91578M

915785

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

3

SUPERVISOR'S USE ONLY

Tuanaki, Kaupae 3, 2019

91578M Te whakahāngai i ngā tikanga pārōnaki hei whakaoti rapanga

9.30 i te ata Rātū 26 Whiringa-ā-rangi 2019
Whiwhinga: Ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pārōnaki hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pārōnaki mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pārōnaki mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tuhia ō mahinga KATOA.

Tirohia mēnā kei a koe te Pukapuka Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Mēnā ka hiahia whārangi atu anō koe mō ō tuhinga, whakamahia te (ngā) whārangi wātea kei muri o tēnei pukapuka, ka āta tohu ai i te tau tūmahi.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–21 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATAHI

MĀ TE KAIMĀKA ANAKE

(a) Kimi pārōnaki mō $y = \sqrt{3x^2 - 1}$.

Hei aha noa te whakarūnā i tō tuhinga.

(b) Whiriwhiria te pāpātanga panoni o te pānga $f(t) = 5\ln(3t - 1)$ ina ko $t = 4$.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

(c) Whiriwhiria te rōnaki o te pātapa ki te ānau $y = \dfrac{e^{2x}}{1 + x^2}$ ki te pūwāhi ina ko $x = 2$.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

QUESTION ONE

ASSESSOR'S USE ONLY

(a) Differentiate $y = \sqrt{3x^2 - 1}$.

You do not need to simplify your answer.

(b) Find the rate of change of the function $f(t) = 5\ln(3t - 1)$ when $t = 4$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) Find the gradient of the tangent to the curve $y = \dfrac{e^{2x}}{1 + x^2}$ at the point where $x = 2$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) Mō (t)ēhea uara o x e heke ana te pānga $y = x^3e^x$?

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

MĀ TE KAIMĀKA ANAKE

(e) Kei te nui haere te rōrahi o tētahi poi.

I te wā tonu he 0.5 m te pūtoru o te poi, he 0.4 $m^2\ s^{-1}$ te pāpātanga o te nui haere o te horahanga mata o te poi.

Whiriwhiria te pāpātanga e nui haere ana te rōrahi o te poi i tēnei wā tonu.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

ASSESSOR'S USE ONLY

(d) For what value(s) of x is the function $y = x^3e^x$ decreasing?

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) The volume of a sphere is increasing.

At the instant when the sphere's radius is 0.5 m, the surface area of the sphere is increasing at a rate of 0.4 $m^2\ s^{-1}$.

Find the rate at which the volume of the sphere is increasing at this instant.

You must use calculus and show any derivatives that you need to find when solving this problem.

TŪMAHI TUARUA

MĀ TE KAIMĀKA ANAKE

(a) Kimi pārōnaki mō $y = (2x - 5)^4$.

Hei aha noa te whakarūnā i tō tuhinga.

(b) Whiriwhiria te rōnaki o te pātapa ki te ānau $y = \tan 2x$ i te pūwāhi o te ānau ina ko $x = \frac{\pi}{6}$.
Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

(c) E tautuhia tawhātia ana tētahi ānau mā ngā whārite $x = \frac{1}{(5-t)^2}$ me $y = 5t - t^2$.

Whiriwhiria te rōnaki o te pātapa ki te ānau i te pūwāhi ina ko $t = 2$.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

QUESTION TWO

ASSESSOR'S USE ONLY

(a) Differentiate $y = (2x - 5)^4$.

You do not need to simplify your answer.

(b) Find the gradient of the tangent to the curve $y = \tan 2x$ at the point on the curve where $x = \frac{\pi}{6}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) A curve is defined parametrically by the equations $x = \frac{1}{(5-t)^2}$ and $y = 5t - t^2$.

