91262M

2

SUPERVISOR'S USE ONLY

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Te Pāngarau me te Tauanga, Kaupae 2, 2019

91262M Te whakahāngai tikanga tuanaki hei whakaoti rapanga

9.30 i te ata Rāpare 21 Whiringa-ā-rangi 2019
Whiwhinga: Rima

Paetae	Kaiaka	Kairangi
Te whakahāngai tikanga tuanaki hei whakaoti rapanga.	Te whakahāngai tikanga tuanaki mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai tikanga tuanaki mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tirohia mēnā kei a koe te Pukapuka Tikanga Tātai L2–MATHMF.

Whakaaturia ngā mahinga KATOA.

Mēnā ka hiahia whārangi atu anō mō ō tuhinga, whakamahia te wāhi wātea kei muri o tēnei pukapuka.

Me mātua whakaatu e koe te whakamahi tuanaki i ō tuhinga mō ngā tūmahi katoa i tēnei pepa.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–27 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

© Mana Tohu Mātauranga o Aotearoa, 2019. Pūmau te mana.
Kia kaua rawa he wāhi o tēnei tuhinga e whakahuatia ki te kore te whakaaetanga tuatahi a te Mana Tohu Mātauranga o Aotearoa.

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATAHI

(a) Ka tohua he pānga f mā te $f(x) = x^4 + 3x^2 - 17$

Whakamahia te tuanaki hei whiriwhiri i te rōnaki o te kauwhata o te pānga kei te pūwāhi $x = 2$.

(b) Whiriwhiria ngā taunga o te(ngā) pūwāhi kei te kauwhata o te pānga $y = 4x^3 - 4x + 4$ ko te pātapa ki te ānau he whakarara ki te rārangi $y - 8x + 6 = 0$.

QUESTION ONE

ASSESSOR'S USE ONLY

(a) A function f is given by $f(x) = x^4 + 3x^2 - 17$

Use calculus to find the gradient of the graph of the function at the point where $x = 2$.

(b) Find the coordinates of the point(s) on the graph of the function $y = 4x^3 - 4x + 4$ where the tangent to the curve is parallel to the line $y - 8x + 6 = 0$.

MĀ TE KAIMĀKA ANAKE

(c) Kei te whakakī a Sophie i tētahi poihau. Ko te rōrahi o te poihau kei te tohua e $V = \frac{4}{3}\pi r^3$

ina ko V te rōrahi o te poihau i te cm^3, ā, ko te r te pūtoru ki te cm.

Kimihia te pūtoru o te poihau ina ko te pāpātanga o te rerekē haere o te rōrahi e ai ki te pūtoru he 25π cm^3 /cm.

(d) Whakamahia te tuanaki hei whiriwhiri i te pānga pūrua o x e whai ana i ngā āhuatanga e whai ake:

- te pāpātanga whakapiki (rōnaki) o te 7 ina ko $x = 0$
- he pūwāhi huringa ina ko $x = 1$
- he uara o −20 ina ko $x = 4$.

ASSESSOR'S USE ONLY

(c) Sophie is blowing up a balloon. The volume of the balloon is given by $V = \frac{4}{3}\pi r^3$

where V is the volume of the balloon in cm^3, and r is the radius in cm.

Find the radius of the balloon when the rate of change of the volume with respect to the radius is $25\pi\ \text{cm}^3/\text{cm}$.

(d) Use calculus to find the quadratic function of x that has the following properties:

- a rate of increase (gradient) of 7 when $x = 0$
- a turning point when $x = 1$
- a value of -20 when $x = 4$.

(e) Ka pā noa ngā kauwhata o

$$g(x) = x^3 - ax^2 + 6 \quad \text{me} \quad h(x) = 2x^2 + bx + 13$$

ina ko $x = -1$ (kia ōrite te pātapa i te pūwāhi whakapā).

Whakamahia te tuanaki hei whiriwhiri i ngā taunga o te pūwāhi whakapā o ngā kauwhata e rua.

MĀ TE KAIMĀKA ANAKE

(e) The graphs of

$$g(x) = x^3 - ax^2 + 6 \quad \text{and} \quad h(x) = 2x^2 + bx + 13$$

just touch when $x = -1$ (so they have a common tangent at the point of contact).

Use calculus to find the coordinates of the point of contact of the two graphs.

