See back cover for an English translation of this cover

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91579M

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Tohua tēnei pouaka mēnā KĀORE koe i tuhituhi i roto i tēnei pukapuka

Tuanaki, Kaupae 3, 2021

91579M Te whakahāngai i ngā tikanga pāwhaitua hei whakaoti rapanga

Ngā whiwhinga: Ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pāwhaitua hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pāwhaitua mā te whakaaro tuhonohono hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pāwhaitua mā te whakaaro waitara hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tuhia ō mahinga KATOA.

Tirohia mēnā kei a koe te Pukapuka o ngā Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Ki te hiahia koe ki ētahi atu wāhi hei tuhituhi whakautu, whakamahia te wāhi wātea kei muri i te pukapuka nei.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–21 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi ki roto i tētahi wāhi kairuku whakahāngai (). Ka tapahia pea tēnei wāhi ina mākahia te pukapuka.

ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

TŪMAHI TUATAHI

(a) Whiriwhiria $\int\left(\frac{x}{3}+\frac{3}{x}\right)\mathrm{d}x$.

(b) Ko te pānga rōnaki o tētahi kōpiko ko $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{8}{x^3}$.

(i) Whiriwhiria te whārite o te kōpiko mēnā ka pā mā te pūwāhi (1,3).

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

(ii) Tātaihia te horahanga e rohea ana e te kōpiko, te tuaka-x, me ngā rārangi $x = 1$ me $x = 2$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

QUESTION ONE

(a) Find $\int\left(\frac{x}{3}+\frac{3}{x}\right)\mathrm{d}x$.

(b) The gradient function of a curve is $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{8}{x^3}$.

(i) Find the equation of the curve if it passes through the point (1,3).

You must use calculus and show the results of any integration needed to solve the problem.

(ii) Find the area enclosed by the curve, the x-axis, and the lines $x = 1$ and $x = 2$.

You must use calculus and show the results of any integration needed to solve the problem.

(c) Ka taea te nekehanga o tētahi ahanoa te whakatauira mā te whārite pārōnaki $a(t) = 2 - \sin 2t$, ina $t \geq 0$.

ko a te whakaterenga ake o te ahanoa, i te m s^{-2}

ā, ko t te wā ki te hēkona.

I $t = 0$, ko te tere o te ahanoa he 1 m s^{-1}, ā, ko te tawhiti i nekehia ai te ahanoa he 3 m.

He aha te tawhiti i nekehia ai te ahanoa i te wā $t = 5$?

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

(c) An object's motion can be modelled by the differential equation $a(t) = 2 - \sin 2t$, where $t \geq 0$.

a is the acceleration of the object, in m s^{-2}

and t is time in seconds.

At $t = 0$, the object has a velocity of 1 m s^{-1} and a displacement of 3 m.

What is the displacement of the object at time $t = 5$?

You must use calculus and show the results of any integration needed to solve the problem.

(d) I turuturu tētahi taika wai.

I te 6 haora mai i te turutanga o te taika, ko te rōrahi o te wai i roto i te taika he 400 rita.

I te 10 haora mai i te turutanga o te taika, ko te rōrahi o te wai i roto i te taika he 256 rita.

Ko te pāpātanga o te turuturu mai o te wai i te taika i tētahi wā he pānga riterite ki te pūtakerua o te rōrahi o te wai i te taika i taua wā tonu.

E hia te nui o te wai i roto i te taika i te wā tonu i tīmata ki te turuturu?

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

(d) A water tank developed a leak.

6 hours after the tank started to leak, the volume of water in the tank was 400 litres.

10 hours after the tank started to leak, the volume of water in the tank was 256 litres.

The rate at which the water leaks out of the tank at any instant is proportional to the square root of the volume of the water in the tank at that instant.

How much water was in the tank at the instant it started to leak?

You must use calculus and show the results of any integration needed to solve the problem.

TŪMAHI TUARUA

(a) Whiriwhiria $\int\left(e^{4x}+4\sqrt{x}\right)dx$.

(b) Mēnā ko $\int_{1}^{5} h(x)\,dx = 6$ he aha te uara o $\int_{1}^{5}\left(h(x)+2\right)dx$?

