3

SUPERVISOR'S USE ONLY

91578M

915785

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Tuanaki, Kaupae 3, 2018

91578M Te whakahāngai i ngā tikanga pārōnaki hei whakaoti rapanga

9.30 i te ata Rātū 13 Whiringa-ā-rangi 2018
Whiwhinga: Ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pārōnaki hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pārōnaki mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pārōnaki mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tuhia ō mahinga KATOA.

Tirohia mēnā kei a koe te Pukapuka Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Mēnā ka hiahia whārangi atu anō koe mō ō tuhinga, whakamahia te (ngā) whārangi wātea kei muri o tēnei pukapuka, ka āta tohu ai i te tau tūmahi.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–27 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATAHI

MĀ TE KAIMĀKA ANAKE

(a) Kimi pāronaki mō $y = 2x^3 + \frac{5}{\left(x^3+2\right)^3}$

Hei aha noa te whakarūnā i tō tuhinga.

(b) Mēnā $f(x) = 3\cos 3x$, whakaaturia ko $9f(x) + f''(x) = 0$.

(c) Whiriwhiria te rōnaki o te ānau $y = \ln\left|\sin^2 x\right|$ i te pūwāhi ko $x = \frac{\pi}{6}$

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

QUESTION ONE

ASSESSOR'S USE ONLY

(a) Differentiate $y = 2x^3 + \dfrac{5}{\left(x^3+2\right)^3}$

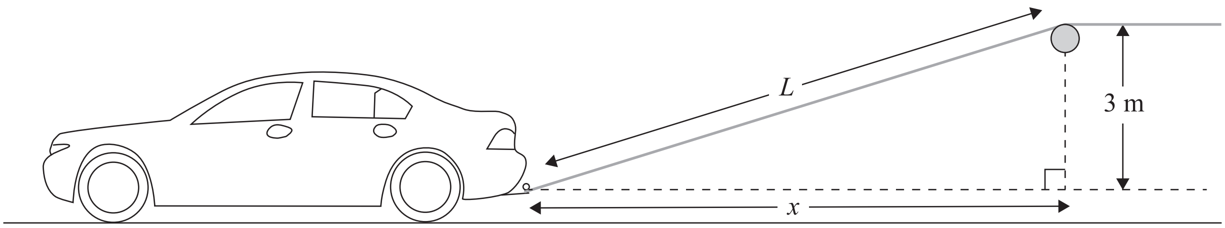
You do not need to simplify your answer.

(b) If $f(x) = 3\cos 3x$, show that $9f(x) + f''(x) = 0$.

(c) Find the gradient of the curve $y = \ln\left|\sin^2 x\right|$ at the point where $x = \dfrac{\pi}{6}$

You must use calculus and show any derivatives that you need to find when solving this problem.

(d)



Kei te tōia tētahi waka mā tētahi taura e here ana ki te pongare i muri i te waka.

Ka haere te taura mā roto i tētahi tauru, ā, ko te wāhanga o runga he 3 m ake te tawhiti mai i te papa tēnā i te pongare.

He x m whakatehuapae te tauru mai i te pongare, e ai ki te hoahoa i runga.

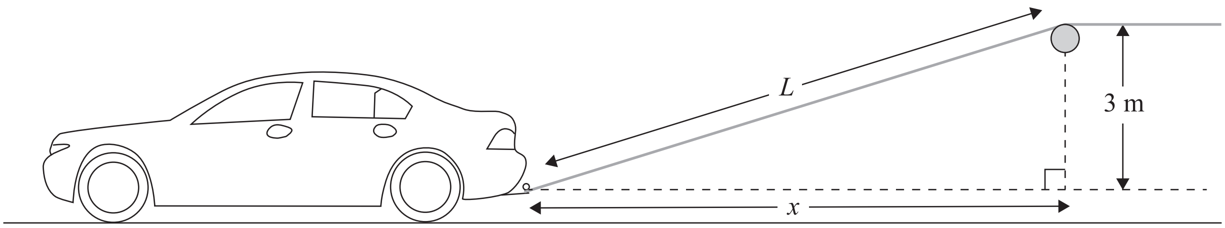
Kei te kumea haerehia te taura ki te tere o te 0.6 m s^{-1}.

