3

91578M

915785

SUPERVISOR'S USE ONLY

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Tuanaki, Kaupae 3, 2017

91578M Te whakahāngai i ngā tikanga pārōnaki hei whakaoti rapanga

9.30 i te ata Rāpare 23 Whiringa-ā-rangi 2017
Whiwhinga: Ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pārōnaki hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pārōnaki mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pārōnaki mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tuhia ō mahinga KATOA.

Tirohia mēnā kei a koe te pukapuka Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Mēnā ka hiahia whārangi atu anō mō ō tuhinga, whakamahia ngā whārangi wātea kei muri o tēnei pukapuka, ka āta tohu ai i ngā tau tūmahi.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–27 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATAHI

(a) Kimi pāronaki mō $y = \sqrt{x} + \tan(2x)$.

(b) Whiriwhiria te rōnaki o te pātapa ki te ānau $y = \dfrac{e^{2x}}{x+2}$ ki te pūwāhi ina ko $x = 0$.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki i rapua e koe ina whakaoti i tēnei rapanga.

ASSESSOR'S USE ONLY

QUESTION ONE

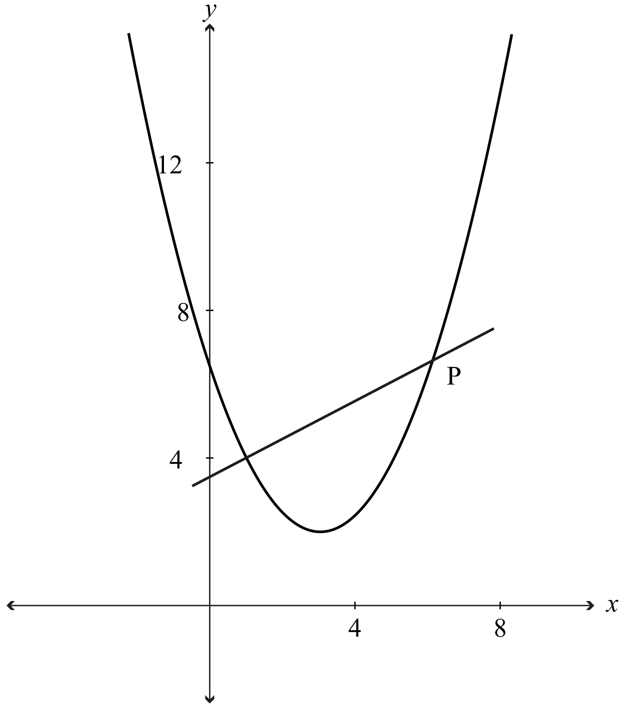
(a) Differentiate $y = \sqrt{x} + \tan(2x)$.

(b) Find the gradient of the tangent to the curve $y = \dfrac{e^{2x}}{x+2}$ at the point where $x = 0$.

You must use calculus and show any derivatives that you need to find when solving this problem.

MĀ TE KAIMĀKA ANAKE

(c) Ko te rārangi hāngai ki te unahi $y = 0.5(x - 3)^2 + 2$ i te pūwāhi (1,4) ka pūtahi anō ki te unahi i te pūwāhi P.

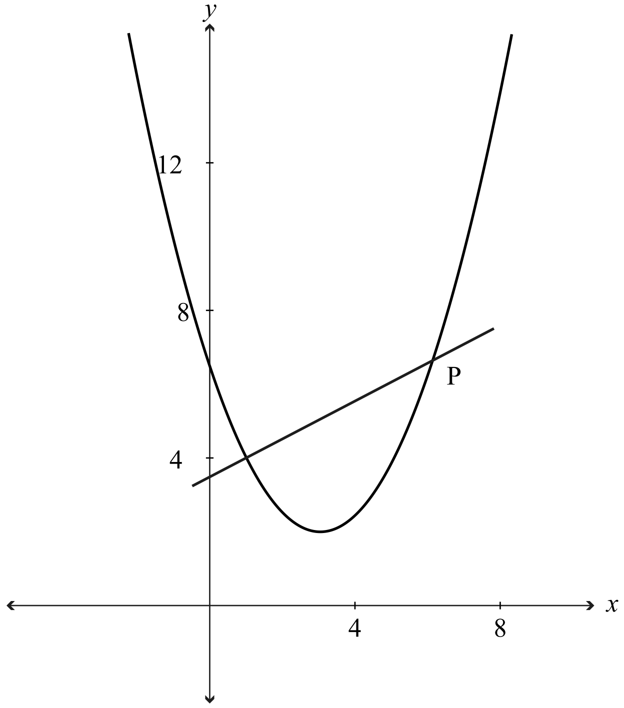


Whiriwhiria te taunga-x o te pūwāhi P.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki i rapua e koe ina whakaoti i tēnei rapanga.

