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MANA TOHU MĀTAURANGA O AOTEAROA

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KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

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Te Pāngarau me te Tauanga (Tauanga), Kaupae 3, 2018

9.30 i te ata Rāpare 22 Whiringa-ā-rangi 2018

TE PUKAPUKA O NGĀ TIKANGA TĀTAI ME NGĀ TŪTOHI
mō 91584M, 91585M me 91586M

Tirohia tēnei pukapuka hei whakatutuki i ngā tūmahi o ō pukapuka Tūmahi, Tuhiinga hoki.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–8 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

KA TAEA TĒNEI PUKAPUKA TE PUPURI HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TE PĀNGARAU ME TE TAUANGA (TAUANGA) – ĒTAHI TIKANGA TĀTAI ME ĒTAHI TŪTOHI WHAITAKE

Ngā Whiriwhiringa Raupapa me ngā Whiriwhiringa Raupapa Kore

$${}^n P_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^n C_r = \frac{n!}{(n-r)!r!}$$

Taurangi Whakapae

$$E[aX + b] = aE[X] + b$$

$$\text{Var}[aX + b] = a^2 \text{Var}[X]$$

$$E[aX + bY] = aE[X] + bE[Y]$$

$$\text{Var}[aX + bY] = a^2 \text{Var}[X] + b^2 \text{Var}[Y]$$

mēnā he wehe kē te X me te Y

Tūponotanga

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Te Tau Toharite me te Taurangitanga o tētahi Taurangi Matapōkere

$$\begin{aligned} \mu &= E(X) & \sigma^2 &= \text{Var}(X) \\ &= \sum x.P(X=x) & \sigma &= \text{SD}(X) \\ & & &= \sqrt{\sum (x - \mu)^2 . P(X=x)} \\ & & &= \sqrt{E(X^2) - [E(X)]^2} \end{aligned}$$

Tuaritanga Ōrite Motukore

Ko te pānga kiato tūponotanga, $f(x)$, mō tētahi tuaritanga ōrite motukore, ko:

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{ina } a \leq x \leq b \\ 0 & \text{i wāhi kē} \end{cases}$$

MATHEMATICS AND STATISTICS (STATISTICS) – USEFUL FORMULAE AND TABLES

Permutations and Combinations

$${}^n P_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^n C_r = \frac{n!}{(n-r)!r!}$$

Expectation Algebra

$$E[aX + b] = aE[X] + b$$

$$\text{Var}[aX + b] = a^2 \text{Var}[X]$$

$$E[aX + bY] = aE[X] + bE[Y]$$

$$\text{Var}[aX + bY] = a^2 \text{Var}[X] + b^2 \text{Var}[Y]$$

if X, Y are independent

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Mean and Variance of a Discrete Random Variable

$$\begin{aligned} \mu &= E(X) & \sigma^2 &= \text{Var}(X) \\ &= \sum x.P(X = x) & \sigma &= \text{SD}(X) \\ & & &= \sqrt{\sum (x - \mu)^2 . P(X = x)} \\ & & &= \sqrt{E(X^2) - [E(X)]^2} \end{aligned}$$

Continuous Uniform Distribution

The probability density function, $f(x)$, for a continuous uniform distribution is defined as:

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq x \leq b \\ 0 & \text{elsewhere} \end{cases}$$

Tuaritanga Huarua

Ko ia tau e whakaatu ana i te tūponotanga ka noho mai tētahi taurangi matapōkere huarua X , ki te ura x , ko n me π hei tawhā.