Ka pā haere tonu ngā wīra o te waka ki te papa.

He aha te tere o te waka ina ko te roa o te taura, L, i waenga i te pongare me te tauru he 5.4 m?

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

(d)



A car is being pulled along by a rope attached to the tow-bar at the back of the car.

The rope passes through a pulley, the top of which is 3 m further from the ground than the tow-bar.

The pulley is x m horizontally from the tow-bar, as shown in the diagram above.

The rope is being winched in at a speed of 0.6 m s^{-1}.

The wheels of the car remain in contact with the ground.

At what speed is the car moving when the length of the rope, L, between the tow-bar and the pulley is 5.4 m?

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) E tautuhia ana tētahi ānau mā ngā whārite tawhā $x = t^3 + 1$
$y = t^2 + 1$

Whakaaturia mai ko $\dfrac{\dfrac{d^2y}{dx^2}}{\left(\dfrac{dy}{dx}\right)^4}$ he aumou.

MĀ TE KAIMĀKA ANAKE

(e) A curve is defined by the parametric equations $x = t^3 + 1$
$y = t^2 + 1$

Show that $\dfrac{\dfrac{d^2y}{dx^2}}{\left(\dfrac{dy}{dx}\right)^4}$ is a constant.

ASSESSOR'S USE ONLY

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUARUA

(a) Kimihia te pārōnaki o $y = 3\sqrt{x} + \operatorname{cosec} 5x$.

(b) E rere ana tētahi korakora i tētahi rārangi torotika. Ka taea te tawhiti, ā-mita, i rere ai te korakora te whakatauira mā te pānga

$$s(t) = \ln(3t^2 + 3t + 1) \qquad t \geq 0$$

ko t te wā ā-hēkona.

Whiriwhiria te terenga o tēnei korakora i muri i te 2 hēkona.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

ASSESSOR'S USE ONLY

QUESTION TWO

(a) Differentiate $y = 3\sqrt{x} + \operatorname{cosec} 5x$.

(b) A particle is travelling in a straight line. The distance, in metres, travelled by the particle may be modelled by the function

$$s(t) = \ln(3t^2 + 3t + 1) \qquad t \geq 0$$

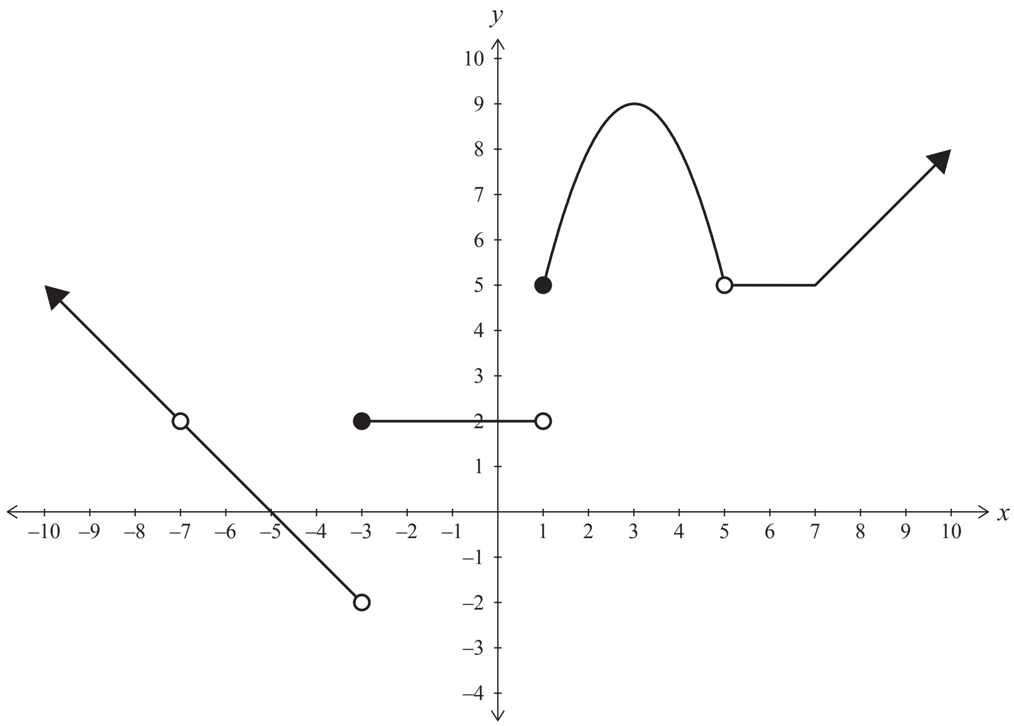
where t is time measured in seconds.