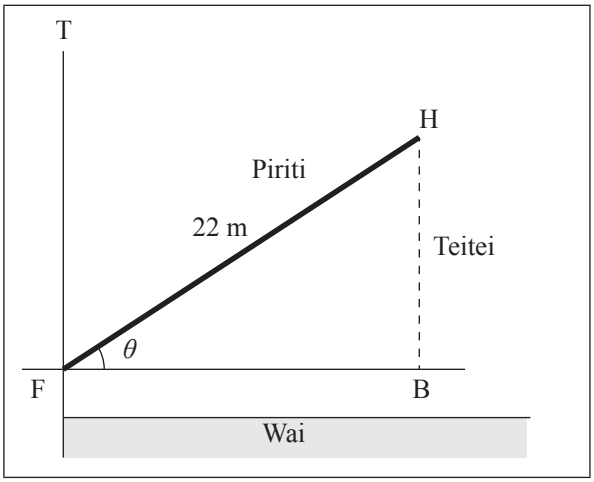
Find the gradient of the tangent to the curve at the point when $t = 2$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) Ka taea te piriti o Wynyard Crossing i Tāmaki Makaurau te hiki me te whakaheke kia taea ai ngā poti teitei te hipa ina tuwhera ana, ā, kia taea ai ngā kaihīkoi te whakawhiti ina kati ana. E rua ngā ringa o te piriti, he 22 mita te roa o ia ringa.

Ina hiki ana te piriti, ko te koki o te ringa o te piriti i runga ake o te huapae ka nui haere ake ki te pāpātanga o te 0.01 rad s^{-1}.

www.youtube.com/watch?v=Q4xrCt-uYPE



Whiriwhiria te pāpātanga e piki haere ana te tetei, BH, ina he 15 mita te teitei ake o H i te huapae, FB.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

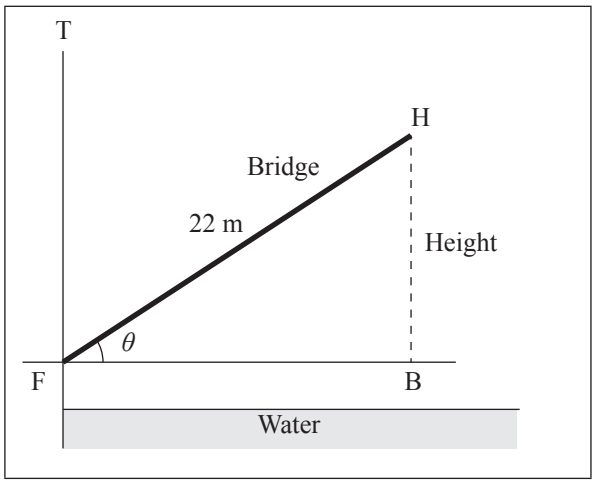
MĀ TE KAIMĀKA ANAKE

ASSESSOR'S USE ONLY

(d) The Wynyard Crossing bridge in Auckland can be raised and lowered to allow tall boats to sail through when open, and pedestrians to walk across when closed. The bridge consists of two arms, each of length 22 metres.

When the bridge is rising, the angle of the bridge arm above the horizontal increases at the rate of 0.01 rad s^{-1}.

www.youtube.com/watch?v=Q4xrCt-uYPE



Find the rate at which the height, BH, is increasing when H is 15 metres above the horizontal, FB.

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) Mēnā ko $y = e^u$ ā ko $u = \sin 2x$ whakaaturia ko

$$\frac{d^2y}{dx^2} = \frac{d^2y}{du^2}\left(\frac{du}{dx}\right)^2 + \frac{dy}{du}\frac{d^2u}{dx^2}$$

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

MĀ TE KAIMĀKA ANAKE

(e) If $y = e^u$ and $u = \sin 2x$ show that

$$\frac{d^2y}{dx^2} = \frac{d^2y}{du^2}\left(\frac{du}{dx}\right)^2 + \frac{dy}{du}\frac{d^2u}{dx^2}$$

You must use calculus and show any derivatives that you need to find when solving this problem.