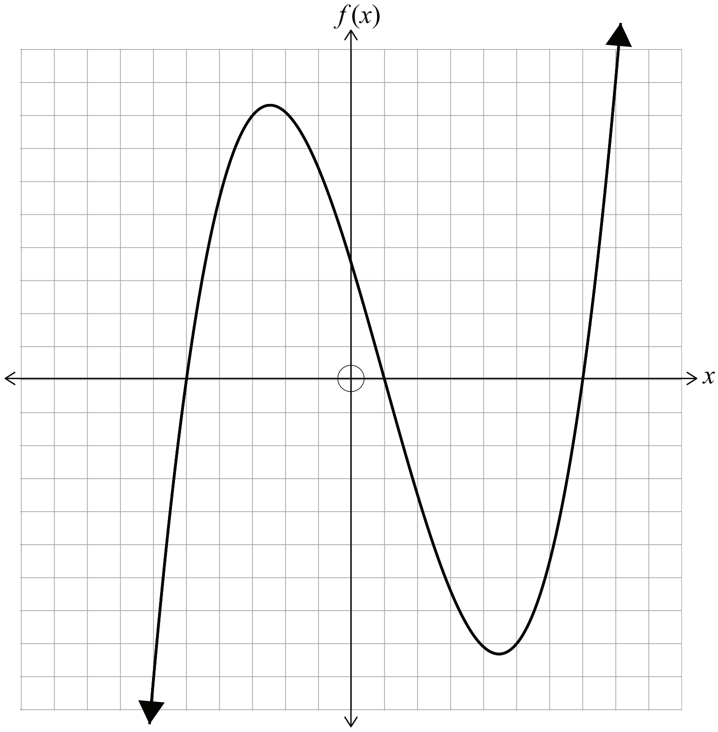
ASSESSOR'S USE ONLY

MĀ TE KAIMĀKA ANAKE

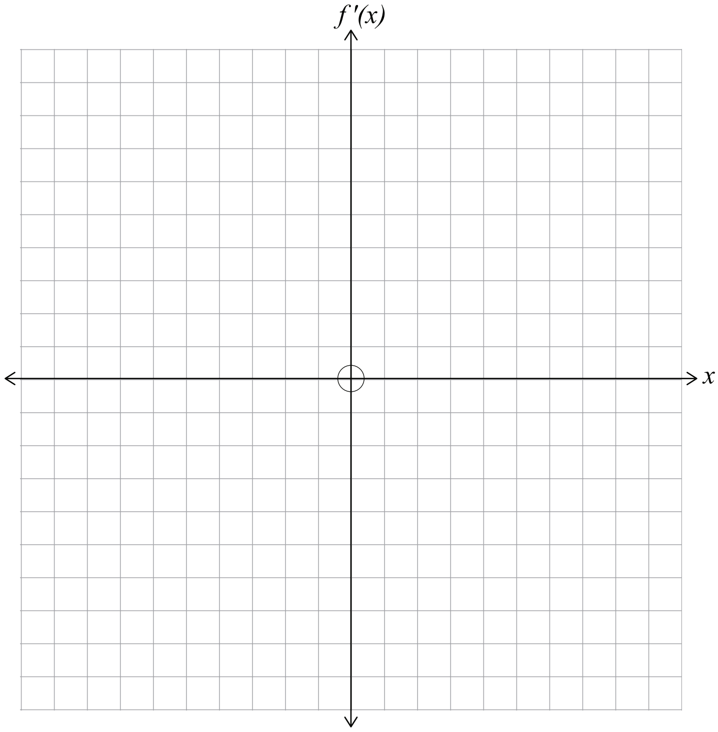
TŪMAHI TUARUA

(a) E whakaatuhia ana te kauwhata o te pānga $y = f(x)$ ki ngā tuaka i raro nei.

He ōrite te āwhata o ngā huinga tuaka e rua.



Tuhia te kauwhata o te pānga rōnaki $y = f'(x)$ ki ngā tuaka o raro.



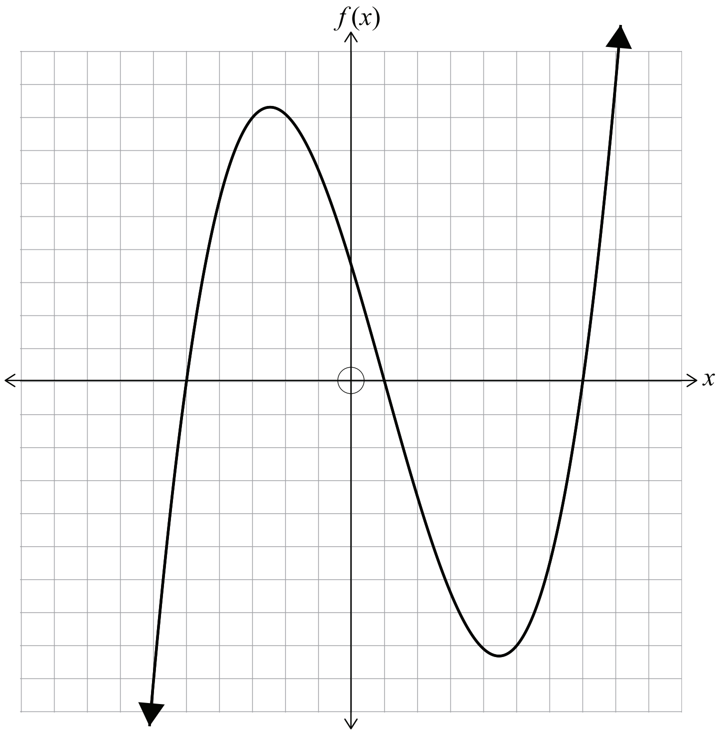
Ki te hiahia koe ki te tuhi anō i tēnei mahinga, whakamahia ngā tukutuku i te whārangi 22.

