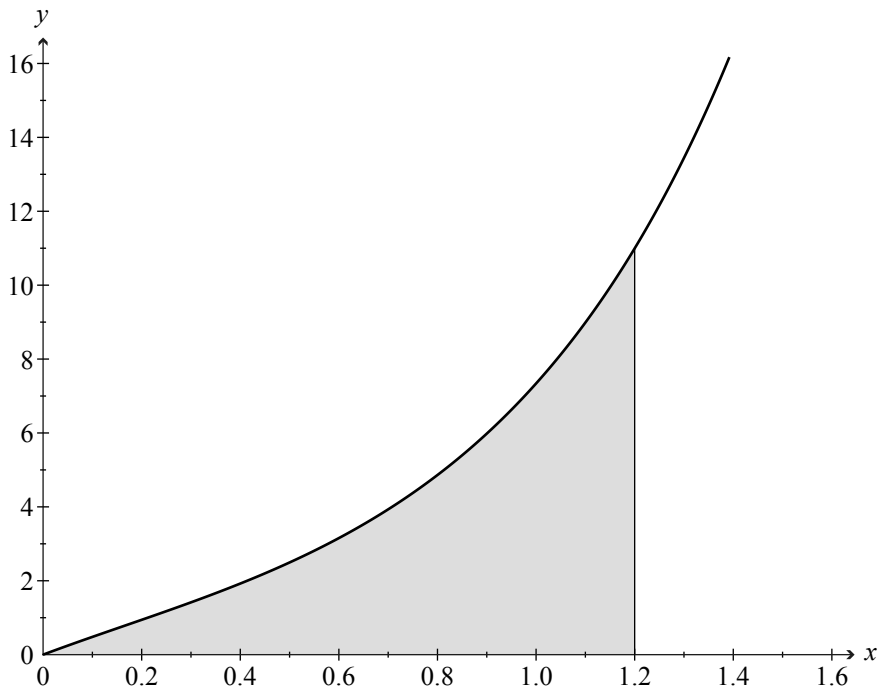
(d) Whakamahia te tikanga pāwhaitua hei tātai i te horahanga e rohea ana e te ānau $y = e^{2x} - \frac{1}{e^{3x}}$ me ngā rārangi $y = 0$, $x = 0$, me $x = 1.2$ (te horahanga kauruku i roto i te hoahoa i raro).



Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i tēnei rapanga.

ASSESSOR'S USE ONLY

(d) Use integration to find the area enclosed between the curve $y = e^{2x} - \frac{1}{e^{3x}}$ and the lines $y = 0$, $x = 0$, and $x = 1.2$ (the area shaded in the diagram below).



You must use calculus and give the results of any integration needed to solve this problem.

(e) He ipu tā Mr Newton, he hinu kei roto, ā, ka waiho e ia ki te karāti. Engari, ka raua e ia te ipu ki runga i tētahi nēra koi kātahi ka tīmata ki te turuturu.

MĀ TE KAIMĀKA ANAKE

Ko te pāpātanga o te heke o te rōrahi o te hinu i roto i te ipu ka tukuna mā te whārite pārōnaki

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -kVt$$

ina ko V te rōrahi o te hinu kei te toe i roto i te ipu i ngā haora e t i muri i te waihotanga o te ipu ki te karāti.

Ko te rōrahi o te hinu i roto i te ipu i te waihotanga ki te karāti he 3000 mL.

I muri i te 20 haora, he 2400 mL te rōrahi o te hinu i roto i te ipu.

E hia te rahi o te hinu, mēnā e toe tonu ana he hinu, kei roto i te ipu i muri i te 96 haora mai i te waihotanga ki te karāti?

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

ASSESSOR'S USE ONLY

(e) Mr Newton has a container of oil and places it in the garage. Unfortunately, he puts the container on top of a sharp nail and it begins to leak.

The rate of decrease of the volume of oil in the container is given by the differential equation

$$\frac{dV}{dt} = -kVt$$

where V is the volume of oil remaining in the container t hours after the container was put in the garage.

The volume of oil in the container when it was placed in the garage was 3000 mL.

After 20 hours, the volume of oil in the container was 2400 mL.