ASSESSOR'S USE ONLY

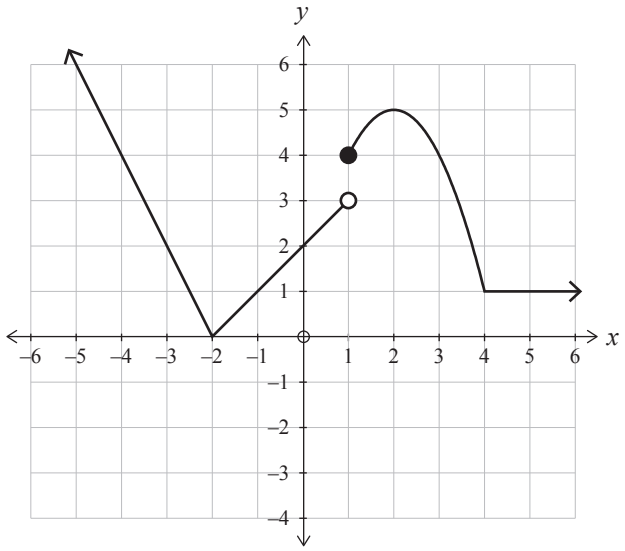
TŪMAHI TUATORU

MĀ TE KAIMĀKA ANAKE

(a) Kimi pārōnaki mō $y = \dfrac{4}{\sin x}$.

Hei aha noa te whakarūnā i tō tuhinga.

(c) E whakaatu ana te kauwhata i raro nei i te pānga $y = f(x)$.



(i) Whiriwhiria te (ngā) uara katoa o x e ū ana ki i ia āhuatanga nei e whai ake:

1. $f'(x) = 0$:

2. kāore e taea te kimi pārōnaki mō $f(x)$:

(ii) He aha te uara o $\lim\limits_{x \to 1} f(x)$?

Āta kōrero mai mēnā kāore rawa he uara.

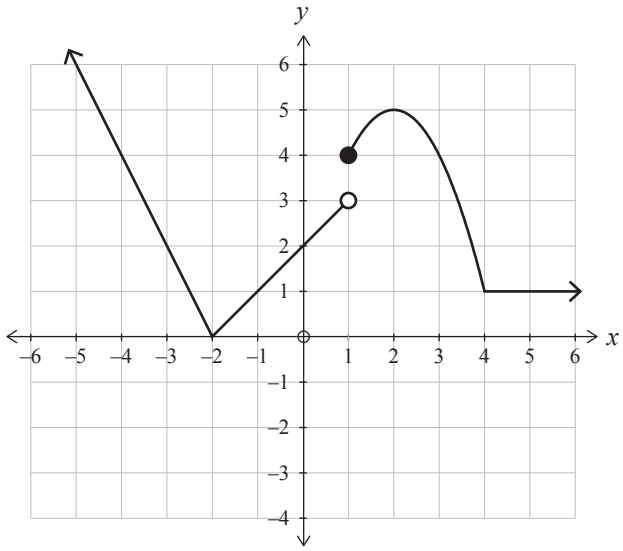
QUESTION THREE

ASSESSOR'S USE ONLY

(a) Differentiate $y = \dfrac{4}{\sin x}$.

You do not need to simplify your answer.

(b) The graph below shows the function $y = f(x)$.



(i) Find all the value(s) of x which meet each of the following conditions:

1. $f'(x) = 0$:

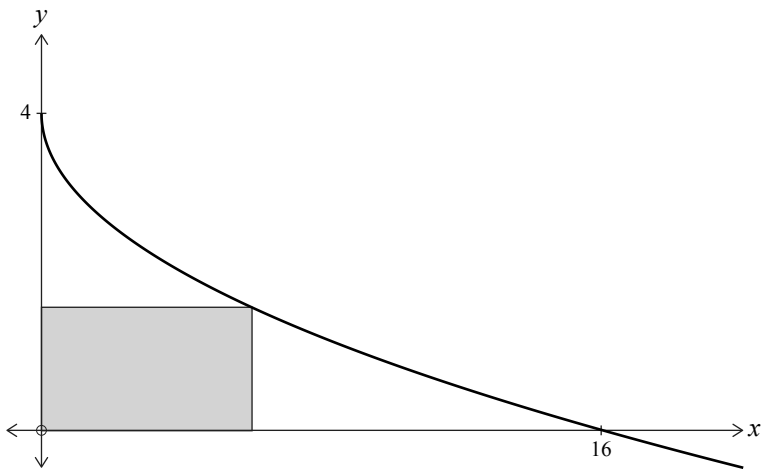
2. $f(x)$ is not differentiable:

(ii) What is the value of $\lim\limits_{x \to 1} f(x)$?