ASSESSOR'S USE ONLY

(c) The normal to the parabola $y = 0.5(x-3)^2 + 2$ at the point (1,4) intersects the parabola again at the point P.



Find the x-coordinate of point P.

You must use calculus and show any derivatives that you need to find when solving this problem.

MĀ TE KAIMĀKA ANAKE

(d) E tautuhia tawhātia ana tētahi ānau mā ngā whārite $x = \sqrt{t+1}$ me te $y = \sin 2t$.

Whiriwhiria te rōnaki o te pātapa ki te ānau i te pūwāhi ina ko $t = 0$.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki i rapua e koe ina whakaoti i tēnei rapanga.

(d) A curve is defined parametrically by the equations $x = \sqrt{t+1}$ and $y = \sin 2t$.

Find the gradient of the tangent to the curve at the point when $t = 0$.

You must use calculus and show any derivatives that you need to find when solving this problem.

ASSESSOR'S USE ONLY

(e) Whiriwhiria ngā uara o *a* me *b*, ina he pūwāhi huringa o te ānau $y = \dfrac{ax - b}{x^2 - 1}$ i (3,1).

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki i rapua e koe ina whakaoti i tēnei rapanga.

MĀ TE KAIMĀKA ANAKE

(e) Find the values of a and b such that the curve $y = \dfrac{ax - b}{x^2 - 1}$ has a turning point at $(3,1)$.

You must use calculus and show any derivatives that you need to find when solving this problem.

ASSESSOR'S USE ONLY

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUARUA

(a) Kimi pāronaki mō $y = 2(x^2 - 4x)^5$.

Hei aha noa te whakarūnā i tō tuhinga.

(b) Ka whakaritea te ōrau o ngā kākano e tinaku ana mā te rahinga o te wai ka waiwaihia atu ki te pārekereke e whakatipuhia ana ngā kākano, ā, ka taea te whakatauira mā te pānga:

$$P(w) = 96\ln(w + 1.25) - 16w - 12$$

ina ko P te ōrau o ngā kākano ka tinaku, ā,
ko w te rahinga wai i te rā ka waiwaihia atu (ngā rita i ia mita pūrua o te pārekereke),
ā, ko $0 \leq w \leq 15$.

Whiriwhiria te rahinga wai tika hei waiwai atu i ia rā kia tino eke rawa atu te ōrau o ngā kākano e tinaku ana.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki i rapua e koe ina whakaoti i tēnei rapanga.

QUESTION TWO

ASSESSOR'S USE ONLY

(a) Differentiate $y = 2(x^2 - 4x)^5$.

You do not need to simplify your answer.

(b) The percentage of seeds germinating depends on the amount of water applied to the seedbed that the seeds are sown in, and may be modelled by the function:

$$P(w) = 96\ln(w + 1.25) - 16w - 12$$

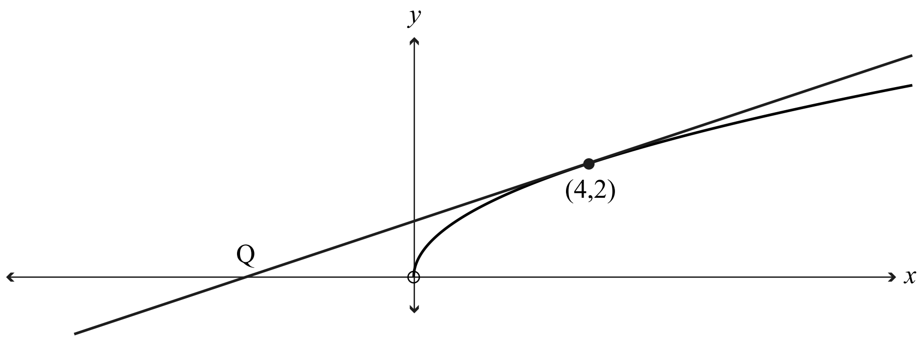
where P is the percentage of seeds that germinate and
w is the daily amount of water applied (litres per square metre of seedbed), with $0 \leq w \leq 15$.