Find the velocity of this particle after 2 seconds.

You must use calculus and show any derivatives that you need to find when solving this problem.

MĀ TE KAIMĀKA ANAKE

(c) E whakaatu ana te hoahoa o raro nei i te kauwhata o te pānga $y = f(x)$.



Mō te pānga i runga ake:

(i) He aha te uara o $f(1)$?

Āta kōrero mai mēnā kāore rawa he uara.

(ii) Mō (t)ēhea uara o x kāore he tepenga o te pānga $f(x)$?

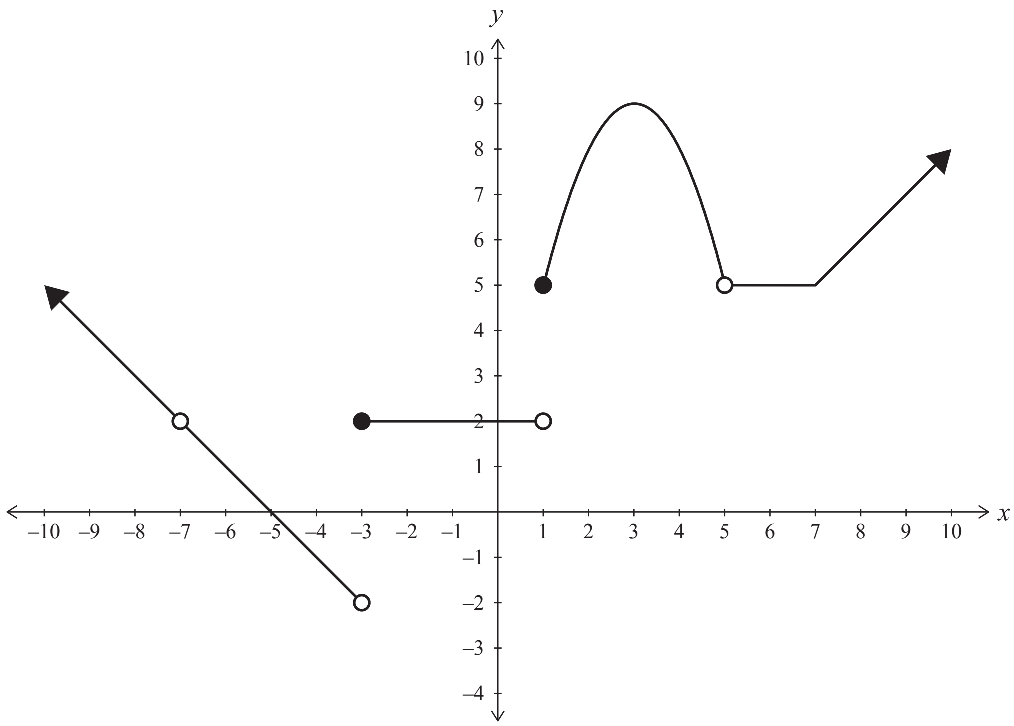
(iii) Whiriwhiria te (ngā) uara katoa o x e ū ana ki ēnei whakaritenga e whai ake:

(1) $f'(x) > 0$:

(2) $f'(x) = 0$, ā, $f''(x) < 0$:

(3) He motukore te $f(x)$ heoi kāore e taea te kimi pārōnaki:

(c) The diagram below shows the graph of the function $y = f(x)$.