QUESTION TWO

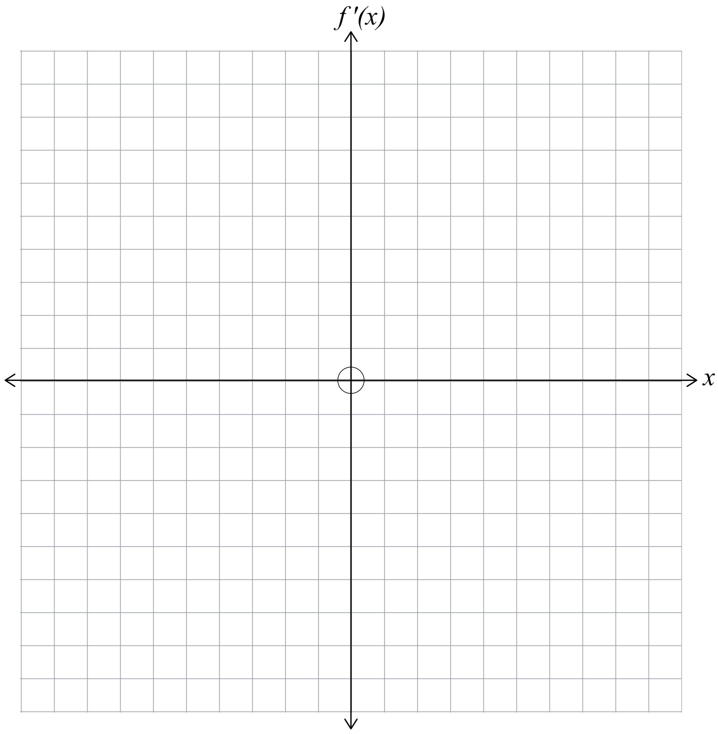
ASSESSOR'S USE ONLY

(a) The graph of a function $y = f(x)$ is shown on the axes below.

Both sets of axes have the same scale.



Sketch the graph of the gradient function $y = f'(x)$ on the axes below.



If you need to redo this question part, use the grids on page 23.

MĀ TE KAIMĀKA ANAKE

(b) Ko te pānga, $p(x)$, ka tohua tana pānga pārōnaki mā te $p'(x) = 5 - 8x^3$.

Ka haukoti te kauwhata o $p(x)$ mā te $(2,-25)$.

Whiriwhiria te whārite o $p(x)$.

(c) Whakaaroarohia te kauwhata o te pānga $f(x) = -2k^2x^3 + 3kx^2 + 12x - 55$, ina ko k tētahi aumou tōrunga.

Whakamahia te tuanaki ki te kimi i ngā kīanga, e ai ki a k, mō te(ngā) awhe o ngā uara o x e piki haere ana tēnei kauwhata.

Kia mārama te parahau i tō(ō) kōwhiringa awhe.

(b) A function, $p(x)$, has a derived function given by $p'(x) = 5 - 8x^3$.

The graph of $p(x)$ passes through $(2,-25)$.

Find the equation of $p(x)$.

(c) Consider the graph of the function $f(x) = -2k^2x^3 + 3kx^2 + 12x - 55$, where k is a positive constant.

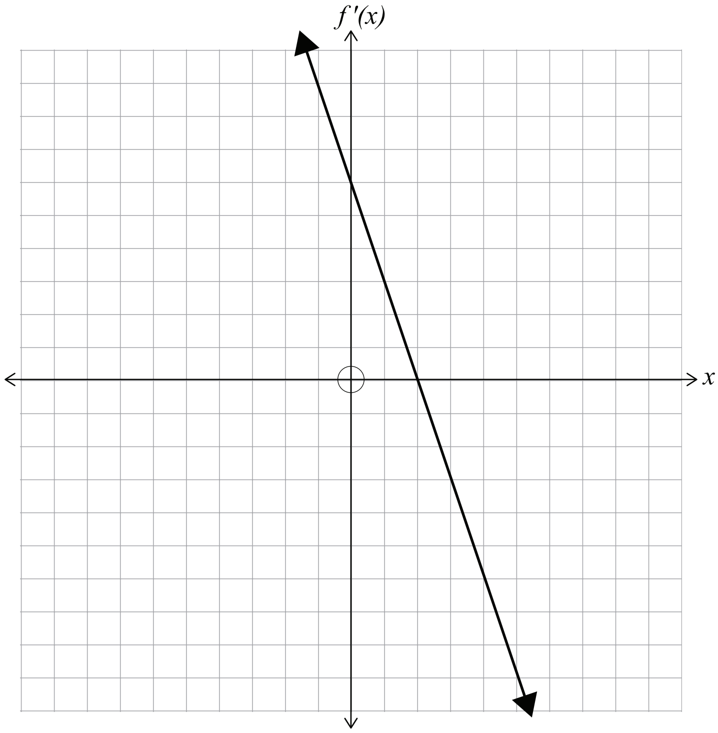
Use calculus to find expressions, in terms of k, for the range(s) of values of x for which this graph is increasing.

Justify your choice of range(s) clearly.