Find the amount of water that should be applied daily to maximise the percentage of seeds germinating.

You must use calculus and show any derivatives that you need to find when solving this problem.

MĀ TE KAIMĀKA ANAKE

(c) Ka tātuhia te pātapa ki te ānau $y = \sqrt{x}$ ki te pūwāhi (4,2).

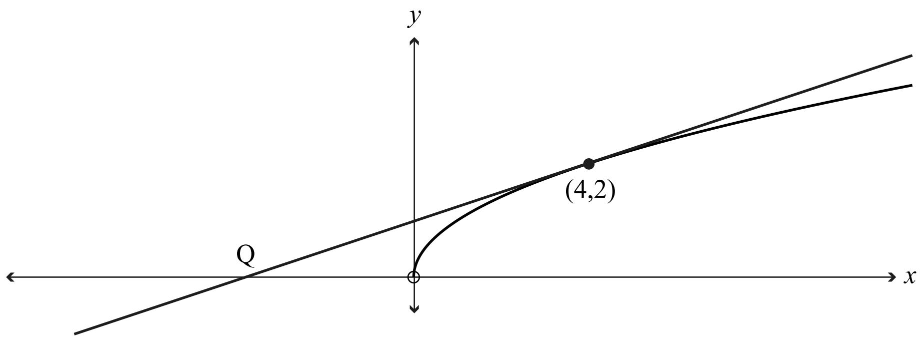


Whiriwhiria ngā taunga o te pūwāhi Q e pūtahi ai te pātapa ki te tuaka *x*.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki i rapua e koe ina whakaoti i tēnei rapanga.

ASSESSOR'S USE ONLY

(c) The tangent to the curve $y = \sqrt{x}$ is drawn at the point (4,2).

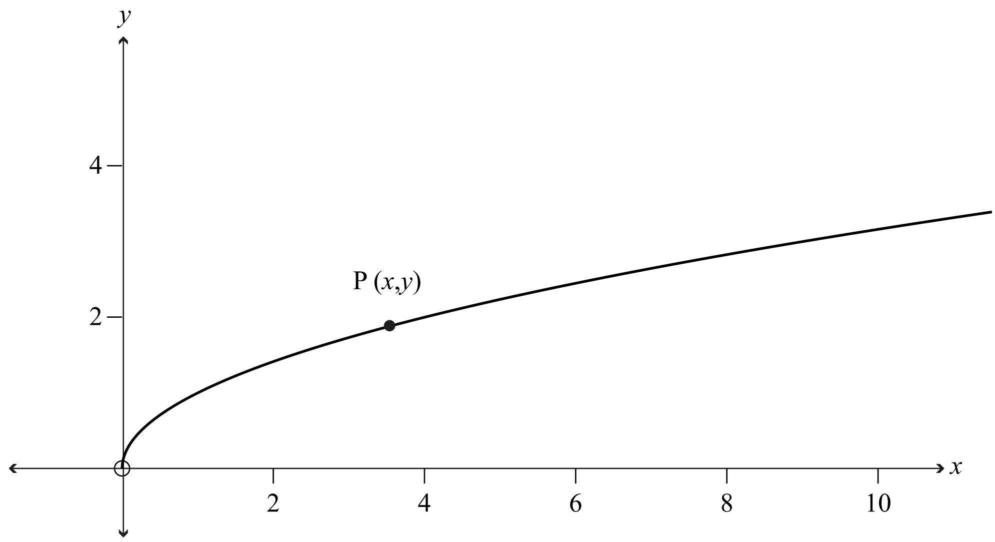


Find the co-ordinates of the point Q where the tangent intersects the x-axis.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) Whiriwhiria ngā taunga o te pūwāhi P (x,y) kei te ānau $y=\sqrt{x}$ e tūtata ana ki te pūwāhi (4,0).

