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91261M

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

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Tohua tēnei pouaka mēnā KĀORE koe i tuhituhi i roto i tēnei pukapuka

Te Pāngarau me te Tauanga, Kaupae 2, 2021

91261M Te whakamahi tikanga taurangi hei whakaoti rapanga

Ngā whiwhinga: Whā

Paetae	Kaiaka	Kairangi
Te whakamahi tikanga taurangi hei whakaoti rapanga.	Te whakamahi tikanga taurangi mā te whakaaro tūhonohono hei whakaoti rapanga.	Te whakamahi tikanga taurangi mā te whakaaro waitara hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tirohia mēnā kei a koe te Puka Tikanga Tātai L2–MATHMF.

Tuhia ō mahinga KATOA.

Ki te hiahia koe ki ētahi atu wāhi hei tuhituhi whakautu, whakamahia te wāhi wātea kei muri i te pukapuka nei.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2-25 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi ki roto i tētahi wāhi kauruku whakahāngai (). Ka tapahia pea tēnei wāhi ina mākahia te pukapuka.

ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

TŪMAHI TUATAHI

(a) Whakarūnāhia ia kīanga, ā, kia tōrunga ngā taupū i tō urupare.

(i) $\dfrac{(3y)^4}{3y^{-1}}$

(ii) $\sqrt[3]{8y^{27}}$

(b) He otinga kei tētahi whārite pūrua o te $x = -\dfrac{2}{3}$ me te $x = 4$.

Kimihia te whārite taketake, ka tuhi i tō whakautu ki te āhua o te $ax^2 + bx + c = 0$, ina ko a, b, me c he tauoti.

QUESTION ONE

(a) Simplify each expression, leaving your answer with positive indices.

(i) $\dfrac{(3y)^4}{3y^{-1}}$

(ii) $\sqrt[3]{8y^{27}}$

(b) A quadratic equation has solutions of $x = -\dfrac{2}{3}$ and $x = 4$.

Find the original equation, giving your answer in the form of $ax^2 + bx + c = 0$, where a, b, and c are whole numbers.

(c) Me kī ko tētahi whārite pūrua kei te āhua $x^2 - 3kx + 2k^2 = 0$, ina ko k he tau pūmau ehara i te kore.

Me whakaatu he huarua ake tētahi otinga i tētahi atu.

(c) Consider a quadratic equation in the form $x^2 - 3kx + 2k^2 = 0$, where k is a non-zero constant.

Show that one solution is twice the other solution.

(d) Whakaaroarohia ngā kōpiko e rua e whai ake nei:

$x^2 = y^2 + 1$ me $y = (x - 1)(x + 1) - 2$

Whiriwhiria ngā taunga o ia pūwāhi pūtahi o ngā kōpiko e rua.

(d) Consider the following two curves:

$x^2 = y^2 + 1$ and $y = (x - 1)(x + 1) - 2$

Find the co-ordinates of each intersection point of the two curves.

TŪMAHI TUARUA

(a) Whakarūnāhia: $\dfrac{x^2 - x - 12}{4x + 12}$

(b) Tuhia $\dfrac{5x}{x-3} - \dfrac{x-4}{x+2}$ hei hautanga kotahi ki tōna āhua rūnā rawa atu.

QUESTION TWO

(a) Simplify: $\dfrac{x^2 - x - 12}{4x + 12}$

(b) Write $\dfrac{5x}{x-3} - \dfrac{x-4}{x+2}$ as a single fraction in its simplest form.

(c) Kei te tūhura a Jessica i tētahi haumi whakaputu. Kei te hiahia ia ki te mōhio e hia te roa ka huarua te uara o tētahi haumitanga o te \$1000 ki te \$2000. Ka hanga ia i te whārite e whai ake nei:

$$2000 = 1000\left(1 + \frac{R}{100}\right)^{D}$$

ina ko R te pāpātanga whakahokinga huamoni mō te haumi, hei ōrau, ā, ko

D te wā ā-tau e huarua ai te uara o te haumitanga.

(i) Mēnā 11 tau te roa e huarua ai tētahi haumitanga, he aha te pāpātanga whakahokinga huamoni?

(ii) Mā te whakarite ko D te tumu o te kīanga: $2000 = 1000\left(1 + \frac{R}{100}\right)^{D}$,

whakaaturia ko $D = \dfrac{\log(2)}{\log(1 + \frac{R}{100})}$

(c) Jessica is investigating a compounding investment. She wants to know how long it would take for an investment of \$1000 to double in value to \$2000. She forms the following equation:

$$2000 = 1000\left(1+\frac{R}{100}\right)^{D}$$

where R is the rate of return on the investment, as a percentage, and

D is the time that the investment would take to double in value, in years.

(i) If an investment takes 11 years to double in value, what is its rate of return?

