91261M

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

2

SUPERVISOR'S USE ONLY

Tohua tēnei pouaka mēnā kāore he tuhituhi i roto i tēnei pukapuka

Te Pāngarau me te Tauanga, Kaupae 2, 2020

91261M Te whakahāngai tūāhua taurangi hei whakaoti rapanga

9.30 i te ata Rāpare 19 Whiringa-ā-rangi 2020
Ngā whiwhinga: Whā

Paetae	Kaiaka	Kairangi
Te whakahāngai tūāhua taurangi hei whakaoti rapanga.	Te whakahāngai tūāhua taurangi mā te whakaaro whai pānga hei whakaoti rapanga.	Te whakahāngai tūāhua taurangi mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tirohia mēnā kei a koe te Puka Tikanga Tātai L2–MATHMF.

Tuhia ō mahinga KATOA.

Ki te hiahia koe ki ētahi atu wāhi hei tuhituhi whakautu, whakamahia te wāhi wātea kei muri i te pukapuka nei.

Me whakaatu e koe ngā mahinga taurangi i tēnei pepa. Ko te tikanga, mā te whakamahi i ngā tikanga o te kimikimi ka tirotiro, te whakautu tika noa iho rānei, ka herea te ākonga ki te taumata Paetae.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2-27 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATAHI

(a) Whakatauwehea $6x^2 + 13x - 15$.

(b) Ka tohua he pānga ko te $f(x) = x^2 + 10x + 22$.

Whakapuakina $f(x)$ ki te āhua pūrua oti, hei tauira $f(x) = (x + a)^2 + b$, ina ko a me b he tau tōpū.

ASSESSOR'S USE ONLY

QUESTION ONE

(a) Factorise $6x^2 + 13x - 15$.

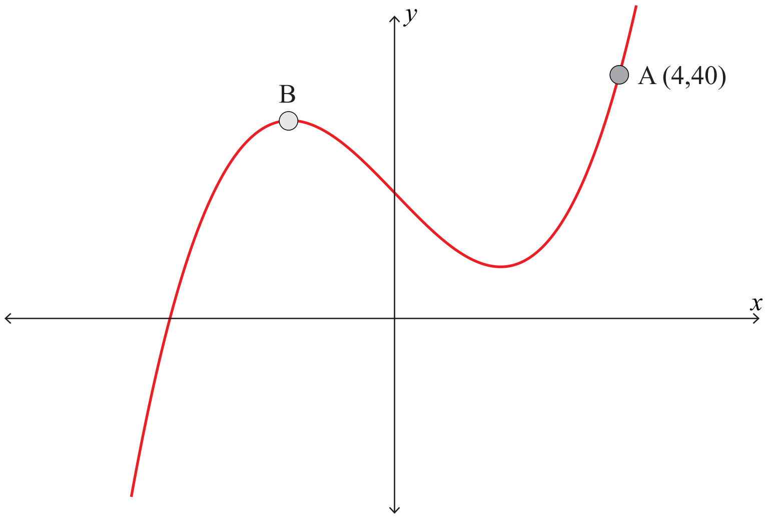
(b) A function is defined as $f(x) = x^2 + 10x + 22$.

Express $f(x)$ in completed square form, i.e. $f(x) = (x + a)^2 + b$, where a and b are integers.

MĀ TE KAIMĀKA ANAKE

(c) I te rautau 16, i te waihanga ngā tohunga pāngarau i tētahi whārite hei whiriwhiri i te whārite pūtoru. I whakamahia e rātou ngā kīanga ki te āhua o $y = x^3 - 12Px + R$, ina ko P me R he tau pūmau tōrunga.

(i) Ko te kauwhata o $y = x^3 - 12Px + R$, mō ētahi uara o P me R, ka hipa mā te pūwāhi A (4,40), ā, kua tātuhia i raro.



Whiriwhiria tētahi kīanga mō P e pā ana ki R.