MĀ TE KAIMĀKA ANAKE

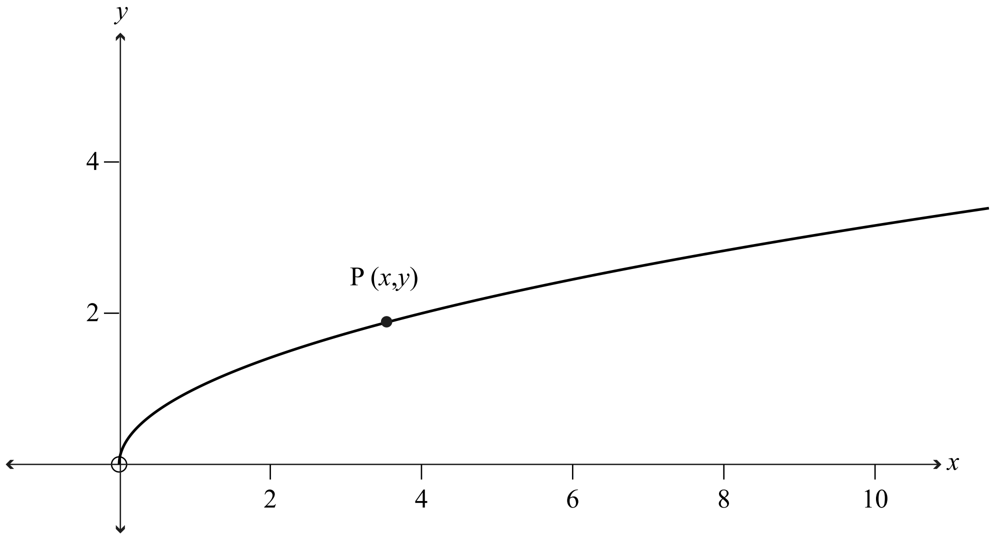


Ehara i te mea me hāpono ko tō otinga te uara mōkito.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki i rapua e koe ina whakaoti i tēnei rapanga.

ASSESSOR'S USE ONLY

(d) Find the coordinates of the point P (x,y) on the curve $y = \sqrt{x}$ that is closest to the point (4,0).

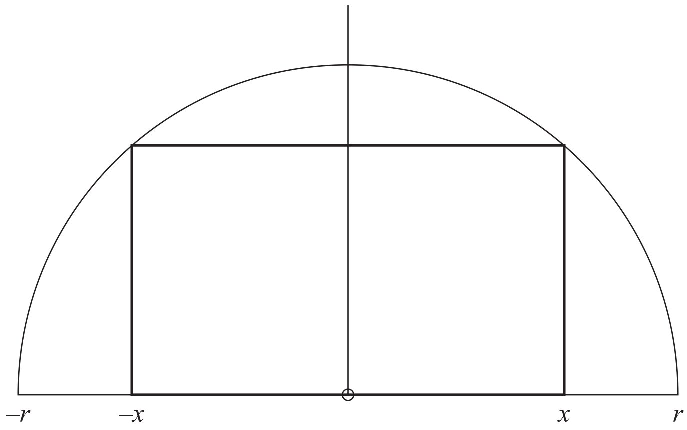


You do not need to prove that your solution is the minimum value.

You must use calculus and show any derivatives that you need to find when solving this problem.

MĀ TE KAIMĀKA ANAKE

(e) Ka tuhia he tapawhā hāngai ki roto i tētahi porowhita haurua o te pūtoro r, e whakaaturia ana i raro.



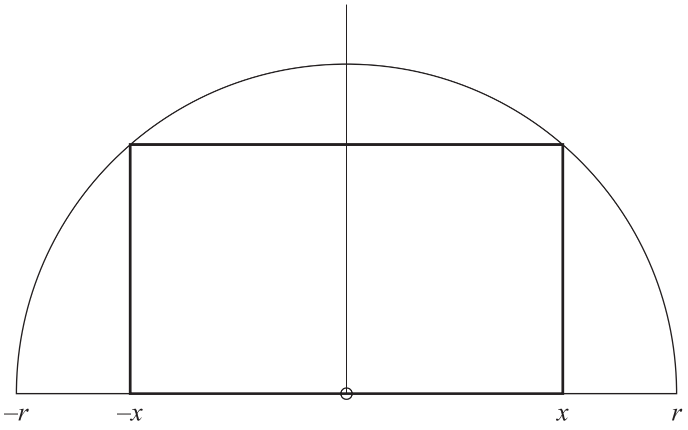
Whakaaturia ka puta te tino horahanga mōrahi i tētahi tapawhā hāngai pēnei ina ko $x = \frac{r}{\sqrt{2}}$.