(c) Whiriwhiria $\int_{0}^{\frac{\pi}{8}} \sin 6x \sin 2x\,dx$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

QUESTION TWO

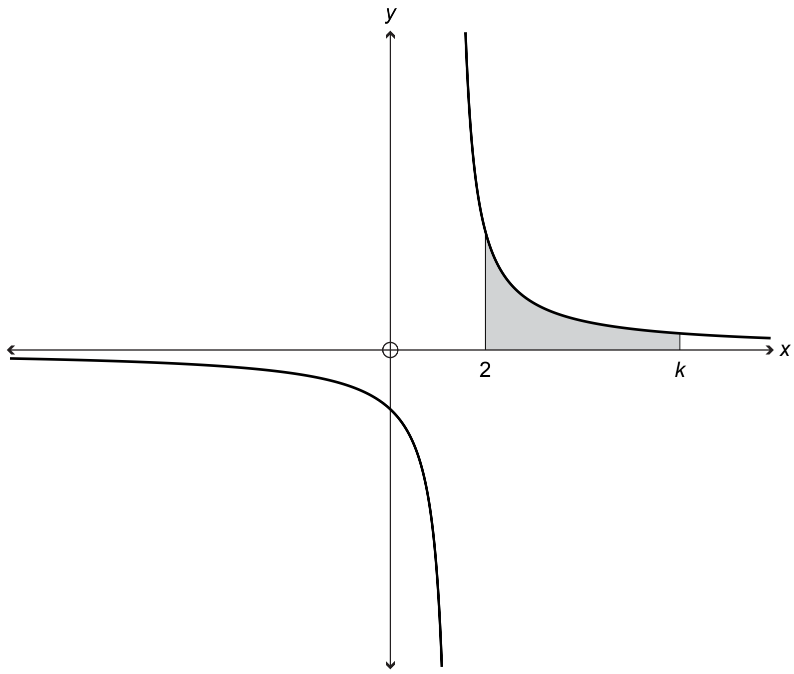
(a) Find $\int \left(e^{4x} + 4\sqrt{x}\right) dx$.

(b) If $\int_1^5 h(x)\,dx = 6$ what is the value of $\int_1^5 \left(h(x) + 2\right) dx$?

(c) Find $\int_0^{\frac{\pi}{8}} \sin 6x \sin 2x \, dx$.

You must use calculus and show the results of any integration needed to solve the problem.

(d) E whakaatu ana te hoahoa i raro nei i tētahi wāhi o te kauwhata o te pānga $g(x) = \dfrac{6}{3x-4}$.

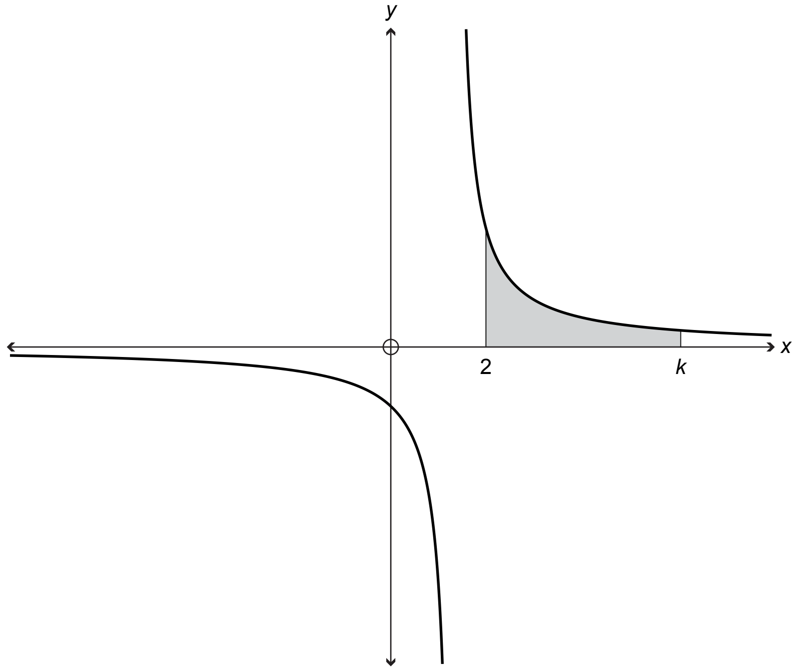


Ko te horahanga o te wāhi kauruku he 4.

Whiriwhiria te uara o k.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

(d) The diagram below shows part of the graph of the function $g(x) = \dfrac{6}{3x-4}$.