How much, if any, of the oil will remain in the container 96 hours after it was placed in the garage?

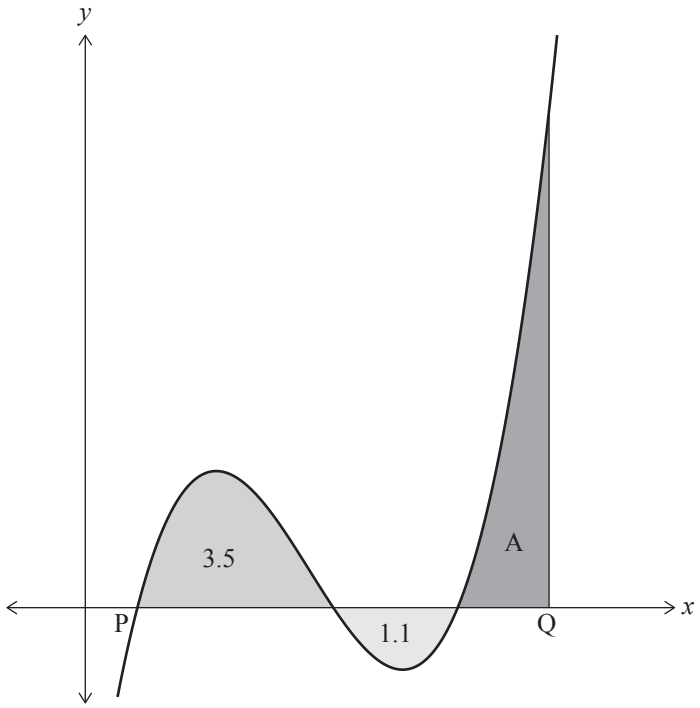
You must use calculus and give the results of any integration needed to solve this problem.

TŪMAHI TUARUA

MĀ TE KAIMĀKA ANAKE

(a) Whiriwhiria $\int \left(5x^2 - 1\right)^2 \mathrm{d}x$.

(b) E whakaatuhia ana te kauwhata o te pānga $y = f(x)$ i raro nei.



Kua tukuna ngā horahanga o ngā wāhi kauruku e rua.

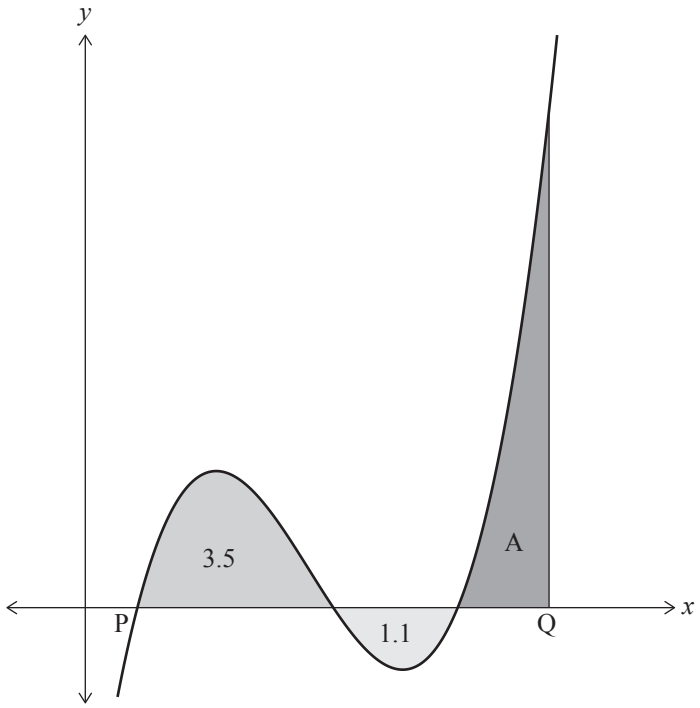
Mēnā ko $\int_{\mathrm{P}}^{\mathrm{Q}} f(x)\,\mathrm{d}x = 9.4$, he aha te horahanga o te wāhi kauruku A?

QUESTION TWO

ASSESSOR'S USE ONLY

(a) Find $\int \left(5x^2 - 1\right)^2 \, dx$.

