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91578M

SUPERVISOR'S USE ONLY

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Tohua tēnei pouaka mēnā kāore he tuhituhi i roto i tēnei pukapuka

Tuanaki, Kaupae 3, 2020

91578M Te whakahāngai i ngā tikanga pārōnaki hei whakaoti rapanga

9.30 i te ata Rāhina 23 Whiringa-ā-rangi 2020
Whiwhinga: Ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pārōnaki hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pārōnaki mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pārōnaki mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tuhia ō mahinga KATOA.

Tirohia mēnā kei a koe te pukapuka Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Mēnā ka hiahia whārangi atu anō koe mō ō tuhinga, whakamahia te (ngā) whārangi wātea kei muri o tēnei pukapuka, ka āta tohu ai i te tau tūmahi.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–23 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATAHI

(a) Kimihia te pārōnaki mō $y = \left(3x - x^2\right)^5$.

Hei aha noa te whakarūnā i ō whakautu.

(b) Whiriwhiria te rōnaki o te pātapa ki te kōpiko $y = 3\sin 2x + \cos 2x$ i te pūwāhi ko $x = \frac{\pi}{4}$.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

(c) Whiriwhiria te uara o x, he pūwāhi tūnoa tō te kauwhata o te pānga $y = \frac{x}{1 + \ln x}$.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

QUESTION ONE

ASSESSOR'S USE ONLY

(a) Differentiate $y = \left(3x - x^2\right)^5$.

You do not need to simplify your answer.

(b) Find the gradient of the tangent to the curve $y = 3\sin 2x + \cos 2x$ at the point where $x = \frac{\pi}{4}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) Find the value of x for which the graph of the function $y = \frac{x}{1 + \ln x}$ has a stationary point.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) Ko te whārite o te kōpiko ko $y = x^2 \cos x$.

Whakaaturia ko te whārite o te pātapa ki te kōpiko i te pūwāhi $\left(\pi, -\pi^2\right)$ ko

$y + 2\pi x = \pi^2$

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

MĀ TE KAIMĀKA ANAKE

(d) A curve has the equation $y = x^2 \cos x$.

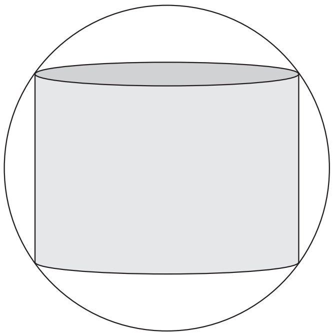
Show that the equation of the tangent to the curve at the point $\left(\pi, -\pi^2\right)$ is

$$y + 2\pi x = \pi^2$$

You must use calculus and show any derivatives that you need to find when solving this problem.

ASSESSOR'S USE ONLY

(e) Kua whakairohia tētahi rango he *h* te teitei me te pūtoro *r*, kei roto i tētahi poi me te pūtoro o te 20 cm, e ai ki te whakaahua.



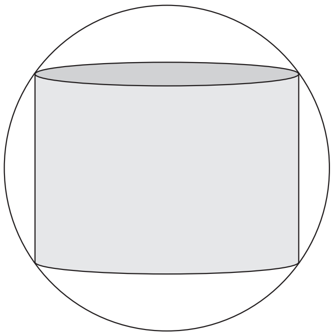
Whiriwhiria te rōrahi mōrahi o te rango ka taea.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

Kāore he tikanga kia hāponotia e koe he mōrahi te rōrahi i tātaihia.

MĀ TE KAIMĀKA ANAKE

(e) A cylinder of height h and radius r is inscribed, as shown to the right, inside a sphere of radius 20 cm.



Find the maximum possible volume of the cylinder.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not need to prove that the volume you have found is a maximum.

ASSESSOR'S USE ONLY

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUARUA

(a) Kimihia te pārōnaki mō $y = \dfrac{\tan x}{x^3}$.

Hei aha noa te whakarūnā i ō whakautu.

(b) Ka whakatauirahia te uara o tētahi waka mā te tikanga tātai

$$V = 17\,000\,e^{-0.25t} + 2\,000\,e^{-0.5t} + 500 \text{ mō } 0 \leq t \leq 20$$

ina ko V te uara o te waka ki ngā tāra ($), ā, ko t te tawhito o te waka ki ngā tau.

Tātaihia te pāpātanga e huri ai te uara o te waka ina eke ki te 8 tau te tawhito.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

ASSESSOR'S USE ONLY

QUESTION TWO

(a) Differentiate $y = \dfrac{\tan x}{x^3}$.

You do not need to simplify your answer.

(b) The value of a car is modelled by the formula

$$V = 17\,000\,\mathrm{e}^{-0.25t} + 2\,000\,\mathrm{e}^{-0.5t} + 500 \quad \text{for } 0 \leq t \leq 20$$

where V is the value of the car in dollars (\$), and t is the age of the car in years.