For the function above:

(i) What is the value of $f(1)$? ________________

State clearly if the value does not exist.

(ii) For what value(s) of x does the function $f(x)$ not have a limit? ________________

(iii) Find all the value(s) of x that meet the following conditions:

(1) $f'(x) > 0$: ________________

(2) $f'(x) = 0$ and $f''(x) < 0$: ________________

(3) $f(x)$ is continuous but not differentiable: ________________

MĀ TE KAIMĀKA ANAKE

(d) Mēnā $y = e^x(2x^2 - x - 1)$, whiriwhiria te (ngā) uara o x mō $\frac{dy}{dx} = 0$.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

(d) If $y = e^x(2x^2 - x - 1)$, find the value(s) of x for which $\frac{dy}{dx} = 0$.

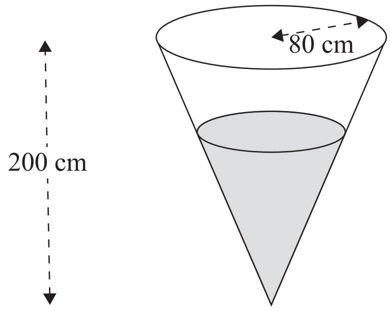
You must use calculus and show any derivatives that you need to find when solving this problem.

ASSESSOR'S USE ONLY

MĀ TE KAIMĀKA ANAKE

(e) Ko te āhua o tētahi taika wai he rite ki tētahi koeko hāngai kōaro.

Ko te teitei o te koeko he 200 cm, ko te pūtoro o te koeko he 80 cm.



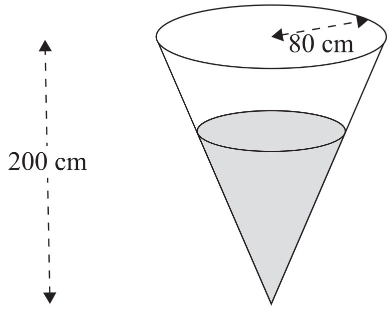
Kei te whakakīa te taika ki te wai ki te pāpātanga o te 150 cm^3 i te hēkona.

He aha te pāpātanga e nui haere ana te horahanga mata o te wai i roto i te taika ina 125 cm te hōhonu o te wai i te taika?

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

(e) A water tank is in the shape of an inverted right-circular cone.

The height of the cone is 200 cm and the radius of the cone is 80 cm.



The tank is being filled with water at a rate of 150 cm^3 per second.

At what rate will the surface area of the water in the tank be increasing when the depth of water in the tank is 125 cm?

You must use calculus and show any derivatives that you need to find when solving this problem.

ASSESSOR'S USE ONLY

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATORU

(a) Kimihia te pārōnaki o $y = \dfrac{e^{2x}}{x^2 + 1}$.

Hei aha noa te whakarūnā i tō tuhinga.

(b) E tautuhia ana tētahi ānau mā ngā whārite tawhā

$x = 5e^{2t}$

$y = 2e^{5t}$

Whiriwhiria te rōnaki o te pātapa ki te ānau i te pūwāhi ina ko $t = 0$.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

QUESTION THREE

ASSESSOR'S USE ONLY

(a) Differentiate $y = \dfrac{e^{2x}}{x^2 + 1}$.

You do not need to simplify your answer.