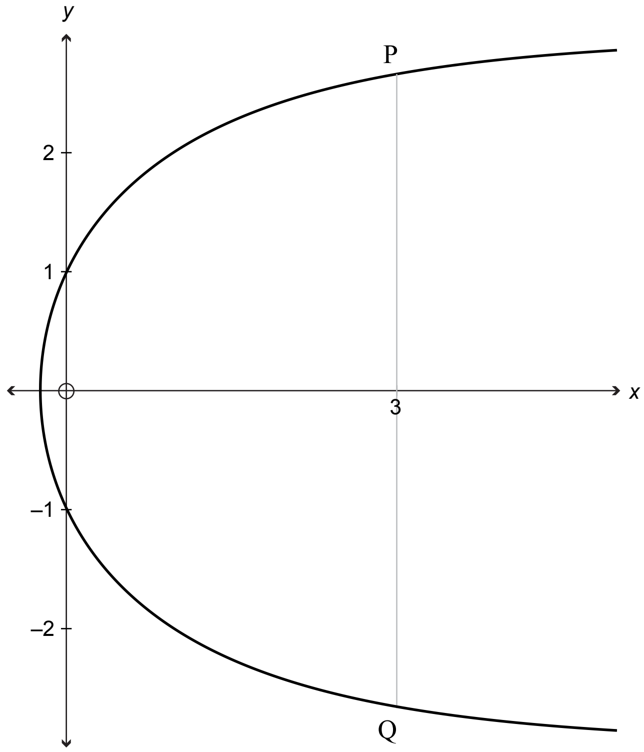
The area of the shaded region is 4.

Find the value of k.

You must use calculus and show the results of any integration needed to solve the problem.

(e) E whakaatu ana te hoahoa i raro nei i te kauwhata o tētahi kōpiko $y = f(x)$,

ka ū ki te whārite pārōnaki $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{ye^{0.5x}}$.



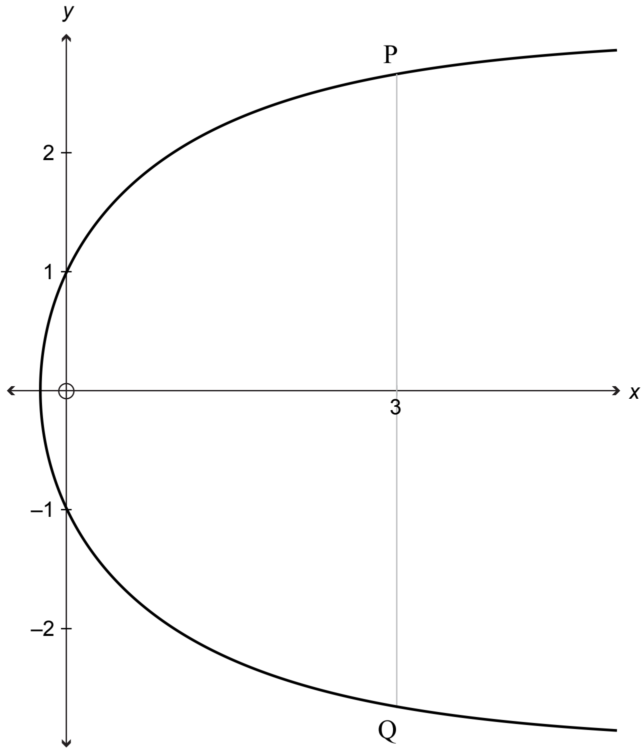
Ko ngā pūwāhi P me Q he pūwāhi i te kauwhata o te kōpiko ko 3 ngā taunga-x.

He aha te tawhiti poutū i waenga i ngā pūwāhi P me Q?

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

(e) The diagram below shows the graph of a curve $y = f(x)$,

which satisfies the differential equation $\frac{dy}{dx} = \frac{2}{ye^{0.5x}}$.



Points P and Q are the points on the graph of the curve that have x-coordinates of 3.

What is the vertical distance between points P and Q?

You must use calculus and show the results of any integration needed to solve the problem.

TŪMAHI TUATORU

(a) Whiriwhiria $\int\left(x+\sqrt{x}\right)^2 \mathrm{d}x$.

