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Level 2 Mathematics and Statistics 2021

91261 Apply algebraic methods in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply algebraic methods in solving problems.	Apply algebraic methods, using relational thinking, in solving problems.	Apply algebraic methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–14 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION ONE

(a) Simplify each expression, leaving your answer with positive indices.

(i) $\dfrac{(3y)^4}{3y^{-1}}$

(ii) $\sqrt[3]{8y^{27}}$

(b) A quadratic equation has solutions of $x = -\dfrac{2}{3}$ and $x = 4$.

Find the original equation, giving your answer in the form of $ax^2 + bx + c = 0$, where a, b, and c are whole numbers.

(c) Consider a quadratic equation in the form $x^2 - 3kx + 2k^2 = 0$, where k is a non-zero constant.

Show that one solution is twice the other solution.

(d) Consider the following two curves:

$x^2 = y^2 + 1$ and $y = (x - 1)(x + 1) - 2$

Find the co-ordinates of each intersection point of the two curves.

QUESTION TWO

(a) Simplify: $\dfrac{x^2 - x - 12}{4x + 12}$

(b) Write $\dfrac{5x}{x-3} - \dfrac{x-4}{x+2}$ as a single fraction in its simplest form.

(c) Jessica is investigating a compounding investment. She wants to know how long it would take for an investment of \$1000 to double in value to \$2000. She forms the following equation:

$$2000 = 1000\left(1+\frac{R}{100}\right)^D$$

where R is the rate of return on the investment, as a percentage, and

D is the time that the investment would take to double in value, in years.

(i) If an investment takes 11 years to double in value, what is its rate of return?

(ii) By making D the subject of the expression: $2000 = 1000\left(1+\frac{R}{100}\right)^D$,

show that $D = \dfrac{\log(2)}{\log(1+\frac{R}{100})}$

In her research, Jessica comes across a simple but **approximate** rule for calculating D, the time that the investment would take to double in value. It is commonly called the 'Rule of 72', and it states that:

$$D = \frac{72}{R}$$

Jessica wonders how close the values of D from the 'Rule of 72' are to those calculated using the actual expression, which is:

$$D = \frac{\log(2)}{\log(1+\frac{R}{100})}$$

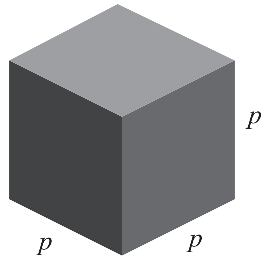
(iii) Show clearly that the value of R for which the 'Rule of 72' exactly calculates D, is the solution to the equation:

$$2^R - \left(1+\frac{R}{100}\right)^{72} = 0$$

You do not need to solve this equation.

QUESTION THREE

Consider a cube with sides of p cm (where $p \neq 0$). The volume of the cube would be p^3 cm^3, and the surface area of the cube would be $6p^2$ cm^2.



Junyang wonders if it is possible to change the dimensions of the cube to make a cuboid that still has the same volume.

(a) First, he tries doubling the length, halving the width, and keeping the height as p, as sketched below.

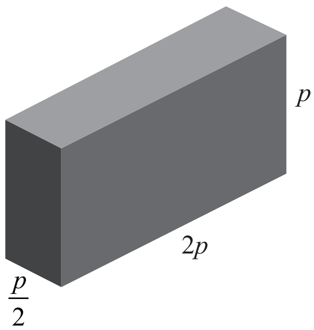
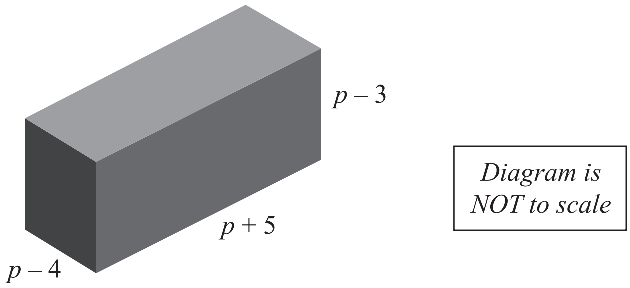


Diagram is NOT to scale

Using algebra, find the volume of this cuboid.

(b) The volume of the cuboid below is given by the expression: $(p - 4)(p + 5)(p - 3)$.

Expand and simplify this expression.



(c) Next, Junyang tries adding an amount, a, to the length and then taking off the same amount from the width, keeping the height the same (see below).

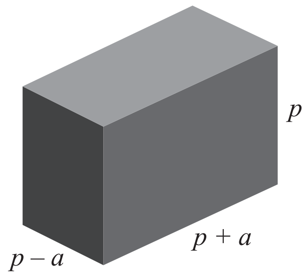


Diagram is NOT to scale

Are there any values of a for which the volume of the cuboid will be the same as the volume of the cube?

(d) Junyang realises that the **surface area** of the cuboid will not be the same as the surface area of the cube unless he also changes the height. He decides to make the height of the cuboid 10 cm.

He wants to find out which value of p would result in the cube having the same surface area as the cuboid. To do this, he needs to form and solve an equation for p.

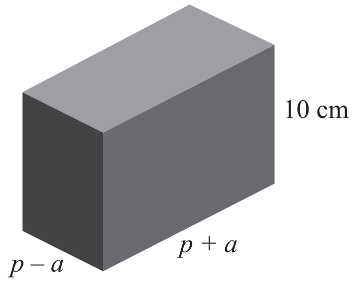


Diagram is NOT to scale

(i) If the surface area of the cube is the same as the surface area of the cuboid, show that $2p^2 - 20p + a^2 = 0$.

Remember that the surface area of the cube is $6p^2$ cm^2.

Please turn over ➤

(ii) As mentioned in part (i), if the surface area of the cube is the same as the surface area of the cuboid, then $2p^2 - 20p + a^2 = 0$.

By using the discriminant (Δ), find the largest possible **whole** number value that *a* could take in this context.

Use this value of *a* to find the dimensions of both the cube and the cuboid.

Explain your reasoning clearly.

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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