(b) A curve is defined parametrically by the parametric equations

$x = 5e^{2t}$

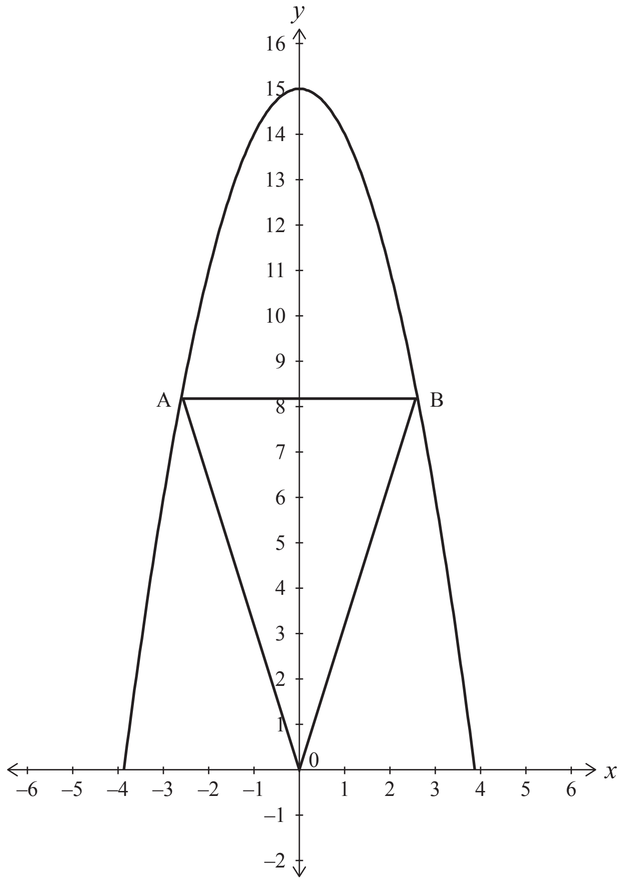
$y = 2e^{5t}$

Find the gradient of the tangent to this curve at the point where $t = 0$.

You must use calculus and show any derivatives that you need to find when solving this problem.

MĀ TE KAIMĀKA ANAKE

(c) E whakaatu ana te hoahoa i raro i te kauwhata o te pānga $y = 15 - x^2$, ā, kua tātuhia tētahi tapatoru waerite OAB i roto.



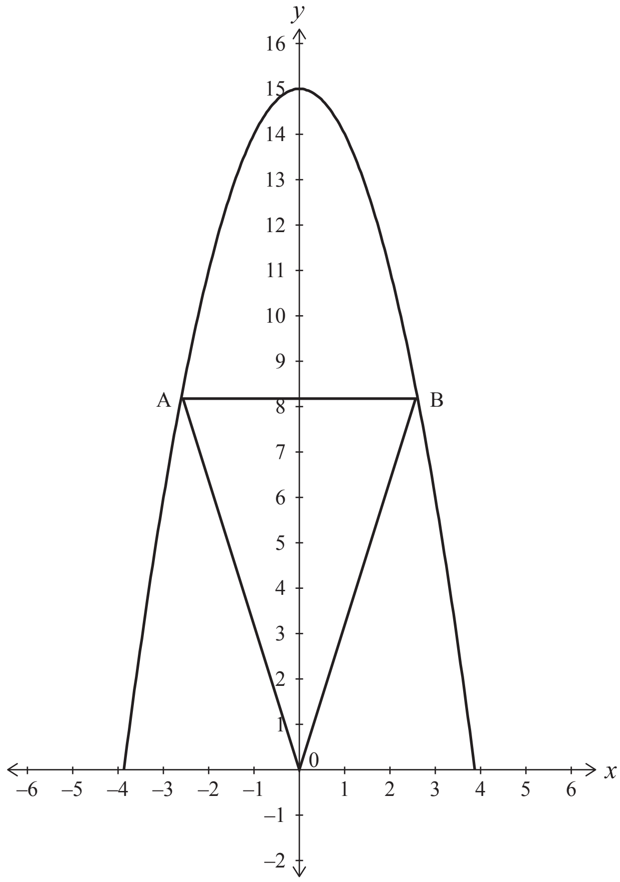
Whiriwhiria te horahanga mōrahi rawa ka taea, A, o te tapatoru.