$$P(X = x) = \binom{n}{x} \pi^x (1 - \pi)^{n-x}$$

$$\mu = n\pi, \quad \sigma = \sqrt{n\pi(1 - \pi)}$$

| π | 0.05 | 0.1 | 0.15 | 1/6 | 0.2 | 0.25 | 0.3 | 1/3 | 0.35 | 0.4 | 0.45 | 0.5 | |
|-------|------|---------------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 4 | 0 | 0.8145 | 0.6561 | 0.5220 | 0.4823 | 0.4096 | 0.3164 | 0.2401 | 0.1975 | 0.1785 | 0.1296 | 0.0915 | 0.0625 |
| | 1 | 0.1715 | 0.2916 | 0.3685 | 0.3858 | 0.4096 | 0.4219 | 0.4116 | 0.3951 | 0.3845 | 0.3456 | 0.2995 | 0.2500 |
| | 2 | 0.0135 | 0.0486 | 0.0975 | 0.1157 | 0.1536 | 0.2109 | 0.2646 | 0.2963 | 0.3105 | 0.3456 | 0.3675 | 0.3750 |
| | 3 | 0.0005 | 0.0036 | 0.0115 | 0.0154 | 0.0256 | 0.0469 | 0.0756 | 0.0988 | 0.1115 | 0.1536 | 0.2005 | 0.2500 |
| | 4 | | 0.0001 | 0.0005 | 0.0008 | 0.0016 | 0.0039 | 0.0081 | 0.0123 | 0.0150 | 0.0256 | 0.0410 | 0.0625 |
| 5 | 0 | 0.7738 | 0.5905 | 0.4437 | 0.4019 | 0.3277 | 0.2373 | 0.1681 | 0.1317 | 0.1160 | 0.0778 | 0.0503 | 0.0313 |
| | 1 | 0.2036 | 0.3281 | 0.3915 | 0.4019 | 0.4096 | 0.3955 | 0.3602 | 0.3292 | 0.3124 | 0.2592 | 0.2059 | 0.1563 |
| | 2 | 0.0214 | 0.0729 | 0.1382 | 0.1608 | 0.2048 | 0.2637 | 0.3087 | 0.3292 | 0.3364 | 0.3456 | 0.3369 | 0.3125 |
| | 3 | 0.0011 | 0.0081 | 0.0244 | 0.0322 | 0.0512 | 0.0879 | 0.1323 | 0.1646 | 0.1811 | 0.2304 | 0.2757 | 0.3125 |
| | 4 | | 0.0005 | 0.0022 | 0.0032 | 0.0064 | 0.0146 | 0.0284 | 0.0412 | 0.0488 | 0.0768 | 0.1128 | 0.1563 |
| | 5 | | | 0.0001 | 0.0001 | 0.0003 | 0.0010 | 0.0024 | 0.0041 | 0.0053 | 0.0102 | 0.0185 | 0.0313 |
| 6 | 0 | 0.7351 | 0.5314 | 0.3771 | 0.3349 | 0.2621 | 0.1780 | 0.1176 | 0.0878 | 0.0754 | 0.0467 | 0.0277 | 0.0156 |
| | 1 | 0.2321 | 0.3543 | 0.3993 | 0.4019 | 0.3932 | 0.3560 | 0.3025 | 0.2634 | 0.2437 | 0.1866 | 0.1359 | 0.0938 |
| | 2 | 0.0305 | 0.0984 | 0.1762 | 0.2009 | 0.2458 | 0.2966 | 0.3241 | 0.3292 | 0.3280 | 0.3110 | 0.2780 | 0.2344 |
| | 3 | 0.0021 | 0.0146 | 0.0415 | 0.0536 | 0.0819 | 0.1318 | 0.1852 | 0.2195 | 0.2355 | 0.2765 | 0.3032 | 0.3125 |
| | 4 | 0.0001 | 0.0012 | 0.0055 | 0.0080 | 0.0154 | 0.0330 | 0.0595 | 0.0823 | 0.0951 | 0.1382 | 0.1861 | 0.2344 |