(ii) By making D the subject of the expression: $2000 = 1000\left(1+\frac{R}{100}\right)^{D}$,

show that $D = \dfrac{\log(2)}{\log(1+\frac{R}{100})}$

I roto i tana rangahau, ka kitea e Jessica tētahi ture māmā, **āwhiwhi** hoki mō te tātai i a *D*, ko te roa o te wā ka huarua te uara. E mōhio whānuitia ana ko te 'Ture o te 72', ā, e kī ana:

$$D = \frac{72}{R}$$

E whakaaroaro ana a Jessica e hia te tata o ngā uara o *D* mai i te 'Ture o te 72' ki ērā kua tātaihia mā te whakamahi i te kīanga ake:

$$D = \frac{\log(2)}{\log(1+\frac{R}{100})}$$

(iii) Kia mārama te whakaatu ko te uara o *R* e tika ai te tātai a te 'Ture o te 72' i te *D*, te otinga o te whārite:

$$2^R - \left(1+\frac{R}{100}\right)^{72} = 0$$

Hei aha te whakaoti i tēnei whārite.

In her research, Jessica comes across a simple but **approximate** rule for calculating *D*, the time that the investment would take to double in value. It is commonly called the 'Rule of 72', and it states that:

$$D = \frac{72}{R}$$

Jessica wonders how close the values of *D* from the 'Rule of 72' are to those calculated using the actual expression, which is:

$$D = \frac{\log(2)}{\log(1 + \frac{R}{100})}$$

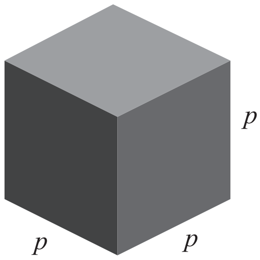
(iii) Show clearly that the value of *R* for which the 'Rule of 72' exactly calculates *D*, is the solution to the equation:

$$2^R - \left(1 + \frac{R}{100}\right)^{72} = 0$$

You do not need to solve this equation.

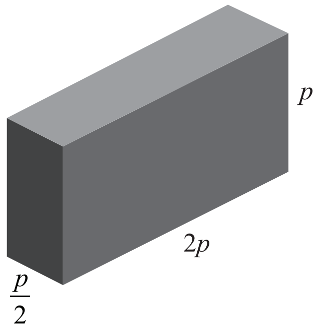
TŪMAHI TUATORU

Me kī ko tētahi mataono rite he p cm ngā tapa (ina $p \neq 0$). Ko p^3 cm^3 te rōrahi o te mataono rite, ā, ko $6p^2$ cm^2 te horahanga o te mataono rite.



E whakaaroaro ana a Junyang mēnā ka taea te huri ngā rahinga o te mataono rite hei hanga i tētahi poro-tapawhā hāngai e ōrite tonu ai te rōrahi.

(a) Tuatahi, ka ngana ia ki te whakahuarua i te roa, ka weherua i te whānui, ka puritia te teitei ko p, e ai ki te tātuhinga i raro.

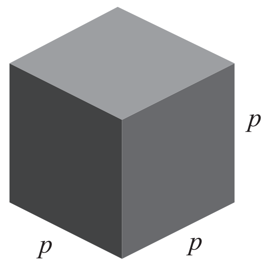


KĀORE i tuhi ā-āwhatatia tēnei hoahoa

Mā te whakamahi i ngā tikanga taurangi, tātaihia te rōrahi o taua poro-tapawhā hāngai.

QUESTION THREE

Consider a cube with sides of p cm (where $p \neq 0$). The volume of the cube would be p^3 cm^3, and the surface area of the cube would be $6p^2$ cm^2.



Junyang wonders if it is possible to change the dimensions of the cube to make a cuboid that still has the same volume.

(a) First, he tries doubling the length, halving the width, and keeping the height as p, as sketched below.

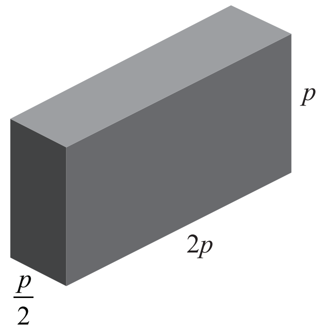
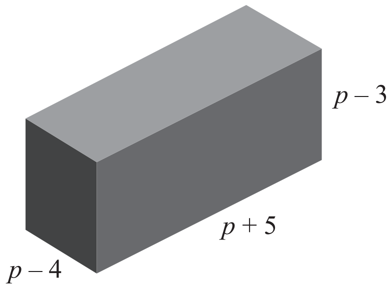


Diagram is NOT to scale

Using algebra, find the volume of this cuboid.

(b) Ka whakaaturia te rōrahi o te poro-tapawhā hāngai i raro nei mā te kīanga: $(p - 4)(p + 5)(p - 3)$.

Whakarohaina, whakarūnāhia hoki taua kīanga.



KĀORE i tuhi ā-āwhatatia tēnei hoahoa

(b) The volume of the cuboid below is given by the expression: $(p - 4)(p + 5)(p - 3)$.

Expand and simplify this expression.