Calculate the rate at which the value of the car is changing when it is 8 years old.

You must use calculus and show any derivatives that you need to find when solving this problem.

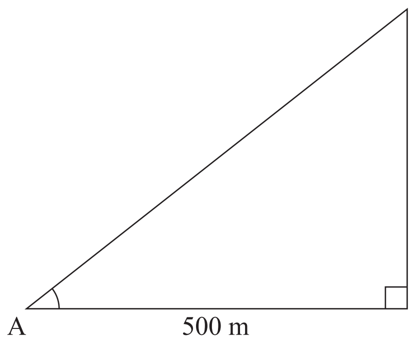
MĀ TE KAIMĀKA ANAKE

(c) Whiriwhiria ngā taunga-x o ngā pūwāhi tūnoa ki te kauwhata o te pānga

$$f(x) = (2x-3)e^{x^2+k}$$

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

(d) Ka whakarewatia poutūtia tētahi tākirirangi. Ko tōna teitei i runga ake i te pūwāhi whakarewa ka tukuna mā te tikanga tātai $h(t) = 4.8t^2$, ina ko h te teitei ā-mita, ā, ko t te wā ā-hēkona mai i te tukutanga.



Kei te mātakitaki tētahi kaimātakitaki i te pūwāhi A i te tākirirangi. E ōrite ana tōna taumata ki te pūwāhi whakarewa o te tākirirangi, me te 500 m mai i te pūwāhi whakarewa.

Kimihia te pāpātanga e piki ai te koki whakarewa o te tākirirangi i A ina eke te tākirirangi ki te 480 m i runga ake o te pūwāhi whakarewa.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

(e) Ka tautuhia tētahi kōpiko mā ngā whārite tawhā $x = \ln(t)$ me $y = 6t^3$ ina ko $t > 0$.

Kei te kōpiko te pūwāhi P, ā, ki te pūwāhi P, $\frac{d^2y}{dx^2} = 2$.

Kimihia ngā tino taunga o te pūwāhi P.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

ASSESSOR'S USE ONLY

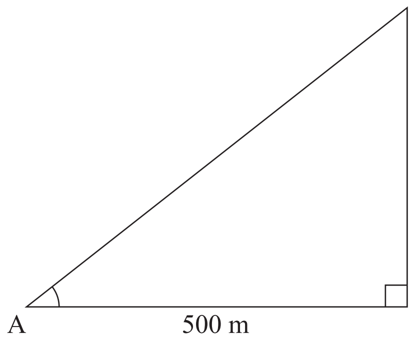
(c) Find the x-coordinates of any stationary points on the graph of the function

$$f(x) = (2x - 3)e^{x^2 + k}$$

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) A rocket is fired vertically upwards. Its height above the launch point is given by the formula $h(t) = 4.8t^2$, where h is the height in metres, and t is the time in seconds from firing.

www.airspacemag.com/as-next/milestone-180968351/



An observer at point A is watching the rocket. She is at the same level as the launch point of the rocket, and 500 m from the launch point.

Find the rate at which the angle of elevation at A of the rocket is increasing when the rocket is 480 m above the launch point.

You must use calculus and show any derivatives that you need to find when solving this problem.

ASSESSOR'S USE ONLY

(e) A curve is defined by the parametric equations $x = \ln(t)$ and $y = 6t^3$ where $t > 0$.

The point P lies on the curve, and at point P, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2$.

Find the exact coordinates of point P.

You must use calculus and show any derivatives that you need to find when solving this problem.

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATORU

(a) Kimihia te pārōnaki mō $y = 3\ln\left(x^2 - 1\right)$.

Hei aha noa te whakarūnā i ō whakautu.

(b) Mō tēhea, ēhea uara rānei o x ko te pātapa ki te kauwhata o te pānga $f(x) = 2x - 2\sqrt{x},\ x > 0$, he 1 te rōnaki?