State clearly if the value does not exist.

MĀ TE KAIMĀKA ANAKE

(c) Kotahi te akitu o tētahi tapawhā hāngai i (0,0), me te akitu kōaro kei te ānau $y = 4 - \sqrt{x}$, ina ko $0 < x < 16$, e whakaaturia ana ki te kauwhata i raro.



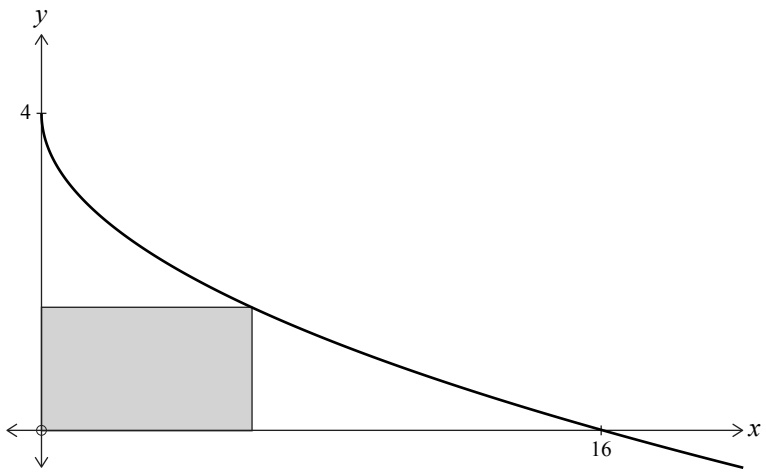
Whiriwhiria te horahanga mōrahi rawa ka taea o te tapawhā hāngai.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

Kāore he tikanga kia hāponotia e koe he mōrahi te horahanga i tātaihia.

ASSESSOR'S USE ONLY

(c) A rectangle has one vertex at (0,0), and the opposite vertex on the curve $y = 4 - \sqrt{x}$, where $0 < x < 16$, as shown on the graph below.



Find the maximum possible area of the rectangle.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not need to prove that the area you have found is a maximum.

(d) Ka whakatauiratia te whakaterenga o tētahi ahanoa mā te pānga

$v = 2e^{t} + 8e^{-t}$, mō $t \geq 0$

ina ko v te tere o te ahanoa i te m s^{-1}
ko t te wā ā-hēkona mai i te tīmatanga o te neke o te ahanoa.

Kimihia mai te wā ko te whakaterenga o te ahanoa he 0.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

MĀ TE KAIMĀKA ANAKE

Ka haere tonu te Tūmahi Tuatoru i te whārangi 18.

ASSESSOR'S USE ONLY

(d) The velocity of an object is modelled by the function

$$v = 2e^{t} + 8e^{-t}, \text{ for } t \geq 0$$

where v is the velocity of the object, in m s^{-1}
and t is the time in seconds since the start of the object's motion.

Find the time when the acceleration of the object is 0.