(b) The graph of a function $y = f(x)$ is shown below.



The areas of two of the shaded regions are given.

If $\int_P^Q f(x)\,dx = 9.4$, what is the area of shaded region A?

MĀ TE KAIMĀKA ANAKE

(c) Ko te whakaterenga o tētahi ahanoa ka tukuna mā $a(t) = 0.2t + 0.3\sqrt{t}$ mō $0 \le t \le 10$.

ina ko a te whakaterenga o te ahanoa i te m s^{-2}
ina ko t te wā, ā-hēkona, mai i te nekehanga o te ahanoa.

I te neke te ahanoa me te tere o te 5 m s^{-1} ina ko $t = 4$.

E hia te tawhiti o te ahanoa mai i tōna wāhi tīmatanga i muri i te 9 hēkona?

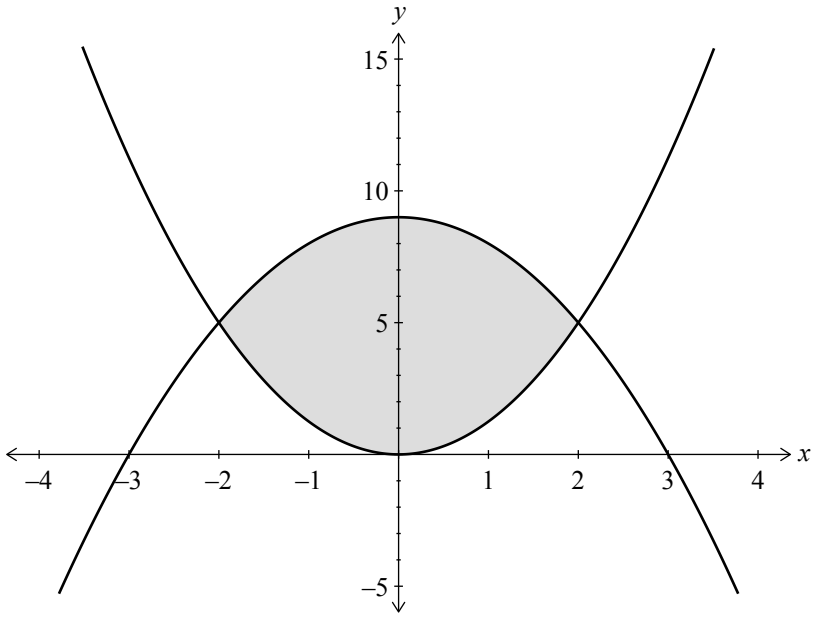
Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

(d) Kimihia te uara o te aumou m ina ko $\int_{m}^{2m} (2x - m)^2 \, dx = 117$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

MĀ TE KAIMĀKA ANAKE

(e) E whakaaturia ana ngā kauwhata o $y = (k - 1)x^2$, $k > 1$ me $y = 9 - x^2$ i te hoahoa i raro.



Ko te wāhi kauruku he 24 te horahanga.

Whiriwhiria te uara o k.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

ASSESSOR'S USE ONLY

(c) The acceleration of an object is given by $a(t) = 0.2t + 0.3\sqrt{t}$ for $0 \le t \le 10$.

where a is the acceleration of the object in m s^{-2}
and t is the time in seconds from when the object started to move.

The object was moving with a velocity of 5 m s^{-1} when $t = 4$.

How far was the object from its starting point after 9 seconds?

