See back cover for an English translation of this cover

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91578M

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Tohua tēnei pouaka mēnā KĀORE koe i tuhituhi i roto i tēnei pukapuka

Tuanaki, Kaupae 3, 2021

91578M Te whakahāngai i ngā tikanga pārōnaki hei whakaoti rapanga

Ngā whiwhinga: Ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pārōnaki hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pārōnaki mā te whakaaro tuhonohono hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pārōnaki mā te whakaaro waitara hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tuhia ō mahinga KATOA.

Tirohia mēnā kei a koe te Pukapuka o ngā Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Ki te hiahia koe ki ētahi atu wāhi hei tuhituhi whakautu, whakamahia te wāhi wātea kei muri i te pukapuka nei.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–21 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi ki roto i tētahi wāhi kairuku whakahāngai (). Ka tapahia pea tēnei wāhi ina mākahia te pukapuka.

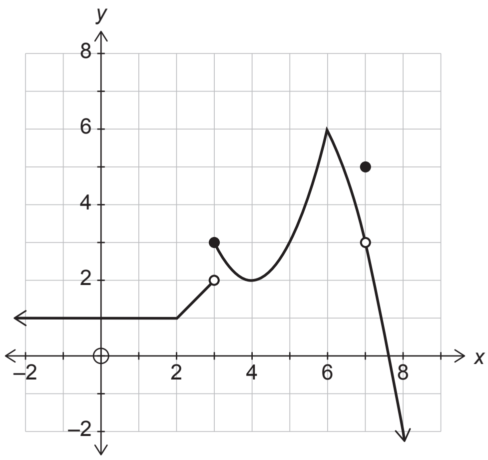
ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

TŪMAHI TUATAHI

(a) Kimi pāronaki mō $y = e^{3x}\sin 2x$.

Hei aha noa te whakarūnā i tō whakautu.

(b) E whakaatu ana te kauwhata i raro nei i te pānga $y = f(x)$.



Mō te pānga i runga ake:

(i) Whiriwhiria te (ngā) uara mō x e ū ana ki ēnei whakaritenga e whai ake:

(1) $f'(x) = 0$:

(2) $f(x)$ he kōpapa whakarunga:

(ii) He aha te uara o $\lim\limits_{x \to 7} f(x)$:

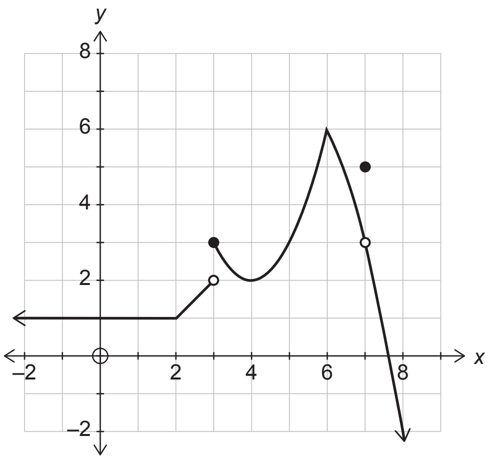
Kia mārama te kōrero mēnā kore rawa he uara.

QUESTION ONE

(a) Differentiate $y = e^{3x}\sin 2x$.

You do not need to simplify your answer.

(b) The graph below shows the function $y = f(x)$.



For the function above:

(i) Find the value(s) of x that meet the following conditions:

(1) $f'(x) = 0$:

(2) $f(x)$ is concave upwards:

(ii) What is the value of $\lim\limits_{x\to 7} f(x)$:

State clearly if the value does not exist.

(c) Ko te whārite mō tētahi kōpiko he $y=(2x+3)e^{x^2}$.

Whiriwhiria te (ngā) taunga-x o tētahi (ētahi) pūwāhi tū i te kōpiko.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

(d) E tautuhia tawhātia ana tētahi kōpiko mā ngā whārite $x=t^2+3t$ me te $y=t^2\ln(2t-3)$, mō $t>\dfrac{3}{2}$.

Whiriwhiria te rōnaki o te pātapa o te kōpiko i te pūwāhi (10,0).

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

(c) A curve has the equation $y = (2x+3)e^{x^2}$.

Find the x-coordinate(s) of any stationary point(s) on the curve.

You must use calculus and show any derivatives that you need to find when solving this problem.

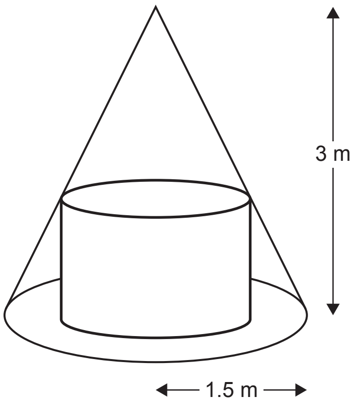
(d) A curve is defined parametrically by the equations $x = t^2 + 3t$ and $y = t^2 \ln(2t-3)$, for $t > \frac{3}{2}$.

Find the gradient of the tangent to the curve at the point (10,0).

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) He 3 m te teitei o te koeko, ā, he 1.5 m te pūtoro.