| | 5 | | 0.0001 | 0.0004 | 0.0006 | 0.0015 | 0.0044 | 0.0102 | 0.0165 | 0.0205 | 0.0369 | 0.0609 | 0.0938 |
| | 6 | | | | | 0.0001 | 0.0002 | 0.0007 | 0.0014 | 0.0018 | 0.0041 | 0.0083 | 0.0156 |
| 7 | 0 | 0.6983 | 0.4783 | 0.3206 | 0.2791 | 0.2097 | 0.1335 | 0.0824 | 0.0585 | 0.0490 | 0.0280 | 0.0152 | 0.0078 |
| | 1 | 0.2573 | 0.3720 | 0.3960 | 0.3907 | 0.3670 | 0.3115 | 0.2471 | 0.2048 | 0.1848 | 0.1306 | 0.0872 | 0.0547 |
| | 2 | 0.0406 | 0.1240 | 0.2097 | 0.2344 | 0.2753 | 0.3115 | 0.3177 | 0.3073 | 0.2985 | 0.2613 | 0.2140 | 0.1641 |
| | 3 | 0.0036 | 0.0230 | 0.0617 | 0.0781 | 0.1147 | 0.1730 | 0.2269 | 0.2561 | 0.2679 | 0.2903 | 0.2918 | 0.2734 |
| | 4 | 0.0002 | 0.0026 | 0.0109 | 0.0156 | 0.0287 | 0.0577 | 0.0972 | 0.1280 | 0.1442 | 0.1935 | 0.2388 | 0.2734 |
| | 5 | | 0.0002 | 0.0012 | 0.0019 | 0.0043 | 0.0115 | 0.0250 | 0.0384 | 0.0466 | 0.0774 | 0.1172 | 0.1641 |
| | 6 | | | 0.0001 | 0.0001 | 0.0004 | 0.0013 | 0.0036 | 0.0064 | 0.0084 | 0.0172 | 0.0320 | 0.0547 |
| | 7 | | | | | 0.0001 | 0.0002 | 0.0005 | 0.0005 | 0.0006 | 0.0016 | 0.0037 | 0.0078 |
| 8 | 0 | 0.6634 | 0.4305 | 0.2725 | 0.2326 | 0.1678 | 0.1001 | 0.0576 | 0.0390 | 0.0319 | 0.0168 | 0.0084 | 0.0039 |
| | 1 | 0.2793 | 0.3826 | 0.3847 | 0.3721 | 0.3355 | 0.2670 | 0.1977 | 0.1561 | 0.1373 | 0.0896 | 0.0548 | 0.0313 |
| | 2 | 0.0515 | 0.1488 | 0.2376 | 0.2605 | 0.2936 | 0.3115 | 0.2965 | 0.2731 | 0.2587 | 0.2090 | 0.1569 | 0.1094 |
| | 3 | 0.0054 | 0.0331 | 0.0839 | 0.1042 | 0.1468 | 0.2076 | 0.2541 | 0.2731 | 0.2786 | 0.2787 | 0.2568 | 0.2188 |
| | 4 | 0.0004 | 0.0046 | 0.0185 | 0.0260 | 0.0459 | 0.0865 | 0.1361 | 0.1707 | 0.1875 | 0.2322 | 0.2627 | 0.2734 |
| | 5 | | 0.0004 | 0.0026 | 0.0042 | 0.0092 | 0.0231 | 0.0467 | 0.0683 | 0.0808 | 0.1239 | 0.1719 | 0.2188 |
| | 6 | | | 0.0002 | 0.0004 | 0.0011 | 0.0038 | 0.0100 | 0.0171 | 0.0217 | 0.0413 | 0.0703 | 0.1094 |
| | 7 | | | | | 0.0001 | 0.0004 | 0.0012 | 0.0024 | 0.0033 | 0.0079 | 0.0164 | 0.0313 |
| | 8 | | | | | | 0.0001 | 0.0002 | 0.0002 | 0.0002 | 0.0007 | 0.0017 | 0.0039 |
| 9 | 0 | 0.6302 | 0.3874 | 0.2316 | 0.1938 | 0.1342 | 0.0751 | 0.0404 | 0.0260 | 0.0207 | 0.0101 | 0.0046 | 0.0020 |