(b) Whakamahia ngā uara i te papatau i raro hei kimi i tētahi āwhiwhitanga ki $\int_1^{2.5} f(x)\,\mathrm{d}x$, mā te whakamahi i te Ture Taparara.

x	1	1.25	1.5	1.75	2	2.25	2.5
$f(x)$	0.8	1.1	1.5	1.9	2.2	2.1	2.4

(c) Whakaaroarotia te whārite pārōnaki $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\sec^2 2x}{y}$.

Mēnā ko $y = 2$ ina $x=\frac{3\pi}{8}$, tātaihia te (ngā) uara o y ina $x=\pi$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

QUESTION THREE

(a) Find $\int\left(x+\sqrt{x}\right)^2 \mathrm{d}x$.

(b) Use the values given in the table below to find an approximation to $\int_1^{2.5} f(x)\,\mathrm{d}x$ using the Trapezium Rule.

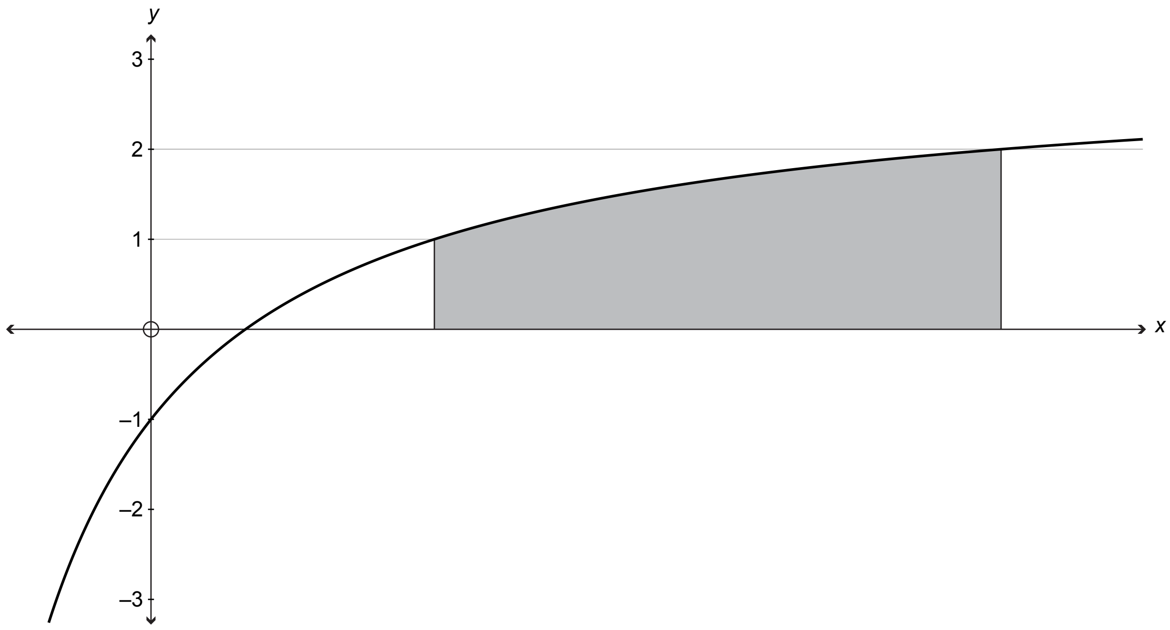
x	1	1.25	1.5	1.75	2	2.25	2.5
$f(x)$	0.8	1.1	1.5	1.9	2.2	2.1	2.4

(c) Consider the differential equation $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\sec^2 2x}{y}$.

Given that $y = 2$ when $x=\dfrac{3\pi}{8}$, find the value(s) of y when $x=\pi$.

You must use calculus and show the results of any integration needed to solve the problem.

(d) E whakaatu ana te hoahoa i raro nei i tētahi wāhi o te kauwhata o te pānga $y = \dfrac{3x-2}{x+2}$.