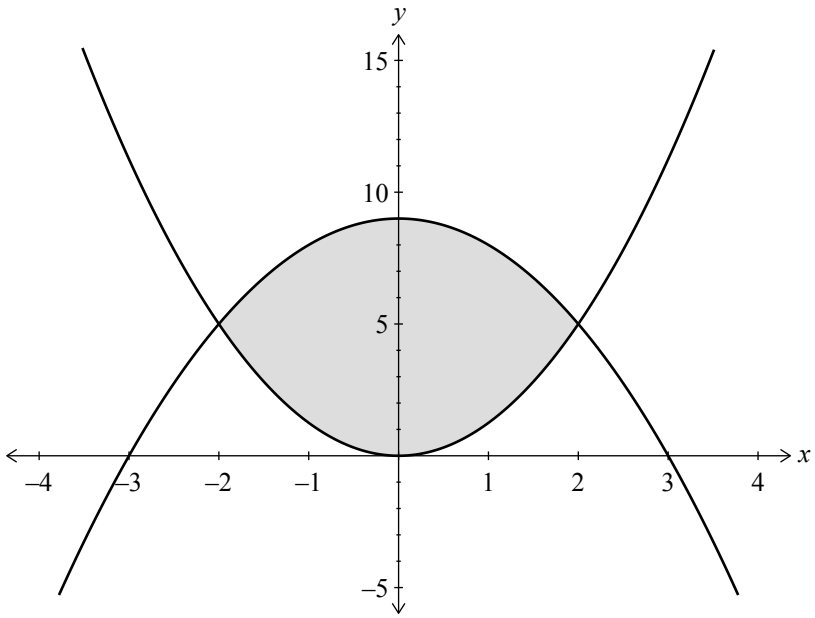
You must use calculus and give the results of any integration needed to solve this problem.

(d) Find the value of the constant m such that $\int_{m}^{2m} (2x - m)^2 \, dx = 117$.

You must use calculus and give the results of any integration needed to solve this problem.

ASSESSOR'S USE ONLY

(e) The graphs of $y = (k - 1)x^2$, $k > 1$ and $y = 9 - x^2$ are shown in the diagram below.



The shaded region has an area of 24.

Find the value of k.

You must use calculus and give the results of any integration needed to solve this problem.

TŪMAHI TUATORU

MĀ TE KAIMĀKA ANAKE

(a) Kimihia te uara o k ina ko $\int_1^4 \left(4 + \frac{k}{x^2}\right) \mathrm{d}x = 0$.

(b) Whakamahia ngā uara i raro hei kimi i tētahi āwhiwhitanga ki $\int_1^4 f(x)\mathrm{d}x$, mā te whakamahi i te Ture a Simpson.

x	1	1.5	2	2.5	3	3.5	4
$f(x)$	1.4	2	3	3.8	2.8	2.2	1.8

QUESTION THREE

ASSESSOR'S USE ONLY

(a) Find the value of k if $\int_1^4 \left(4 + \frac{k}{x^2}\right) \mathrm{d}x = 0$.

(b) Use the values given below to find an approximation to $\int_1^4 f(x)\,\mathrm{d}x$, using Simpson's rule.

x	1	1.5	2	2.5	3	3.5	4
$f(x)$	1.4	2	3	3.8	2.8	2.2	1.8

(c) Whakamahia te tikanga pāwhaitua hei tātai i te horahanga e rohea ana e ngā kauwhata o ngā pānga $y = 2 - x^2$ me $y = -x$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

MĀ TE KAIMĀKA ANAKE

(d) Whiriwhiria $\int \left(\frac{e^{3x} - x^2}{e^{3x} - x^3} \right) dx$.

Ka haere tonu te Tūmahi Tuatoru i te whārangi 18.

ASSESSOR'S USE ONLY

(c) Use integration to find the area enclosed between the graphs of the functions $y = 2 - x^2$ and $y = -x$.

You must use calculus and give the results of any integration needed to solve this problem.

(d) Find $\int \left(\frac{e^{3x} - x^2}{e^{3x} - x^3} \right) dx$.

Question Three continues on page 19.

(e) Mēnā ko $\sec x \cdot \dfrac{dy}{dx} = e^{y+\sin x}$ me $y = -1$ ina ko $x = 0$, kimihia te uara o y ina ko $x = \dfrac{\pi}{2}$.

MĀ TE KAIMĀKA ANAKE

(e) If $\sec x \cdot \dfrac{dy}{dx} = e^{y+\sin x}$ and $y = -1$ when $x = 0$, find the value of y when $x = \dfrac{\pi}{2}$.

ASSESSOR'S USE ONLY

He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

English translation of the wording on the front cover

Level 3 Calculus, 2016

91579 Apply integration methods in solving problems

9.30 a.m. Wednesday 23 November 2016
Credits: Six

91579M

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–21 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.