| | 1 | 0.2985 | 0.3874 | 0.3679 | 0.3489 | 0.3020 | 0.2253 | 0.1556 | 0.1171 | 0.1004 | 0.0605 | 0.0339 | 0.0176 |
| | 2 | 0.0629 | 0.1722 | 0.2597 | 0.2791 | 0.3020 | 0.3003 | 0.2668 | 0.2341 | 0.2162 | 0.1612 | 0.1110 | 0.0703 |
| | 3 | 0.0077 | 0.0446 | 0.1069 | 0.1302 | 0.1762 | 0.2336 | 0.2668 | 0.2731 | 0.2716 | 0.2508 | 0.2119 | 0.1641 |
| | 4 | 0.0006 | 0.0074 | 0.0283 | 0.0391 | 0.0661 | 0.1168 | 0.1715 | 0.2048 | 0.2194 | 0.2508 | 0.2600 | 0.2461 |
| | 5 | | 0.0008 | 0.0050 | 0.0078 | 0.0165 | 0.0389 | 0.0735 | 0.1024 | 0.1181 | 0.1672 | 0.2128 | 0.2461 |
| | 6 | | 0.0001 | 0.0006 | 0.0010 | 0.0028 | 0.0087 | 0.0210 | 0.0341 | 0.0424 | 0.0743 | 0.1160 | 0.1641 |
| | 7 | | | 0.0001 | 0.0001 | 0.0003 | 0.0012 | 0.0039 | 0.0073 | 0.0098 | 0.0212 | 0.0407 | 0.0703 |
| | 8 | | | | | 0.0001 | 0.0004 | 0.0009 | 0.0013 | 0.0035 | 0.0083 | 0.0176 | |
| | 9 | | | | | | 0.0001 | 0.0001 | 0.0001 | 0.0003 | 0.0008 | 0.0020 | |
| 10 | 0 | 0.5987 | 0.3487 | 0.1969 | 0.1615 | 0.1074 | 0.0563 | 0.0282 | 0.0173 | 0.0135 | 0.0060 | 0.0025 | 0.0010 |
| | 1 | 0.3151 | 0.3874 | 0.3474 | 0.3230 | 0.2684 | 0.1877 | 0.1211 | 0.0867 | 0.0725 | 0.0403 | 0.0207 | 0.0098 |
| | 2 | 0.0746 | 0.1937 | 0.2759 | 0.2907 | 0.3020 | 0.2816 | 0.2335 | 0.1951 | 0.1757 | 0.1209 | 0.0763 | 0.0439 |
| | 3 | 0.0105 | 0.0574 | 0.1298 | 0.1550 | 0.2013 | 0.2503 | 0.2668 | 0.2601 | 0.2522 | 0.2150 | 0.1665 | 0.1172 |
| | 4 | 0.0010 | 0.0112 | 0.0401 | 0.0543 | 0.0881 | 0.1460 | 0.2001 | 0.2276 | 0.2377 | 0.2508 | 0.2384 | 0.2051 |
| | 5 | 0.0001 | 0.0015 | 0.0085 | 0.0130 | 0.0264 | 0.0584 | 0.1029 | 0.1366 | 0.1536 | 0.2007 | 0.2340 | 0.2461 |
| | 6 | | 0.0001 | 0.0012 | 0.0022 | 0.0055 | 0.0162 | 0.0368 | 0.0569 | 0.0689 | 0.1115 | 0.1596 | 0.2051 |
| | 7 | | | 0.0001 | 0.0002 | 0.0008 | 0.0031 | 0.0090 | 0.0163 | 0.0212 | 0.0425 | 0.0746 | 0.1172 |
| | 8 | | | | | 0.0001 | 0.0004 | 0.0014 | 0.0030 | 0.0043 | 0.0106 | 0.0229 | 0.0439 |
| | 9 | | | | | | 0.0001 | 0.0003 | 0.0005 | 0.0016 | 0.0042 | 0.0098 | |
| 10 | | (ko ērā atu o ngā tau katoa < 0.0001) | | | | | | | | | | | |
| | | | | | | | | | | 0.0001 | 0.0003 | 0.0010 | |