Ehara i te mea me hāpono koe ko tō otinga te horahanga mōrahi.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki i rapua e koe ina whakaoti i tēnei rapanga.

(e) A rectangle is inscribed in a semi-circle of radius r, as shown below.

ASSESSOR'S USE ONLY



Show that the maximum possible area of such a rectangle occurs when $x = \frac{r}{\sqrt{2}}$.

You do not need to prove that your solution gives the maximum area.

You must use calculus and show any derivatives that you need to find when solving this problem.

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATORU

(a) Kimihia te pāronaki mō $y = x\ln(3x - 1)$.

Hei aha noa te whakarūnā i tō tuhinga.

(b) Whiriwhiria te rōnaki o te ānau $y = \frac{1}{x} - \frac{1}{x^2}$ i te pūwāhi $\left(2, \frac{1}{4}\right)$.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki i rapua e koe ina whakaoti i tēnei rapanga.

ASSESSOR'S USE ONLY

QUESTION THREE

(a) Differentiate $y = x\ln(3x - 1)$.

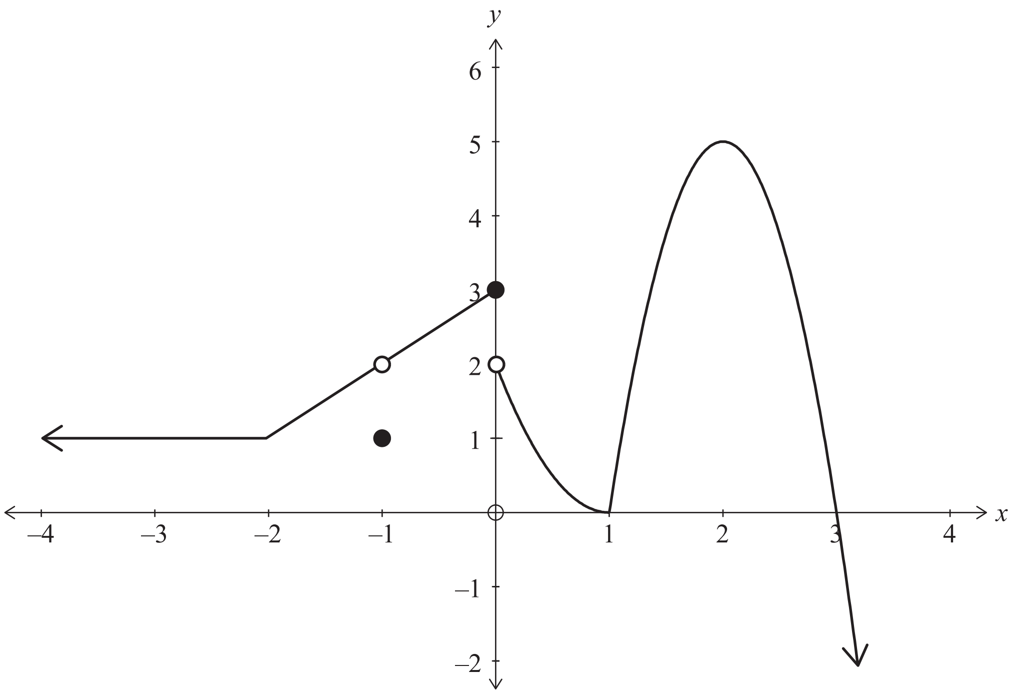
You do not need to simplify your answer.

(b) Find the gradient of the curve $y = \frac{1}{x} - \frac{1}{x^2}$ at the point $\left(2, \frac{1}{4}\right)$.

You must use calculus and show any derivatives that you need to find when solving this problem.

MĀ TE KAIMĀKA ANAKE

(c) E tohu ana te kauwhata i raro nei i te pānga $y = f(x)$.