ASSESSOR'S USE ONLY

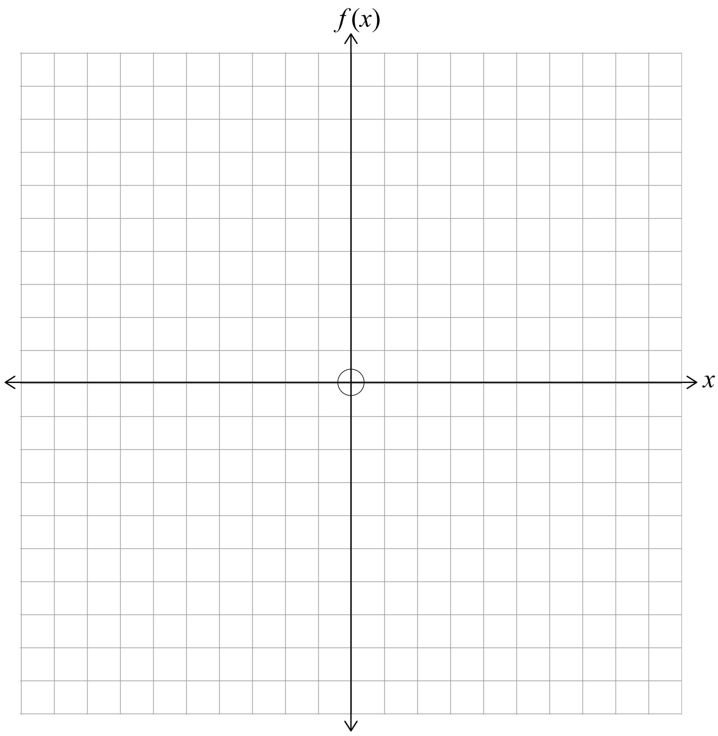
MĀ TE KAIMĀKA ANAKE

(d) E whakaatu ana te hoahoa o raro i te kauwhata o te pānga rōnaki $y = f'(x)$ o tētahi pānga $y = f(x)$. He ōrite te āwhata o ngā huinga tuaka e rua.



Ka haukoti te kauwhata o te pānga mā te pūtake (0,0).

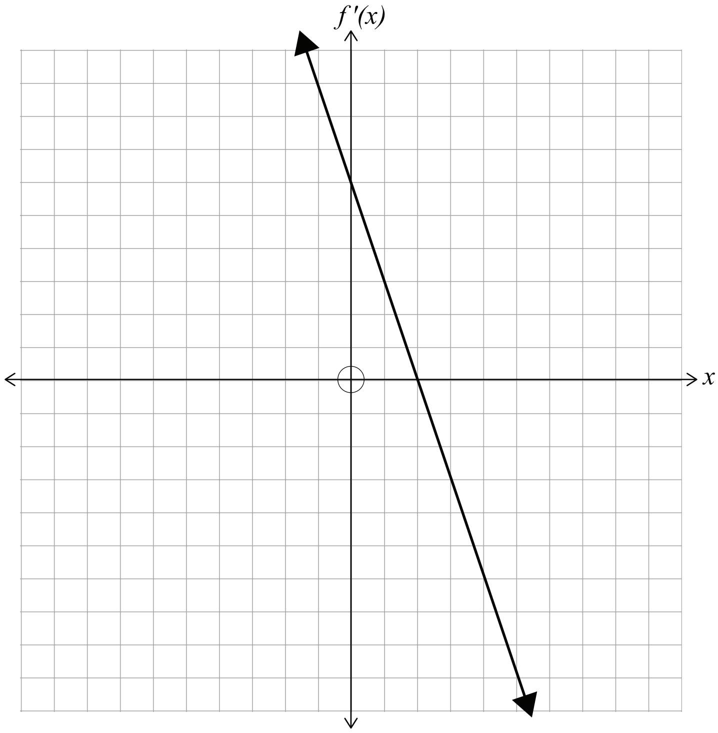
Ki ngā tuaka o raro, tuhia te kauwhata o te pānga $f(x)$.



Ki te hiahia koe ki te tuhi anō i tēnei mahinga, whakamahia ngā tukutuku i te whārangi 24.

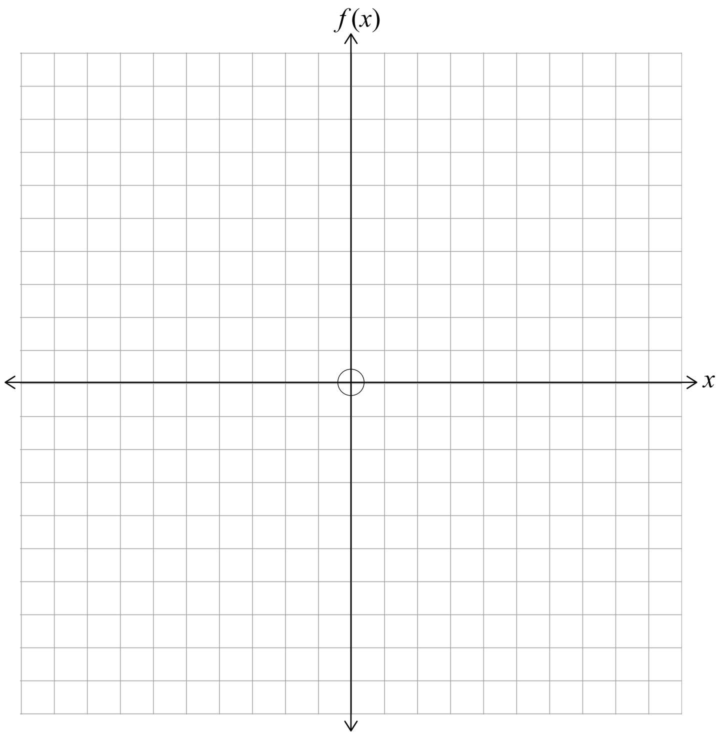
ASSESSOR'S USE ONLY

(d) The diagram below shows the graph of the gradient function $y = f'(x)$ of a function $y = f(x)$. Both sets of axes have the same scale.