(ii) I te pūwāhi B e tika ana ko $3x^2 - 12P = 0$.

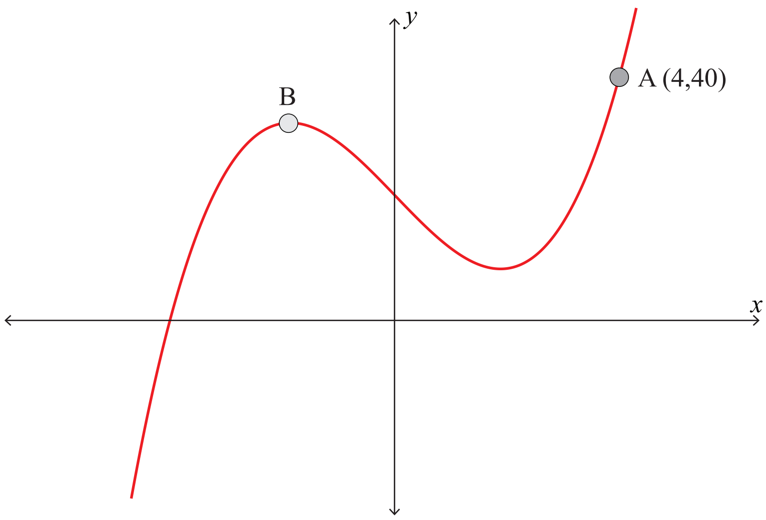
Mā te whakamahi i te taurangi, whakaaturia ko $x = -2P^{0.5}$ i B.

MĀ TE KAIMĀKA ANAKE

ASSESSOR'S USE ONLY

(c) In the 16th century, mathematicians were developing a formula to solve any cubic equation. They used expressions in the form of $y = x^3 - 12Px + R$, where P and R are positive constants.

(i) The graph of $y = x^3 - 12Px + R$, for some values of P and R, passes through the point A (4,40) and is sketched below.



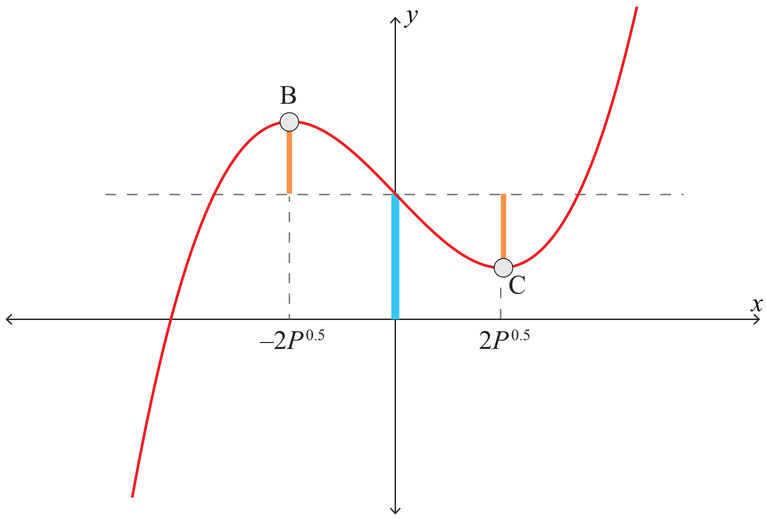
Find an expression for P in terms of R.