Mō te pānga i runga ake:

(i) Whiriwhiria te (ngā) uara mō x e ū ana ki ēnei whakaritenga e whai ake:

(1) $f'(x) = 0$: ________

(2) He motukore te $f(x)$ heoi kāore e taea te kimi pārōnaki: ________

(3) Kāore e motukore te $f(x)$: ________

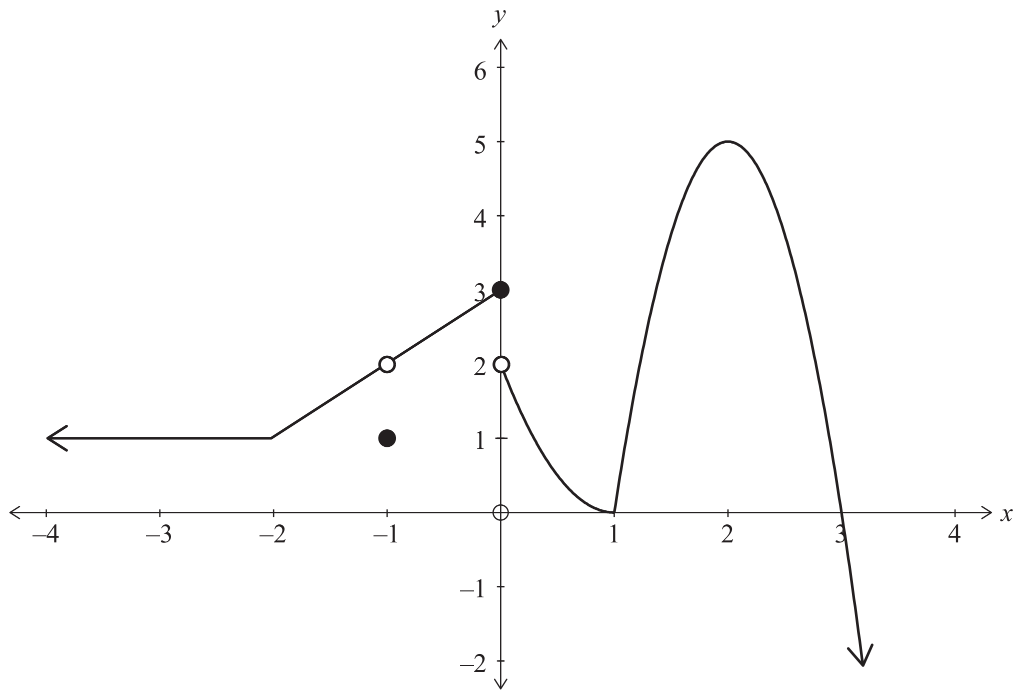
(4) $f''(x) < 0$: ________

(ii) He aha te uara o $\lim\limits_{x \to -1} f(x)$? ________

Āta kōrero mai mēnā kāore rawa he uara.

ASSESSOR'S USE ONLY

(c) The graph below shows the function $y = f(x)$.



For the function above:

(i) Find the value(s) of x that meet the following conditions:

(1) $f'(x) = 0$: ______

(2) $f(x)$ is continuous but not differentiable: ______

(3) $f(x)$ is not continuous: ______

(4) $f''(x) < 0$: ______

(ii) What is the value of $\lim_{x \to -1} f(x)$? ______

State clearly if the value does not exist.

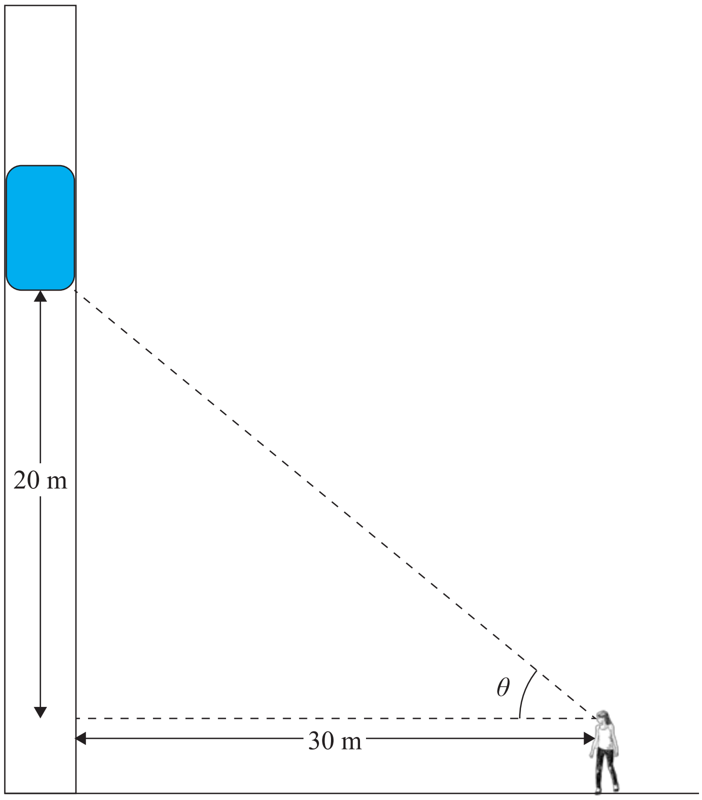
MĀ TE KAIMĀKA ANAKE

(d) He ararewa ā-waho kei tētahi whare. E piki ana te ararewa ki te tere aumou o te 2 m s^{-1}.