The graph of the function passes through the origin (0,0).

On the axes below, sketch the graph of the function $f(x)$.



If you need to redo this question part, use the grids on page 25.

(e) E haere ana tētahi motokā ki te tere aumou o te 28 m s^{-1} i tētahi rori torotika me te whakatata haere ki tētahi kokonga. Ka whakamahia e te kaitaraiwa ngā pereki ka heke te tere ki te pāpātanga aumou o te 4 m s^{-2} kia tae rā anō te motokā ki te kokonga me te tere o te 10 m s^{-1}.

Whakamahia te tuanaki ki te whiriwhiri e hia te tawhiti o te motokā mai i te kokonga i te wā tuatahi i whakamahia ngā pereki.

Parahautia tō tuhinga.

(e) A car travelling at a constant speed of 28 m s^{-1} on a straight road is approaching a corner. The driver applies the brakes and decelerates at a constant rate of 4 m s^{-2} until the car reaches the corner with a speed of 10 m s^{-1}.

Use calculus to find how far the car was from the corner when the driver first applied the brakes.

Justify your answer.

ASSESSOR'S USE ONLY

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATORU

(a) Kei te whakahaerehia e Matiu tana motokā whakahaere-mamao, ā, kei te neke whakamua, whakamuri i tētahi ara torotika. Ko te tawhiti o te motokā mai i te pūwāhi P i te ara he s mita, he t hēkona mai i te wehenga i P, ko te $s(t) = 6t - t^2$.

(i) Whiriwhiria te tere tīmata o te motokā.

(ii) E hia te roa mai i te wehenga i P ka huri te ahunga o te motokā?

(iii) E hia te tere o te motokā ina tae atu ki P i te wā tuarua?

ASSESSOR'S USE ONLY

QUESTION THREE

(a) Matiu is operating his remote-controlled car, which is moving back and forth along a straight track. The car's distance s metres from a point P on the track, t seconds after leaving P, is given by $s(t) = 6t - t^2$.

(i) Find the initial speed of the car.

(ii) How long after leaving P does the car change direction?

(iii) How fast is the car moving when it reaches P for the second time?

MĀ TE KAIMĀKA ANAKE

(b) Ka tukuna tētahi pūhera he poro-tapawhā hāngai te āhua me tētahi motuhanga tapawhā rite mā te pōhi. Ko te tapeke roa o te poro me te paenga o te motuhanga tapawhā rite he 100 cm.

KĀORE i tuhi ā-āwhatatia tēnei hoahoa

Whiriwhiria te rōrahi mōrahi rawa o te pūhera ka taea.

Whakamāramahia mai he pēhea koe i mōhio ai ko tō whakautu te rōrahi mōrahi rawa, ehara ko te rōrahi iti rawa.

(b) A parcel in the shape of a rectangular cuboid with a square cross section is to be sent through the post. The sum of the length of the cuboid and the perimeter of the square cross section is to be 100 cm.

Diagram is NOT to scale

Find the maximum possible volume of the parcel.

Explain how you know that your answer is the maximum, not the minimum, volume.

(c) E rua ngā pūwāhi, A me B, kei te kauwhata o te pānga $f(x) = x^3 - 3x^2 - 4x$ ina pā ana te pātapa ki te kauwhata mā te pūtake.

Whiriwhiria ngā taunga o ngā pūwāhi A me B me te whārite o ia pātapa.

MĀ TE KAIMĀKA ANAKE

(c) There are two points, A and B, on the graph of the function $f(x) = x^3 - 3x^2 - 4x$ where the tangent to the graph passes through the origin.

Find the coordinates of points A and B and the equation of each tangent.