Tuaritanga Poisson

Ko ia tau e whakaatu ana i te tūponotanga ka noho mai tētahi taurangi matapōkere Poisson X , ki te ura o x , ko te λ hei tawhā.

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$\mu = \lambda, \quad \sigma = \sqrt{\lambda}$$

| x | λ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|-----|-----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0 | | 0.9048 | 0.8187 | 0.7408 | 0.6703 | 0.6065 | 0.5488 | 0.4966 | 0.4493 | 0.4066 | 0.3679 |
| 1 | | 0.0905 | 0.1637 | 0.2222 | 0.2681 | 0.3033 | 0.3293 | 0.3476 | 0.3595 | 0.3659 | 0.3679 |
| 2 | | 0.0045 | 0.0164 | 0.0333 | 0.0536 | 0.0758 | 0.0988 | 0.1217 | 0.1438 | 0.1647 | 0.1839 |
| 3 | | 0.0002 | 0.0011 | 0.0033 | 0.0072 | 0.0126 | 0.0198 | 0.0284 | 0.0383 | 0.0494 | 0.0613 |
| 4 | | | 0.0001 | 0.0003 | 0.0007 | 0.0016 | 0.0030 | 0.0050 | 0.0077 | 0.0111 | 0.0153 |
| 5 | | | | | 0.0001 | 0.0002 | 0.0004 | 0.0007 | 0.0012 | 0.0020 | 0.0031 |
| 6 | | | | | | | | 0.0001 | 0.0002 | 0.0003 | 0.0005 |
| 7 | | | | | | | | | | | 0.0001 |
| x | λ | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2.0 |
| 0 | | 0.3329 | 0.3012 | 0.2725 | 0.2466 | 0.2231 | 0.2019 | 0.1827 | 0.1653 | 0.1496 | 0.1353 |
| 1 | | 0.3662 | 0.3614 | 0.3543 | 0.3452 | 0.3347 | 0.3230 | 0.3106 | 0.2975 | 0.2842 | 0.2707 |
| 2 | | 0.2014 | 0.2169 | 0.2303 | 0.2417 | 0.2510 | 0.2584 | 0.2640 | 0.2678 | 0.2700 | 0.2707 |
| 3 | | 0.0738 | 0.0867 | 0.0998 | 0.1128 | 0.1255 | 0.1378 | 0.1496 | 0.1607 | 0.1710 | 0.1804 |
| 4 | | 0.0203 | 0.0260 | 0.0324 | 0.0395 | 0.0471 | 0.0551 | 0.0636 | 0.0723 | 0.0812 | 0.0902 |
| 5 | | 0.0045 | 0.0062 | 0.0084 | 0.0111 | 0.0141 | 0.0176 | 0.0216 | 0.0260 | 0.0309 | 0.0361 |
| 6 | | 0.0008 | 0.0012 | 0.0018 | 0.0026 | 0.0035 | 0.0047 | 0.0061 | 0.0078 | 0.0098 | 0.0120 |
| 7 | | 0.0001 | 0.0002 | 0.0003 | 0.0005 | 0.0008 | 0.0011 | 0.0015 | 0.0020 | 0.0027 | 0.0034 |
| 8 | | | | 0.0001 | 0.0001 | 0.0001 | 0.0002 | 0.0003 | 0.0005 | 0.0006 | 0.0009 |
| 9 | | | | | | | | 0.0001 | 0.0001 | 0.0001 | 0.0002 |
| x | λ | 2.2 | 2.4 | 2.6 | 2.8 | 3.0 | 3.2 | 3.4 | 3.6 | 3.8 | 4.0 |
| 0 | | 0.1108 | 0.0907 | 0.0743 | 0.0608 | 0.0498 | 0.0408 | 0.0334 | 0.0273 | 0.0224 | 0.0183 |
| 1 | | 0.2438 | 0.2177 | 0.1931 | 0.1703 | 0.1494 | 0.1304 | 0.1135 | 0.0984 | 0 | |

English translation of the wording on the front cover

Level 3 Mathematics and Statistics (Statistics), 2018

9.30 a.m. Thursday 22 November 2018

FORMULAE AND TABLES BOOKLET for 91584, 91585 and 91586

Refer to this booklet to answer the questions in your Question and Answer booklets.

Check that this booklet has pages 2–7 in the correct order and that none of these pages is blank.

YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.