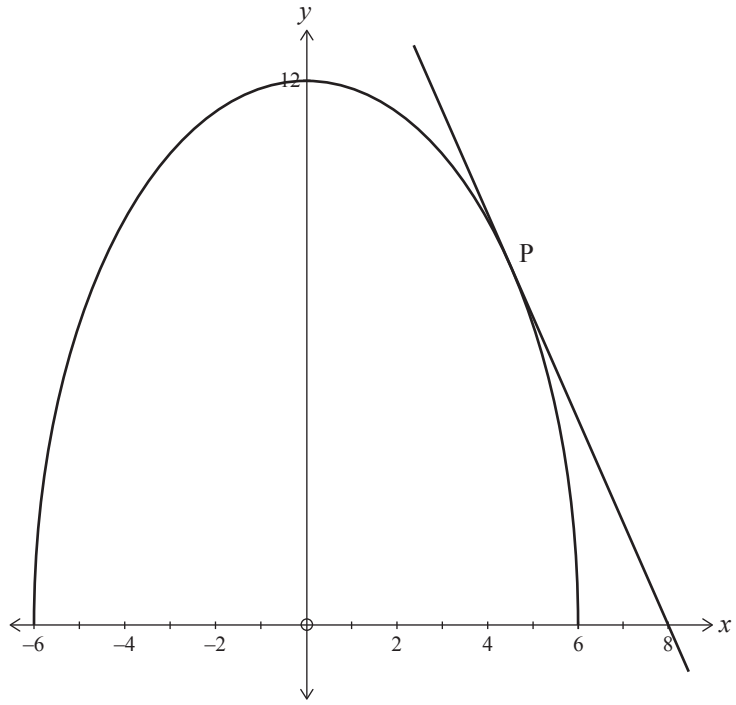
You must use calculus and show any derivatives that you need to find when solving this problem.

Question Three continues on page 19.

MĀ TE KAIMĀKA ANAKE

(e) E whakaatu ana te kauwhata i raro i te pānga $y = 2\sqrt{36 - x^2}$, me te pātapa ki taua pānga i pūwāhi P.

Ka haukoti te pātapa i te tuaka-*x* i te pūwāhi (8,0).

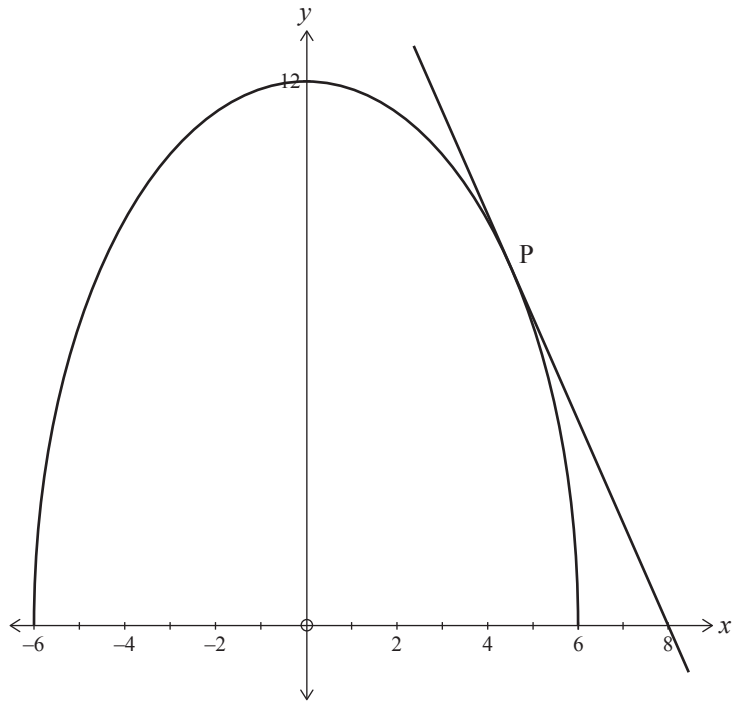


Whiriwhiria te taunga-*x* o te pūwāhi P.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

ASSESSOR'S USE ONLY

(e) The graph below shows the function $y = 2\sqrt{36 - x^2}$, and the tangent to that function at point P. The tangent intersects the x-axis at the point (8,0).



Find the x-coordinate of point P.

You must use calculus and show any derivatives that you need to find when solving this problem.

He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

English translation of the wording on the front cover

Level 3 Calculus, 2019

91578 Apply differentiation methods in solving problems

9.30 a.m. Tuesday 26 November 2019
Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

91578M

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–21 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.