ASSESSOR'S USE ONLY

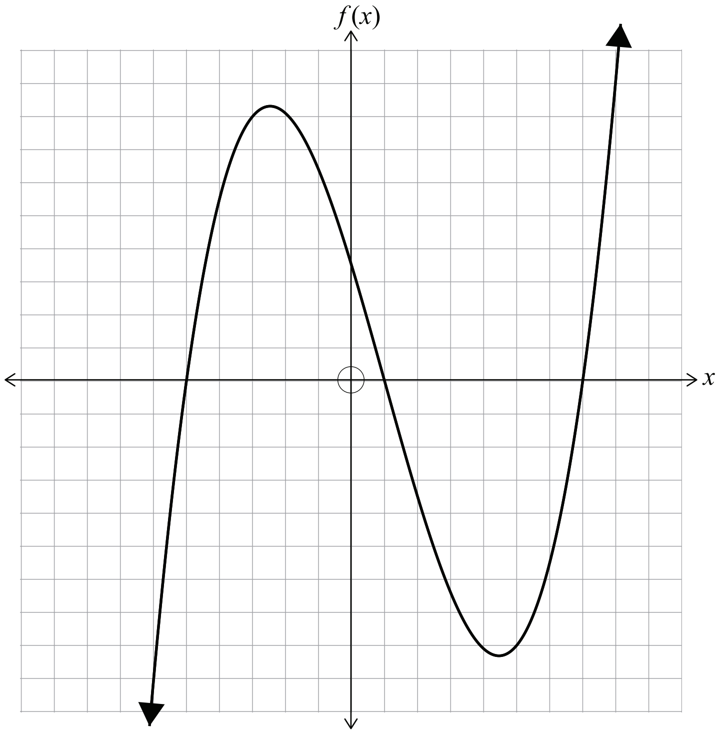
HE TUKUTUKU TĀPIRI

Ki te hiahia koe ki te tuhi anō i tō urupare ki te Tūmahi Tuarua (a), whakamahia te tukutuku i raro nei. Kia mārama te tohu ko tēhea te tuhinga ka hiahia koe kia mākahia.

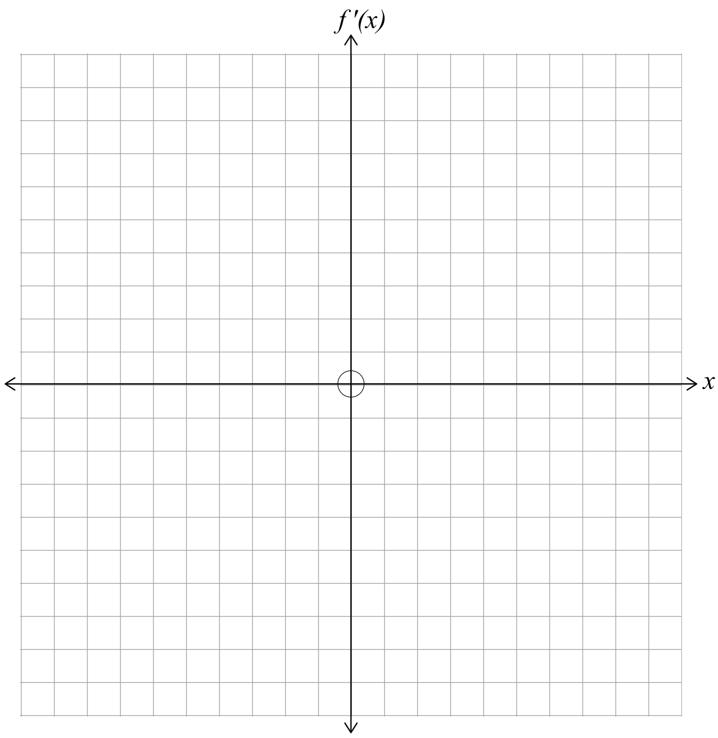
TŪMAHI TUARUA

(a) E whakaatuhia ana te kauwhata o te pānga $y = f(x)$ ki ngā tuaka i raro nei.

He ōrite te āwhata o ngā huinga tuaka e rua.



Tuhia te kauwhata o te pānga rōnaki $y = f'(x)$ ki ngā tuaka o raro.



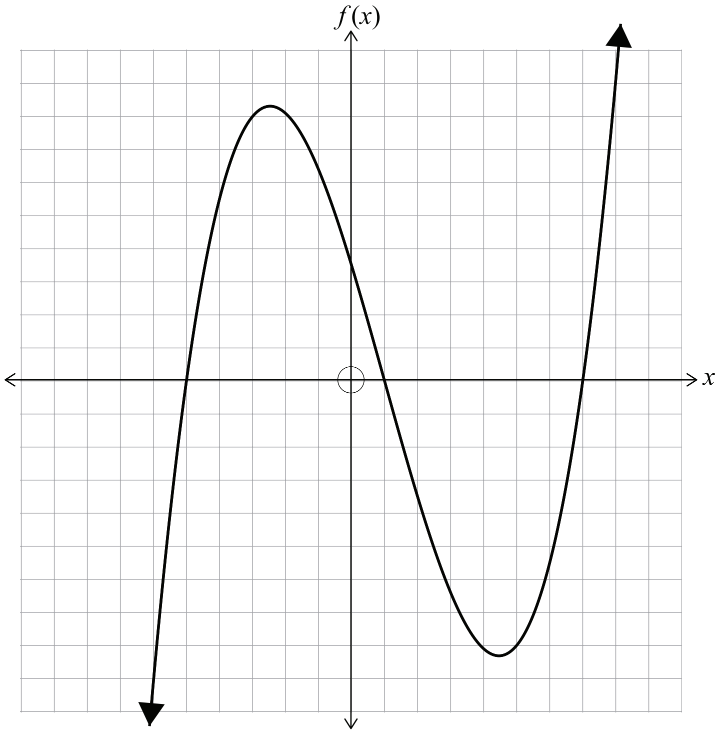
SPARE GRIDS

If you need to redo Question Two (a), use the grid below. Make sure it is clear which answer you want marked.