Kua tuhia he rango i roto i te koeko e whakaaturia ana i te hoahoa o raro.



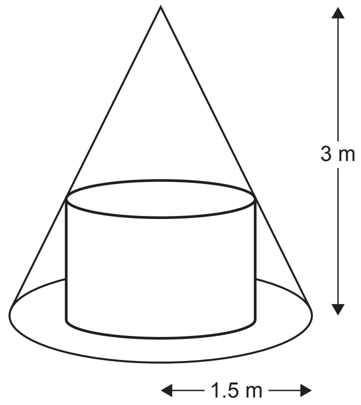
He ōrite te pokapū o te pūtake o te rango ki te pūtake o te koeko.

Hāponotia ko te rōrahi mōrahi o te rango he π m^3.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

(e) A cone has a height of 3 m and a radius of 1.5 m.

A cylinder is inscribed in the cone, as shown in the diagram below.



The base of the cylinder has the same centre as the base of the cone.

Prove that the maximum volume of the cylinder is π m^3.

You must use calculus and show any derivatives that you need to find when solving this problem.

TŪMAHI TUARUA

(a) Kimi pāronaki mō $f(x)=\left(1-x^2\right)^5$.

Hei aha noa te whakarūnā i tō whakautu.

(b) Ko te whārite mō tētahi kōpiko he $y=\dfrac{x^2}{x+1}$.

Whiriwhiria te (ngā) taunga-x o tētahi (ētahi) pūwāhi tū i te kōpiko.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

(c) Ko te whārite o tētahi kōpiko ko $y = (x^2 + 3x + 2)\cos 3x$.

Whiriwhiria te whārite o te rārangi hāngai ki te kōpiko i te pūwāhi e pātahi ana te kōpiko ki te tuaka-y.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

QUESTION TWO

(a) Differentiate $f(x) = \left(1 - x^2\right)^5$.

You do not need to simplify your answer.

(b) A curve has the equation $y = \dfrac{x^2}{x+1}$.

Find the x-coordinate(s) of any stationary point(s) on the curve.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) A curve has the equation $y = (x^2 + 3x + 2)\cos 3x$.

Find the equation of the normal to the curve at the point where the curve crosses the y-axis.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) E piki haere ana te rōrahi o tētahi poihau rite i te pāpātanga pūmau o te 60 cm^3 i te hēkona.

Tātaihia te pāpātanga whakapiki o te pūtoro ina ko te pūtoro he 15 cm.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

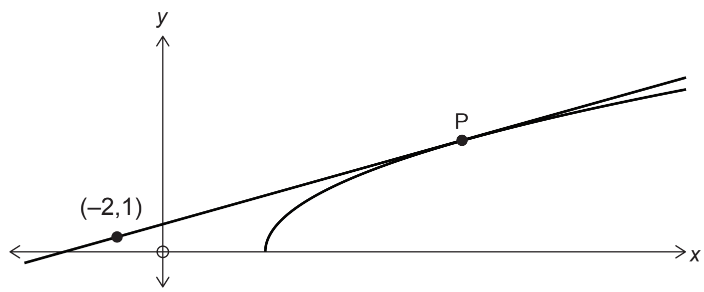
(d) The volume of a spherical balloon is increasing at a constant rate of 60 cm^3 per second.

Find the rate of increase of the radius when the radius is 15 cm.

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) E whakaatu ana te kauwhata i raro nei i te kōpiko $y = \sqrt{2x - 4}$, me te pātapa ki taua kōpiko i te pūwāhi P.

Ka pā te pātapa ki te pūwāhi (-2,1).

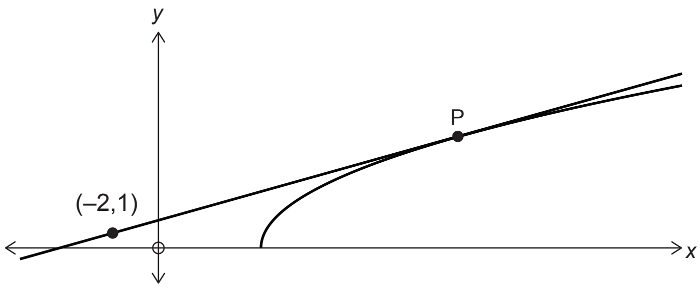


Whiriwhiria ngā taunga o te pūwāhi P.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

(e) The graph below shows the curve $y = \sqrt{2x - 4}$, and the tangent to the curve at point P.

The tangent passes through the point (–2,1).



Find the coordinates of point P.

You must use calculus and show any derivatives that you need to find when solving this problem.

TŪMAHI TUATORU

(a) Kimi pāronaki mō $y = \dfrac{\cot x}{x^2 + 1}$.

Hei aha noa te whakarūnā i tō whakautu.

(b) Kei te kauwhata o te pānga $y = 4\sqrt{x} - x + 2$, ina ko $x > 0$, he pūwāhi tū noa i te pūwāhi Q.

Whiriwhiria ngā taunga o te pūwāhi Q.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

(c) Mō ēhea uara o x e piki ai te pānga $y = \dfrac{x}{x^2 + 4}$?

