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91267



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# Level 2 Mathematics and Statistics 2021 91267 Apply probability methods in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability methods in solving problems.	Apply probability methods, using relational thinking, in solving problems.	Apply probability methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

## You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2-MATHF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–15 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (
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). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

### **QUESTION ONE**

New Zealand is well known for its café culture, and the availability of coffee has increased significantly over the last 20 years. However, there is growing concern that teenagers are consuming more caffeine, with potential negative effects on sleep behaviours and performance at school.

(a) A school was concerned about this issue and conducted an online, anonymous survey of 300 of its senior students. It asked if students drink coffee regularly (at least 3 times a week) and if they have any sleep-related issues. The results are summarised in the table below.

Table 1: Coffee drinking and sleep issues for school students

	Sleep issues	No sleep issues	Total
Drink coffee regularly	68	54	122
Don't drink coffee regularly	75	103	178
Total	143	157	300

(i)	According to the results of this survey, what proportion of the senior students drink coffee regularly?
(ii)	What is the probability that a randomly selected student from this survey who has sleep issues also drinks coffee regularly?

(iii)	Jack looked at the data for students with sleep issues and noticed that fewer of them drink coffee regularly. He claims that this means that it is less likely for coffee drinkers to have sleep issues than those who don't drink coffee.
	Explain Jack's reasoning AND justify why he is wrong.
	Include appropriate calculations to support your answer.

**Table 1: Coffee drinking and sleep issues for school students** (repeated from page 2)

	Sleep issues	No sleep issues	Total
Drink coffee regularly	68	54	122
Don't drink coffee regularly	75	103	178
Total	143	157	300

(b) It is well known that many of the popular energy drinks consumed by teenagers in New Zealand contain caffeine in even higher doses than coffee.

The same survey also asked about the consumption of energy drinks. It found that:

- 36 students didn't drink coffee but did drink energy drinks.
- Only 7 of these students (who drank energy drinks but not coffee) had no sleep issues.
- (i) Use these facts AND the information given in Table 1 to complete the table below.

**Table 2: Effect of caffeine on sleep for school students** 

	Have sleep issues	No sleep issues	Total
Drink coffee	68	54	122
Drink energy drinks (but not coffee)			
Don't drink either coffee or energy drinks			
Total	143	157	300

(ii)	If there were 850 senior students in the school who don't drink either coffee or energy drinks, how many would be expected to have sleep issues?

)	A newspaper article claimed that teenagers in New Zealand who consume drinks containing caffeine regularly are nearly twice as likely to have sleep-related issues as those who don't consume drinks containing caffeine.
	Discuss whether <b>this school's survey</b> provides sufficient evidence to support the validity of this claim.

#### **QUESTION TWO**

The use of plant-based "milks" in New Zealand has become much more popular in recent years. Cafés also now offer their customers a number of alternative milk options such as soy, almond, or coconut milk.

- (a) Jungwoo owns a café and has collected data of his coffee sales over the last few weeks to find out how much he should stock alternative milk products.
  - 40% of his coffee sales were to younger customers.
  - One quarter of younger customers requested an alternative milk type.
  - 80% of older customers ordered a coffee with cow's milk.
  - Of the younger customers who ordered a coffee with alternative milk, 15% ordered soy milk.
  - Half of the older customers who ordered a coffee with alternative milk, ordered soy milk.

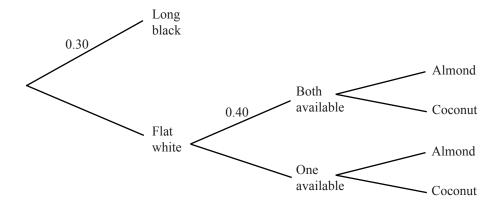
ind the proba	ability that a customer in this café is younger and orders a coffee with
ind the proba	ability that a randomly chosen customer orders coffee with soy milk.

)	Out of the customers who request an alternative milk, what percentage order a coffee with milk?
	The probability that a randomly selected customer ordered an almond milk coffee is 0.066 Of those who requested alternative milks, there were equal proportions of those ordering almond milk in each age range (younger or older), while the rest of the customers ordered coffee with coconut milk.
	How many times more likely is it for a customer to order a coffee with cow's milk than coconut milk?

(b) Kathy prefers to drink a flat white coffee (with milk) but is allergic to cow's milk and dislikes soy milk. When she goes to a café for the first time, if it does not have an alternative milk available, she orders a long black coffee (with no milk). She finds this happens about 30% of the time.

She has found that, out of cafés that offer alternative milks, about 40% have both almond milk and coconut milk available. If the café has both almond and coconut milk available, she flips a coin and chooses either one randomly.

Kathy has also noticed that if a café has only one non-soy alternative milk available, it is twice as likely to be almond as coconut milk.



Overall, is it more likely for Kathy to have a long black, an almond-milk flat white, or a coconut-milk flat white?

Support your answer with relevant calcula	ations.	

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## **QUESTION THREE**

A café manager notices that the amount of time a customer waits to receive their takeaway coffee varies. He collects some data over a week and finds the mean time waited is 7 minutes, with an estimated standard deviation of 1.5 minutes.

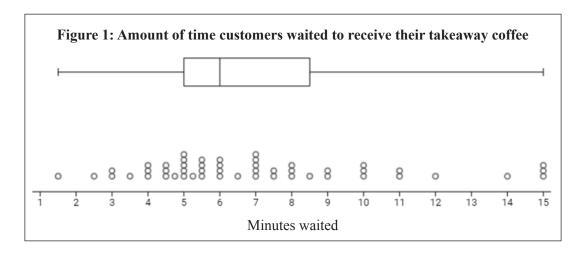
Assume that a normal distribution can be used to model the waiting time for a takeaway coffee at this café.

Working and/or diagrams must be shown.

a)	Find the probability that a customer receives their coffee within 5 minutes.
)	One day, the café has 150 customers who order takeaway coffee over the afternoon period.
	How many of these would you expect to have to wait for between 8 and 10 minutes for their takeaway coffee?

(c)	The manager noticed that when a particular staff member was rostered on, 30% of customers waited less than 5 minutes to receive their coffee. The mean time this staff member takes to make and deliver a customer's coffee is still approximately 7 minutes.			
	The manager suggested this means the staff member should be rewarded for good service.			
	Use the facts above to calculate an estimate of the standard deviation of time taken to receive a coffee from this staff member, and discuss if you agree with the manager's suggestion.			

(d) Some of the data that the café manager collected over the past week are shown in Figure 1 below. (n = 50)



(i) Calculate the median, quartiles, and interquartile range for the data in Figure 1 and for a normal distribution model with mean of 7 minutes and standard deviation of 1.5 minutes. Enter your answers in the table below.

Statistic	Data from Figure 1	Normal model
Median		
Lower quartile		
Upper quartile		
Interquartile range		

(ii)	At the start of this question you are told to assume that a normal distribution model (with mean 7 minutes and standard deviation 1.5 minutes) can be used to model the waiting times for a takeaway coffee at this café.  Is this assumption appropriate?  Use the numerical evidence calculated in part (i) and refer to the centre, spread, and shape of the data in Figure 1 to support your answer.		

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QUESTION NUMBER	With the question number(s) if applicable.	
NUMBER		

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