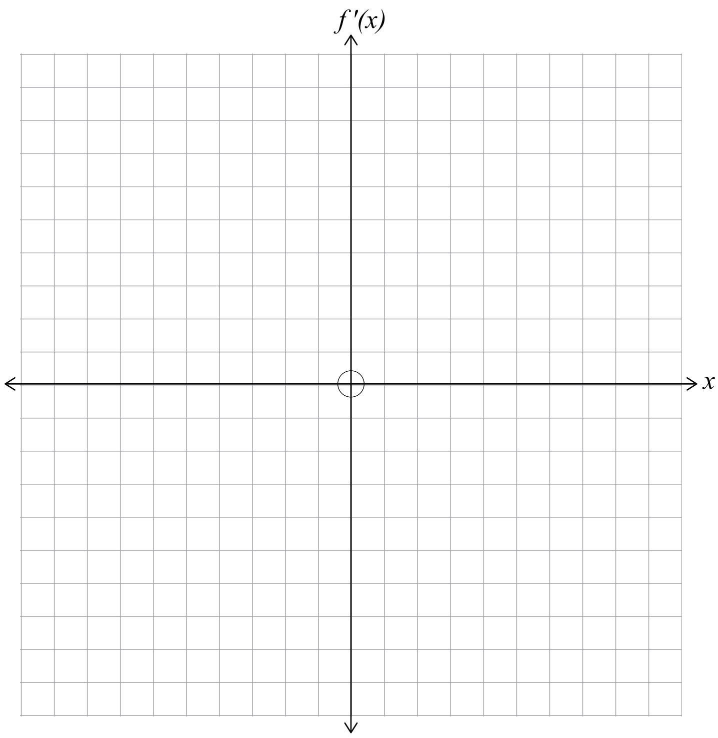
QUESTION TWO

(a) The graph of a function $y = f(x)$ is shown on the axes below.

Both sets of axes have the same scale.



Sketch the graph of the gradient function $y = f'(x)$ on the axes below.



ASSESSOR'S USE ONLY

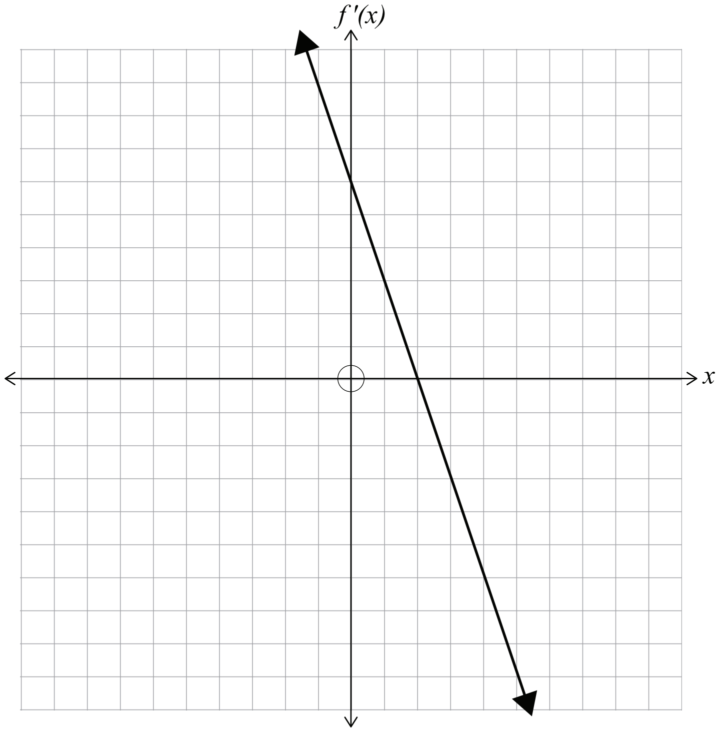
MĀ TE KAIMĀKA ANAKE

Ki te hiahia koe ki te tuhi anō i tō urupare ki te Tūmahi Tuarua (d), whakamahia te tukutuku i raro nei. Kia mārama te tohu ko tēhea te tuhinga ka hiahia koe kia mākahia.

TŪMAHI TUARUA

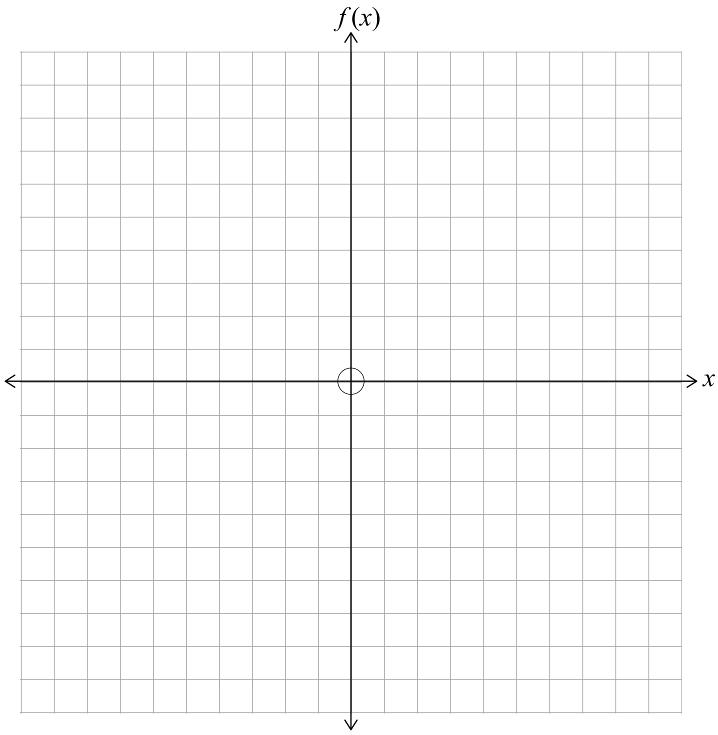
(d) E whakaatu ana te hoahoa o raro i te kauwhata o te pānga rōnaki $y = f'(x)$ o tētahi pānga $y = f(x)$.