Kei te tū noa a Sarah, e mātaki ana i te ararewa i tētahi pūwāhi o te 30 m mai i te pito whakararo o te ararewa.

Me kī ko te koki rewa o te papa ararewa mai i te taumata karu o Sarah he θ.

www.alibaba.com/product-detail/Sicher-external-elevator_60136882005.html



Whiriwhiria te pāpātanga o te piki o te koki rewa ina he 20 m i runga ake te papa ararewa i te taumata karu o Sarah.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki i rapua e koe ina whakaoti i tēnei rapanga.

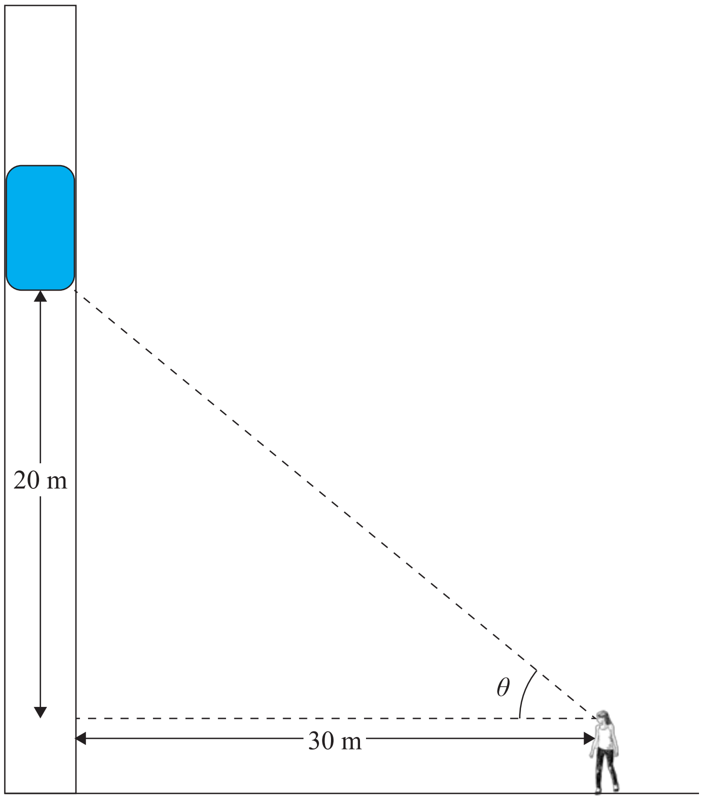
ASSESSOR'S USE ONLY

(d) A building has an external elevator. The elevator is rising at a constant rate of 2 m s^{-1}.

Sarah is stationary, watching the elevator from a point 30 m away from the base of the elevator shaft.

Let the angle of elevation of the elevator floor from Sarah's eye level be θ.

www.alibaba.com/product-detail/Sicher-external-elevator_60136882005.html



Find the rate at which the angle of elevation is increasing when the elevator floor is 20 m above Sarah’s eye level.

You must use calculus and show any derivatives that you need to find when solving this problem.

MĀ TE KAIMĀKA ANAKE

(e) Mō te pānga $y = e^x \cos kx$:

(i) Whiriwhiria $\frac{dy}{dx}$ me $\frac{d^2y}{dx^2}$.

(ii) Whiriwhiria te (ngā) uara katoa o k, kia tika ai te whārite $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$ mō ngā uara katoa o x e pā ana ki te pānga $y = e^x \cos kx$.

(e) For the function $y = e^x \cos kx$:

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(ii) Find all the value(s) of k such that the function $y = e^x \cos kx$ satisfies the equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$ for all values of x.

ASSESSOR'S USE ONLY

He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

English translation of the wording on the front cover

Level 3 Calculus, 2017

91578 Apply differentiation methods in solving problems

9.30 a.m. Thursday 23 November 2017
Credits: Six

91578M

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.