ASSESSOR'S USE ONLY

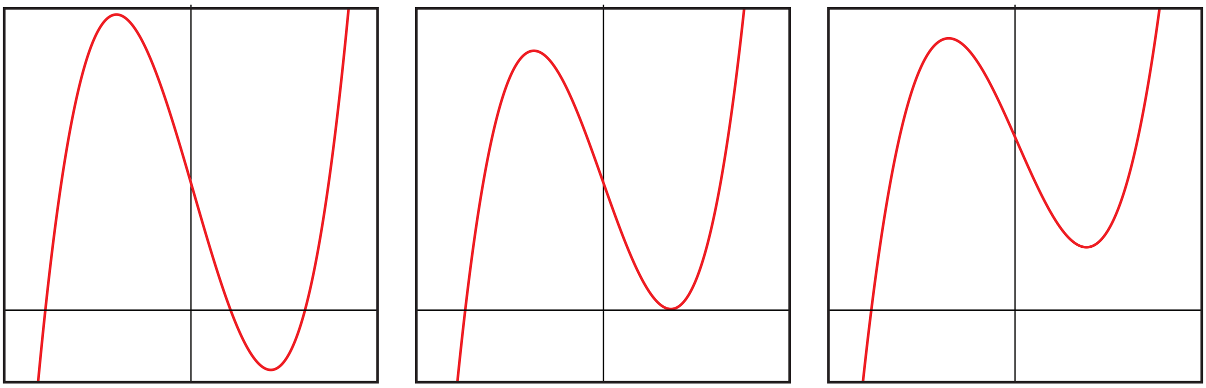
(ii) At point B it is true that $3x^2 - 12P = 0$.

Using algebra, show that $x = -2P^{0.5}$ at B.

(iii) Me whai whakaaro anō ki te kōpiko me te whārite $y = x^3 - 12Px + R$. Ina he rerekē ngā uara o P me R, ka huri te āhua o te kōpiko, ā, he rerekē ngā roa o ngā rārangi karaka me te rārangi kikorangi o waenga (i raro). Engari, nā te hangarite, ka noho rite tonu te roa o ngā rārangi karaka e rua tētahi ki tētahi.



E whakaaturia ana i raro ko ētahi tauira o ngā kauwhata i riro mai i ngā tūmomo uara o P me R.



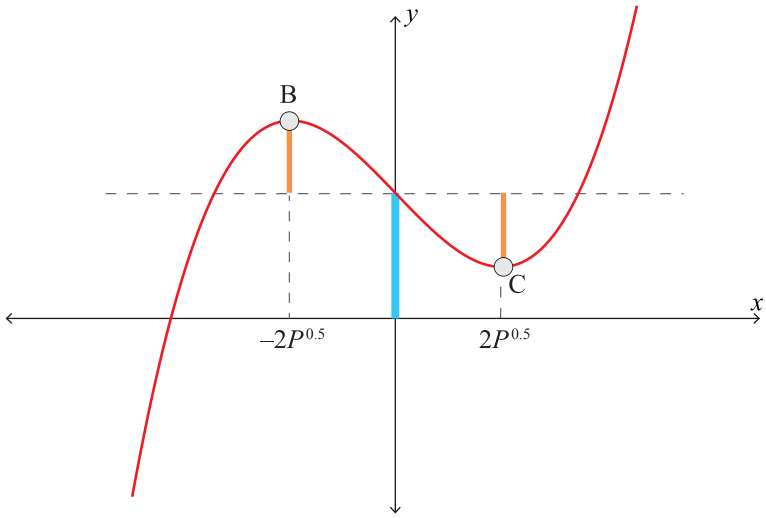
Mō ētahi pahekotanga o P me R, e toru ngā pātahitanga o te kōpiko i te tuaka-x. Ka pēnei mēnā he roa ake ia rārangi karaka i te rārangi kikorangi.

Whiriwhiria te āhuatanga me mātua whakatutuki i P me R mō te kauwhata kia whai haukotinga-x motuhake e toru.