He ōrite te āwhata o ngā huinga tuaka e rua.



Ka haukoti te kauwhata o te pānga mā te pūtake (0,0).

Ki ngā tuaka o raro, tuhia te kauwhata o te pānga $f(x)$.



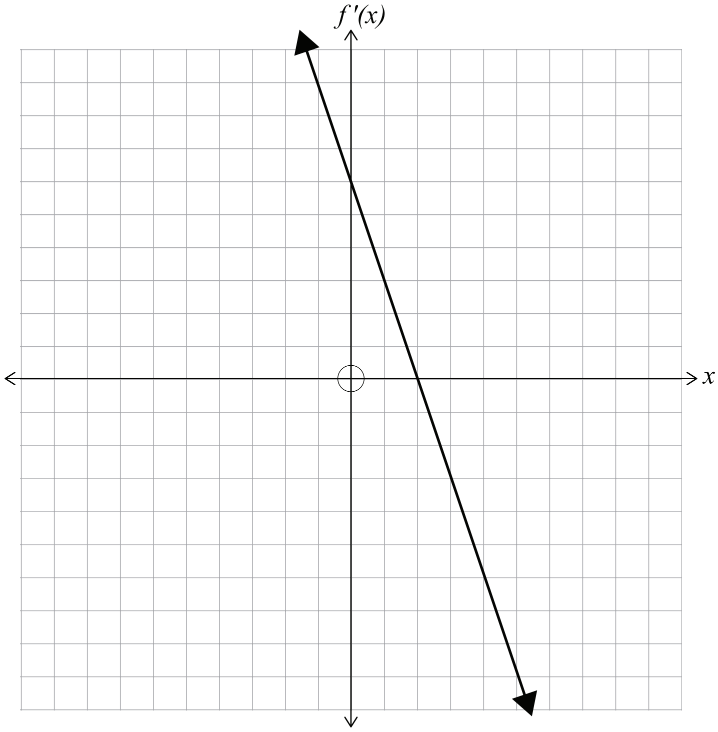
If you need to redo Question Two (d), use the grid below. Make sure it is clear which answer you want marked.

ASSESSOR'S USE ONLY

QUESTION TWO

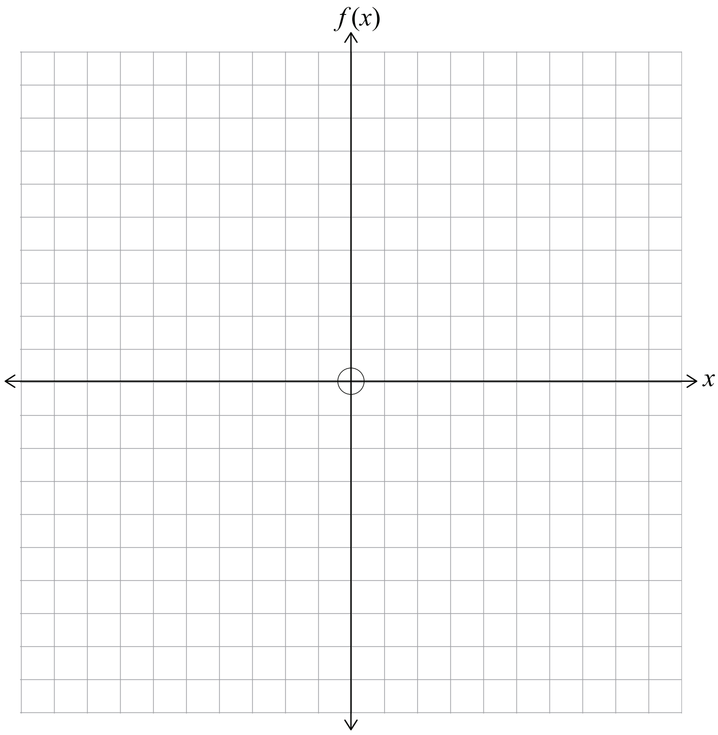
(d) The diagram below shows the graph of the gradient function $y = f'(x)$ of a function $y = f(x)$.

Both sets of axes have the same scale.



The graph of the function passes through the origin (0,0).

On the axes below, sketch the graph of the function $f(x)$.



He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

English translation of the wording on the front cover

Level 2 Mathematics and Statistics, 2019

91262 Apply calculus methods in solving problems

9.30 a.m. Thursday 21 November 2019
Credits: Five

91262M

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Booklet L2–MATHMF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

You must show the use of calculus in answering all questions in this paper.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.