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

QUESTION THREE

(a) Differentiate $y = \dfrac{\cot x}{x^2 + 1}$.

You do not need to simplify your answer.

(b) The graph of the function $y = 4\sqrt{x} - x + 2$, where $x > 0$, has a stationary point at point Q.

Find the coordinates of point Q.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) For what values of x is the function $y = \dfrac{x}{x^2 + 4}$ increasing?

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) Kei tētahi kōpiko te whārite $y = \dfrac{4x+k}{4x-k}$, ina ko k he uara pūmau, ā, ko $x \neq \dfrac{k}{4}$.

Ka noho te pūwāhi P i te kōpiko, ā, ko 3 te taunga-x.

Ko te rōnaki o te pātapa ki te kōpiko i te pūwāhi P he $\dfrac{-8}{27}$.

Tātaihia te (ngā) uara ka taea mō k.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

Ka haere tonu te Tūmahi Tuatoru i te whārangi 18.

(d) A curve has the equation $y = \dfrac{4x+k}{4x-k}$, where k is a constant and $x \neq \dfrac{k}{4}$.

The point P lies on the curve and has an x-coordinate of 3.

The gradient of the tangent to the curve at P is $\dfrac{-8}{27}$.

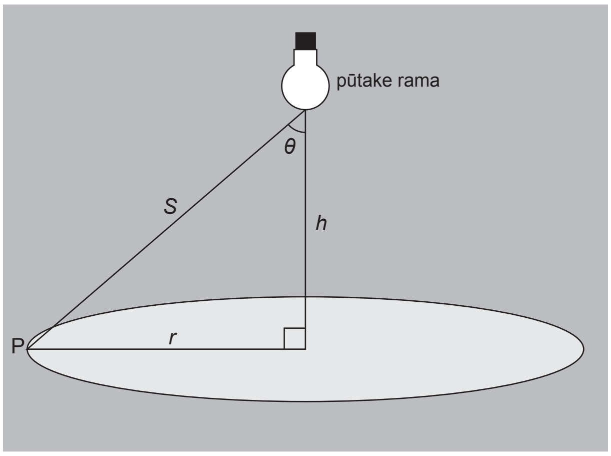
Find the possible value(s) of k.

You must use calculus and show any derivatives that you need to find when solving this problem.

Question Three continues on page 19.

(e) Ka whakatārewahia he rama i runga ake o te pokapū o tētahi tēpu porohita me te pūtoro r.

Ka taea te whakatika te teitei, h, o te rama i runga ake o te tēpu.



Kei runga te pūwāhi P i te paenga o te tēpu.

I te pūwāhi P he pānga riterite tō te whakamārama I ki te whenu o te koki θ i te hoahoa i runga nei, ā, he kōaro te pānga ki te pūrua o te tawhiti, S, mai i te rama.

arā, $I = \dfrac{k\cos\theta}{S^2}$, ko k he uara pūmau.

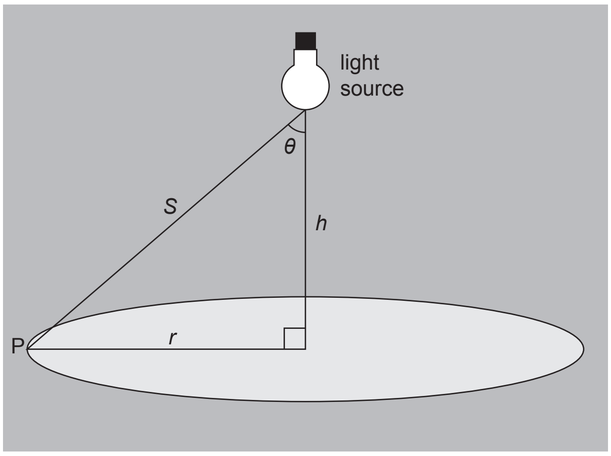
Hāponotia he mōrahi te whakamārama i te paenga o te tēpu ina $h = \dfrac{r}{\sqrt{2}}$.

Ehara i te mea me hāpono koe ka whakaputa tō otinga i te uara mōrahi.

Me mātua whakamahi te tuanaki me te whakaatu i ngā pārōnaki me rapu e koe ina whakaoti i tēnei rapanga.

(e) A lamp is suspended above the centre of a round table of radius r.

The height, h, of the lamp above the table is adjustable.



Point P is on the edge of the table.

At point P the illumination I is directly proportional to the cosine of angle θ in the above diagram, and inversely proportional to the square of the distance, S, to the lamp.

i.e. $I = \dfrac{k\cos\theta}{S^2}$, where k is a constant.

Prove that the edge of the table will have maximum illumination when $h = \dfrac{r}{\sqrt{2}}$.

You do not need to prove that your solution gives the maximum value.

You must use calculus and show any derivatives that you need to find when solving this problem.

He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

English translation of the wording on the front cover

Level 3 Calculus 2021

91578M Apply differentiation methods in solving problems

Credits: Six

91578M

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–21 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area ( ). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.