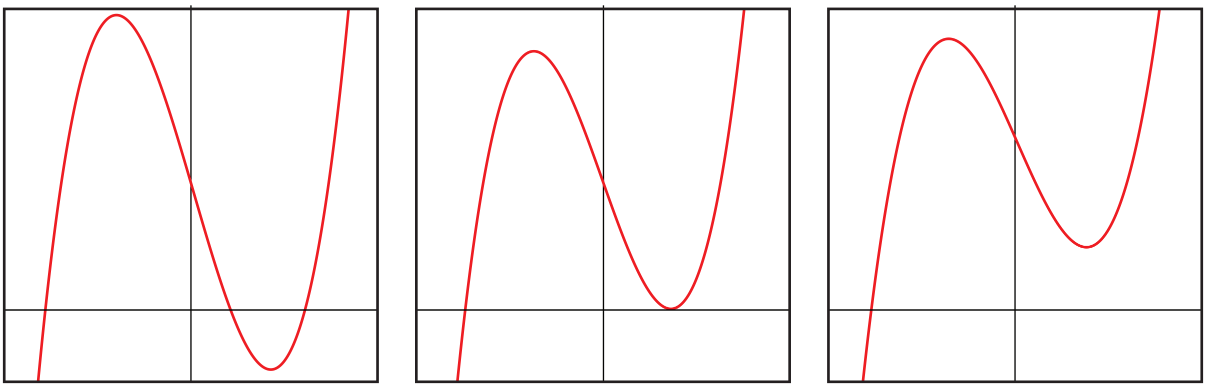
Tuhia tō whakautu ki te āhua ko ngā taupū o P me R he tau tōpū tōrunga.

MĀ TE KAIMĀKA ANAKE

(iii) Consider again the curve with the equation $y = x^3 - 12Px + R$. As the values of P and R vary, the shape of the curve changes, and the lengths of the orange lines and of the central blue line (below) vary. However, by symmetry, the two orange lines remain the same length as each other.



Some examples of the graphs obtained from various values of P and R are illustrated below.



For some combinations of P and R, the curve can intersect the x-axis three times. This will happen if each orange line is longer than the blue line.

Find the condition that P and R must satisfy for the graph to have three distinct x-intercepts.

Write your answer in a form where the exponents of P and R are both positive integers.

ASSESSOR'S USE ONLY

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUARUA

(a) Tuhia hei taupū kōaro ki te āhua māmā rawa: $\log(9y) + \log(4) - \log(3y)$.

(b) Whakaotia ia whārite e whai ake nei:

(i) $\log_x(36) = 2$.

(ii) $\log_5(x) + \log_5(2x) = 4$.

ASSESSOR'S USE ONLY

QUESTION TWO

(a) Write as a single logarithm in simplest form: $\log(9y) + \log(4) - \log(3y)$.

(b) Solve each of the following equations:

(i) $\log_x(36) = 2$.

(ii) $\log_5(x) + \log_5(2x) = 4$.

MĀ TE KAIMĀKA ANAKE

(c) Whakaotihia te whārite: $\frac{5x+4}{x+4} - \frac{3x-4}{2x+1} = 2$.

ASSESSOR'S USE ONLY

(c) Solve the equation: $\frac{5x+4}{x+4}-\frac{3x-4}{2x+1}=2$.

(d) Me whai whakaaro ki ngā unahi e rua:

- Ka tohua te Unahi Tuatahi mā te $y = ax^2 + bx + c$, ā,
- Ka tohua te Unahi Tuarua mā te $y = dx^2 + ex + c$, ina ko a, b, c, d, me e he tau pūmau.

Whakamahia te taurangi hei whiriwhiri i ngā whakatiki kei ngā uara o a, b, c, d, me e hei whakarite ka tūtaki ngā unahi e rua i ngā pūwāhi motuhake e rua.

MĀ TE KAIMĀKA ANAKE

(d) Consider two parabolas:

- Parabola One given by $y = ax^2 + bx + c$ and
- Parabola Two given by $y = dx^2 + ex + c$, where a, b, c, d, and e are constants.

Use algebra to determine the restrictions on the values of a, b, c, d, and e that would ensure that the parabolas meet at two distinct points.

ASSESSOR'S USE ONLY

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATORU

(a) Whakaotihia te whārite $3^{4x} = 30$.

(b) Me whai whakaaro ki te pānga $W = (x+2)^{\frac{2}{5}}$, ina ko x he **tauoti**.