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

ASSESSOR'S USE ONLY

QUESTION THREE

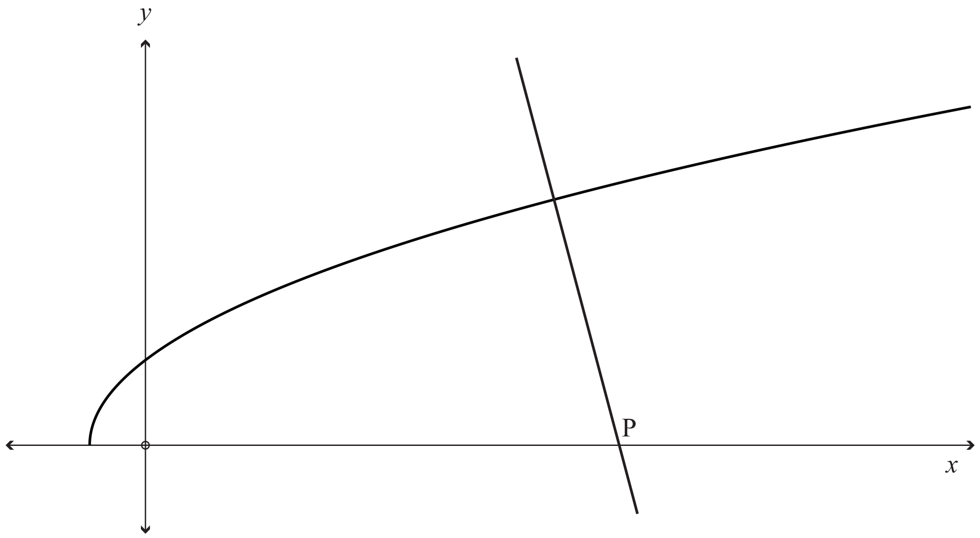
(a) Differentiate $y = 3\ln\left(x^2 - 1\right)$.

You do not need to simplify your answer.

(b) For what value(s) of x does the tangent to the graph of the function $f(x) = 2x - 2\sqrt{x},\ x > 0$, have a gradient of 1?

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) Ko te rārangi hāngai ki te kauwhata o te pānga $y = \sqrt{2x+1}$ i te pūwāhi $(4,3)$ ka pūtahi i te taunga-x i te pūwāhi P.

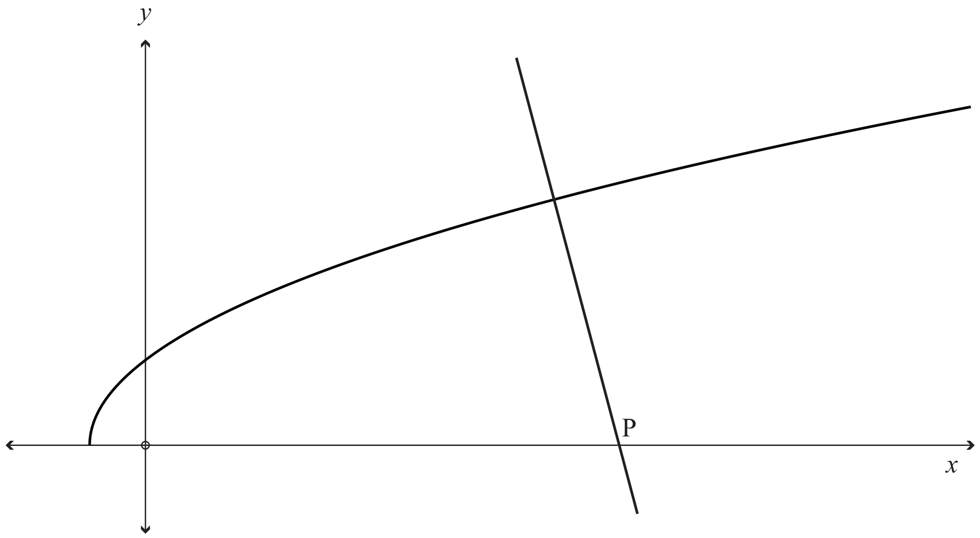


Whiriwhiria te taunga-x o te pūwāhi P.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

ASSESSOR'S USE ONLY

(c) The normal to the graph of the function $y = \sqrt{2x+1}$ at the point $(4,3)$ intersects the x-axis at point P.



Find the x-coordinate of point P.

You must use calculus and show any derivatives that you need to find when solving this problem.

MĀ TE KAIMĀKA ANAKE

(d) E rua ngā pūwāhi tūnoa kei te kauwhata o te pānga $y = \frac{1}{x-3} + x,\ x \neq 3$.

Whiriwhiria ngā taunga-x o ngā pūwāhi tūnoa, ka whakatau mēnā he tihi pātata (local maxima), he whāruarua pātata (local minima) rānei.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

Ka haere tonu te Tūmahi Tuatoru i te whārangi 20.

ASSESSOR'S USE ONLY

(d) The graph of the function $y = \dfrac{1}{x-3} + x,\ \ x \neq 3$, has two stationary points.

Find the x-coordinates of the stationary points, and determine whether they are local maxima or local minima.

You must use calculus and show any derivatives that you need to find when solving this problem.

Question Three continues on page 21.

(e) Ko te whārite o tētahi kōpiko he $y=(3x+2)e^{-2x}$.

Hāponotia ko $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=0$.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

MĀ TE KAIMĀKA ANAKE

(e) A curve has the equation $y=(3x+2)e^{-2x}$.

Prove that $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=0$.

You must use calculus and show any derivatives that you need to find when solving this problem.

ASSESSOR'S USE ONLY

He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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English translation of the wording on the front cover

Level 3 Calculus 2020

91578M Apply differentiation methods in solving problems

9.30 a.m. Monday 23 November 2020
Credits: Six

91578M

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–23 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.