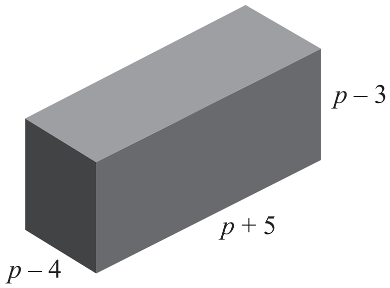
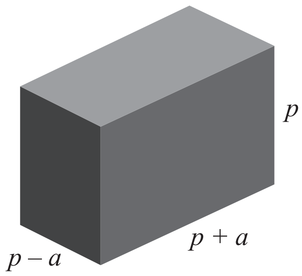


Diagram is NOT to scale

(c) I muri mai, ka whakamātau a Junyang ki te tāpiri i tētahi rahinga, *a*, ki te roa me te tango i te rahinga ōrite mai i te whānui, kia ōrite tonu ai te teitei (tirohia i raro).



KĀORE i tuhi ā-āwhatatia tēnei hoahoa

He uara anō ō *a* ko te rōrahi o te poro-tapawhā hāngai he ōrite ki te rōrahi o te mataono rite?

(c) Next, Junyang tries adding an amount, a, to the length and then taking off the same amount from the width, keeping the height the same (see below).

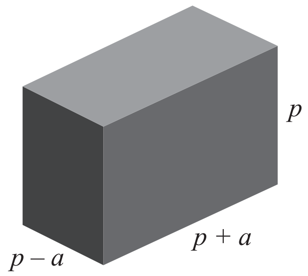
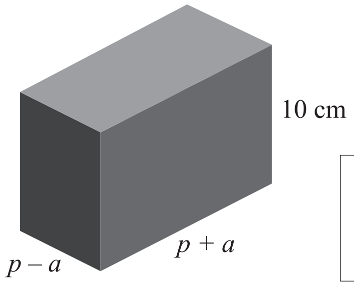


Diagram is NOT to scale

Are there any values of a for which the volume of the cuboid will be the same as the volume of the cube?

(d) Ka kite a Junyang kāore e ōrite te **horahanga mata** o te poro-tapawhā hāngai ki te horahanga mata o te mataono rite, engari ia mēnā ka whakarerekēhia e ia te teitei. Ka whakaritea e ia ko te teitei o te poro-tapawhā hāngai kia 10 cm.

Kei te hiahia ia ki te mōhio ko tēhea te uara o p e ōrite ai te horahanga mata o te mataono rite ki tō te poro-tapawhā hāngai. Kia oti ai tēnei, me hanga ia me te whakaoti i tētahi whārite mō p.



KĀORE i tuhi ā-āwhatatia tēnei hoahoa

(i) Mēnā he ōrite te horahanga mata o te mataono rite ki te horahanga mata o te poro-tapawhā hāngai, me whakaatu ko $2p^2 - 20p + a^2 = 0$.

Kia maumahara ko te horahanga mata o te mataono rite he $6p^2$ cm^2.

(d) Junyang realises that the **surface area** of the cuboid will not be the same as the surface area of the cube unless he also changes the height. He decides to make the height of the cuboid 10 cm.

He wants to find out which value of p would result in the cube having the same surface area as the cuboid. To do this, he needs to form and solve an equation for p.

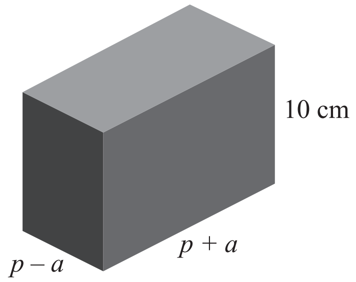


Diagram is NOT to scale

(i) If the surface area of the cube is the same as the surface area of the cuboid, show that $2p^2 - 20p + a^2 = 0$.

Remember that the surface area of the cube is $6p^2$ cm^2.

(i) E ai ki te wāhanga (i), mēnā he ōrite te horahanga mata o te mataono rite ki te horahanga mata o te poro-tapawhā hāngai, kāti ko $2p^2 - 20p + a^2 = 0$.

Mā te whakamahi i te kīanga whakawā (discriminant) (Δ), tātaihia te uara **tauoti** rahi rawa ka taea e a te whakaatu i tēnei horopaki.

Whakamahia tēnei uara o a hei tātai i ngā rahinga o te mataono rite me te poro-tapawhā hāngai.

Āta whakamāramahia ō whakaaro whaitake.

(ii) As mentioned in part (i), if the surface area of the cube is the same as the surface area of the cuboid, then $2p^2 - 20p + a^2 = 0$.

By using the discriminant (Δ), find the largest possible **whole** number value that *a* could take in this context.

Use this value of *a* to find the dimensions of both the cube and the cuboid.

Explain your reasoning clearly.

He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

English translation of the wording on the front cover

Level 2 Mathematics and Statistics 2021

91261M Apply algebraic methods in solving problems

Credits: Four

91261M

Achievement	Achievement with Merit	Achievement with Excellence
Apply algebraic methods in solving problems.	Apply algebraic methods, using relational thinking, in solving problems.	Apply algebraic methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHMF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–25 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.