(i) Me kī ko x te kaupapa o te ture $W = (x+2)^{\frac{2}{5}}$.

(ii) Mō ēhea uara o x ka iti iho ngā uara o te pānga i te 20?

QUESTION THREE

ASSESSOR'S USE ONLY

(a) Solve the equation $3^{4x} = 30$.

(b) Consider the function $W = (x+2)^{\frac{2}{5}}$, where x is a **whole number**.

(i) Make x the subject of the formula $W = (x+2)^{\frac{2}{5}}$.

(ii) For what values of x will the function have values less than 20?

MĀ TE KAIMĀKA ANAKE

(c) He hokohoko kōtui a Zahra. Ka kite a Zahra ka nui ake te utu o te kōtui, ka iti iho ngā kōtui ka hokona. Ka whakamātauhia e Zahra ki te whakapiki i te utu o tētahi kōtui mā te $2 i ia rā (ka tīmata atu i te $7), ā, ka tuhia e ia e hia ngā kōtui ka hokona e ia i ia rā. Koinei tana mahi mō te 6 rā, ā, ka kite ia ko te maha o ngā kōtui i hokona i ia rā i te tīmatanga he 98, ā, ka heke mā te 3 i ia rā.

Ko te rahinga o ngā moni i whiwhi i a Zahra i ia rā mō ngā kōtui, arā, ko ngā whiwhinga moni, kua mau ki te papatau i raro.

Rā, *d*	**1**	**2**	**3**	**4**	**5**	**6**
Utu o tētahi kōtui ($) = $2d + 5$	7	9	11	13	15	17
Maha o ngā kōtui i hokona = $101 - 3d$	98	95	92	89	86	83
Whiwhinga moni ($)	686	855	1012	1157	1290	1411

(i) Mēnā ka noho tika tonu ngā tauira katoa, he rā anō e rite pū ana ngā whiwhinga moni ki te $445? Whakamahia te taurangi hei parahau i tō whakautu me te whakamārama i ō whakataunga.

(c) Zahra sells zips. Zahra notices that the higher the price of a zip, the fewer zips are sold. As an experiment, Zahra increases the price of a zip by \$2 each day (starting at \$7) and keeps a record of how many zips are sold each day. She does this for 6 days and finds that the number of zips sold each day started at 98 and is dropping by 3 each day.

The total amount of money Zahra received each day for zips, the turnover, is also recorded in the table below.

Day, d	**1**	**2**	**3**	**4**	**5**	**6**
Price of a zip (\$) = $2d + 5$	7	9	11	13	15	17
Number of zips sold = $101 - 3d$	98	95	92	89	86	83
Turnover (\$)	686	855	1012	1157	1290	1411

(i) If all the patterns continue to be valid, is there any day on which the turnover is exactly \$445? Use algebra to justify your answer and explain your conclusions.

(ii) Ka kite a Zahra ehara ia tauoti i te uara whiwhinga moni mō tētahi rā.

Mā te whakamahi i te taurangi, whiriwhiria kia toru ngā āhuatanga i te iti rawa me tutuki i tētahi tauoti *k* kia noho ai pea hei whiwhinga moni i tētahi rā.

MĀ TE KAIMĀKA ANAKE

ASSESSOR'S USE ONLY

(ii) Zahra realises that not every whole number is a possible turnover value for a given day.

Using algebra, find at least three conditions a whole number *k* must satisfy for it to be a possible turnover for a given day.

He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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English translation of the wording on the front cover

Level 2 Mathematics and Statistics 2020

91261 Apply algebraic methods in solving problems

9.30 a.m. Thursday 19 November 2020
Credits: Four

91261M

Achievement	Achievement with Merit	Achievement with Excellence
Apply algebraic methods in solving problems.	Apply algebraic methods, using relational thinking, in solving problems.	Apply algebraic methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHMF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

You are required to show algebraic working in this paper. Guess-and-check methods or correct answer(s) only, will generally limit grades to Achievement.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.