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91577M

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

SUPERVISOR'S USE ONLY

Tohua tēnei pouaka mēnā kāore he tuhituhi i roto i tēnei pukapuka

Tuanaki, Kaupae 3, 2020

91577M Te whakahāngai i te taurangi o ngā tau matatini hei whakaoti rapanga

9.30 i te ata Rāhina 23 Whiringa-ā-rangi 2020
Whiwhinga: Rima

Paetae	Kaiaka	Kairangi
Te whakahāngai i te taurangi o ngā tau matatini hei whakaoti rapanga.	Te whakahāngai i te taurangi o ngā tau matatini mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai i te taurangi o ngā tau matatini mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tuhia ō mahinga KATOA.

Tirohia mēnā kei a koe te pukapuka Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Mēnā ka hiahia whārangi atu anō koe mō ō tuhinga, whakamahia te (ngā) whārangi wātea kei muri o tēnei pukapuka, ka āta tohu ai i te tau tūmahi.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–15 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATAHI

MĀ TE KAIMĀKA ANAKE

(a) Mēnā ko $s = 2 + 3\mathrm{i}$, ā, ko $t = 3 + k\mathrm{i}$, whiriwhiria te uara o k ina $st = 21 - \mathrm{i}$.

(b) Whiriwhiria te (ngā) uara o r kia kotahi anake te otinga o te whārite $x^2 + 4rx + r = 0$.

(c) Whakaotia te whārite e whai ake mō x e pā ana ki g.

$$2\sqrt{x} - 5 = \sqrt{4x - g}$$

QUESTION ONE

ASSESSOR'S USE ONLY

(a) If $s = 2 + 3\mathrm{i}$ and $t = 3 + k\mathrm{i}$, find the value of k if $st = 21 - \mathrm{i}$.

(b) Find the value(s) of r such that the equation $x^2 + 4rx + r = 0$ has only one solution.

(c) Solve the following equation for x in terms of g.

$$2\sqrt{x} - 5 = \sqrt{4x - g}$$

(d) Tuhia $\dfrac{k+k\mathrm{i}}{1-\mathrm{i}}+\dfrac{2k}{1+\mathrm{i}}$ ki tana āhua tino māmā.

(e) Mēnā ko $T=\dfrac{a-b\mathrm{i}}{a+b\mathrm{i}}$, ā, he tau pūmau tūturu a me b, hāponotia ko $\dfrac{1+T^2}{2T}=\dfrac{a^2-b^2}{a^2+b^2}$.

MĀ TE KAIMĀKA ANAKE

ASSESSOR'S USE ONLY

(d) Write $\frac{k+k\text{i}}{1-\text{i}}+\frac{2k}{1+\text{i}}$ in its simplest possible form.

(e) Given that $T=\frac{a-b\text{i}}{a+b\text{i}}$, where a and b are real constants, prove that $\frac{1+T^2}{2T}=\frac{a^2-b^2}{a^2+b^2}$.

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUARUA

(a) Mēnā ko $x - 2$ he pānga o $2x^3 + qx^2 - 17x - 10$, whiriwhiria te uara o q.

(b) Whiriwhiria ngā uara katoa o k ka taea ina ko $|5 + 3k\mathrm{i}| = 13$.

(c) Ko tētahi o ngā otinga o $2z^3 - 15z^2 + bz - 30 = 0$ ko $z = 3 + \mathrm{i}$ (ko b he tau tūturu).

Whiriwhiria ngā otinga kē, me te uara o b.

QUESTION TWO

ASSESSOR'S USE ONLY

(a) Given that $x - 2$ is a factor of $2x^3 + qx^2 - 17x - 10$, find the value of q.

(b) Find all possible values of k given that $|5 + 3k\mathrm{i}| = 13$.

(c) One of the solutions of $2z^3 - 15z^2 + bz - 30 = 0$ is $z = 3 + \mathrm{i}$ (b is a real number).

Find the other solutions, and the value of b.

MĀ TE KAIMĀKA ANAKE

(d) Mēnā ko $u = p + p\mathrm{i}$, ā, ko $v = -q + q\mathrm{i}$, ina p me q he tau pūmau tūturu tōrunga, whiriwhiria a $\arg\left(\frac{u}{v}\right)$.

(e) Whiriwhiria te whārite ā-taunga tukutuku (Cartesian equation) o te huanui e whakaahuahia ana e $|z + \mathrm{i}|^2 + |z - \mathrm{i}|^2 = 10$.

Tuhia tō otinga ki te āhua $x^2 + y^2 = k$.

(d) Given that $u = p + p\,\text{i}$ and $v = -q + q\,\text{i}$, where p and q are both positive real constants, find $\arg\left(\dfrac{u}{v}\right)$.

(e) Find the Cartesian equation of the locus described by $|z + \text{i}|^2 + |z - \text{i}|^2 = 10$.
Write your solution in the form $x^2 + y^2 = k$.

ASSESSOR'S USE ONLY

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATORU

(a) Mēnā ko $u = 12k^3\text{cis}(\pi)$ me $v = 2k\text{cis}\left(\frac{\pi}{3}\right)$, tuhia te tino uara o $\frac{u}{v}$ ki te āhua ahuroa.

(b) Ina ko $z = 5 - \text{i}$ ā, $w = -2 + 3\text{i}$, whakaaturia ko $|z|^2 = 2|w|^2$.

(c) Mēnā ko $z = a + b\text{i}$, ina a me b he tau tūturu ehara i te kore, whakaaturia ko $\frac{z\bar{z}}{z+\bar{z}}$ he tau tūturu.

QUESTION THREE

ASSESSOR'S USE ONLY

(a) If $u = 12k^3\text{cis}(\pi)$ and $v = 2k\text{cis}\left(\dfrac{\pi}{3}\right)$, write the exact value of $\dfrac{u}{v}$ in polar form.

(b) If $z = 5 - \text{i}$ and $w = -2 + 3\text{i}$, show that $|z|^2 = 2|w|^2$.

(c) Given that $z = a + b\text{i}$, where a and b are non-zero real numbers, show that $\dfrac{z\bar{z}}{z+\bar{z}}$ is a real number.

MĀ TE KAIMĀKA ANAKE

(d) Whakaotia te whārite $z^4 = -16k^8$, ina ko k he tau pūmau tūturu.

Tuhia ō otinga ki te āhua ahuroa e pā ana ki k.

(e) Mō ngā tau matatini u me v, hāponotia mēnā ko $|u + v| = |u - v|$, kāti he pohewa noa iho te $\dfrac{u}{v}$.

(d) Solve the equation $z^4 = -16k^8$, where k is a real constant.

Give your solutions in polar form in terms of k.

(e) For complex numbers u and v, prove that if $|u + v| = |u - v|$, then $\frac{u}{v}$ is purely imaginary.

ASSESSOR'S USE ONLY

He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

English translation of the wording on the front cover

Level 3 Calculus 2020

91577 Apply the algebra of complex numbers in solving problems

9.30 a.m. Monday 23 November 2020
Credits: Five

91577M

Achievement	Achievement with Merit	Achievement with Excellence
Apply the algebra of complex numbers in solving problems.	Apply the algebra of complex numbers, using relational thinking, in solving problems.	Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–15 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.