Me kī he mōrahi tō whakautu.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

(c) The diagram below shows the graph of the function $y = 15 - x^2$, inside which an isosceles triangle OAB has been drawn.

ASSESSOR'S USE ONLY



Find the maximum possible area, A, of the triangle.

You may assume that your answer is a maximum.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) Whiriwhiria te whārite o te pātapa ki te ānau $y = x^2 \ln x$ ki te pūwāhi ina ko $x = \text{e}$.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

MĀ TE KAIMĀKA ANAKE

Ka haere tonu te Tūmahi Tuatoru i te whārangi 22.

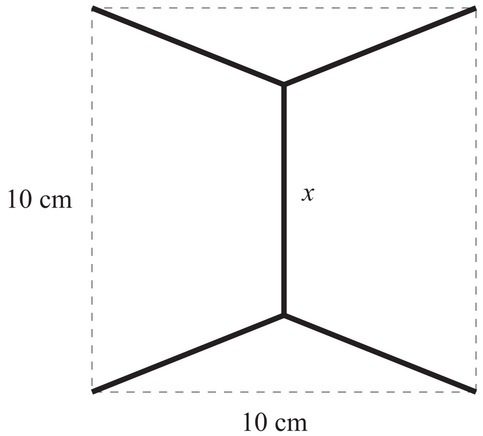
ASSESSOR'S USE ONLY

(d) Find the equation of the tangent to the curve $y = x^2 \ln x$ at the point where $x = \mathrm{e}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

Question Three continues on page 23.

(e)



He mea waihanga te āhua i runga mai i te waea. Kua whai rārangi hangarite poutū me te huapae.

Kei ngā akitu o tētahi tapawhā rite ngā pito o te āhua me tētahi taha he 10 cm te roa, e ai ki te hoahoa i runga ake.

Ko te roa o te waea e rere ana mā te pokapū o te āhua he x cm.

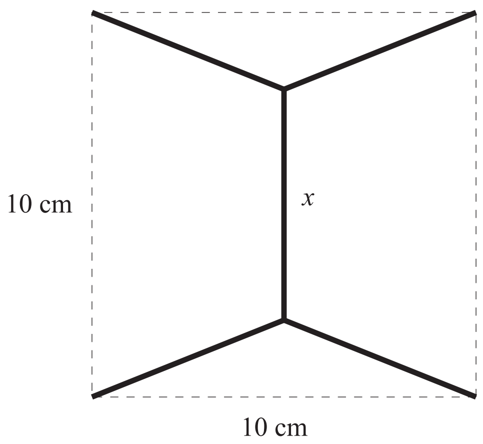
Whiriwhiria te (ngā) uara o x e taea ai te āhua te waihanga mai i te waea tino poto rawa.

Ehara i te mea me hāpono ko te roa he mōkito.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

MĀ TE KAIMĀKA ANAKE

(e)



The above shape is made from wire. It has both vertical and horizontal lines of symmetry.

The ends of the shape are at the vertices of a square with a side length of 10 cm, as shown in the diagram above.

The length of the piece of wire through the centre of the shape is x cm.

Find the value(s) of x that enables the shape to be made with the minimum length of wire.

You do not need to prove that the length is a minimum.

You must use calculus and show any derivatives that you need to find when solving this problem.

ASSESSOR'S USE ONLY

He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

English translation of the wording on the front cover

Level 3 Calculus, 2018

91578 Apply differentiation methods in solving problems

9.30 a.m. Tuesday 13 November 2018
Credits: Six

91578M

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.