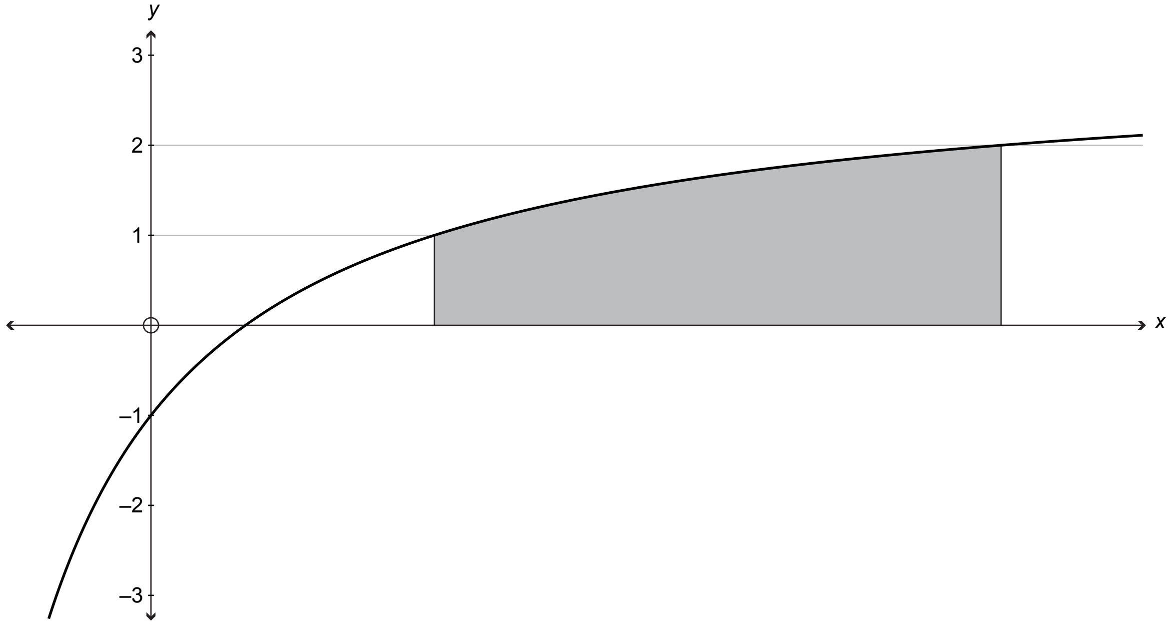


Tātaihia te horahanga o te wāhi kauruku.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

Ka haere tonu te Tūmahi Tuatoru i te whārangi 18.

(d) The diagram below shows part of the graph of the function $y = \dfrac{3x-2}{x+2}$.

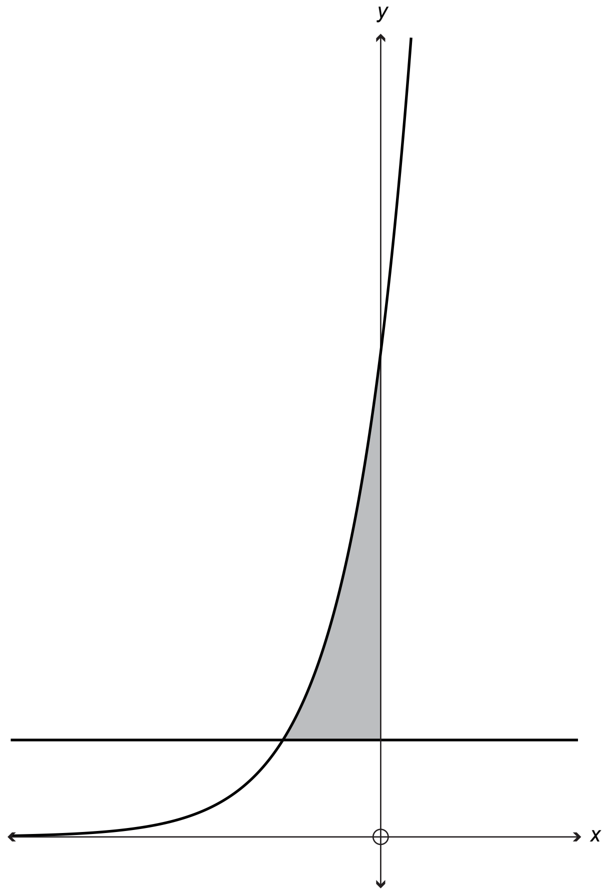


Find the shaded area.

You must use calculus and show the results of any integration needed to solve the problem.

Question Three continues on page 19.

(e) E whakaatu ana te kauwhata i raro nei i ngā pānga $y = \left(ke^{x}\right)^{2}$ me $y = k$, ina ko k he pūmau nui atu i te 1.

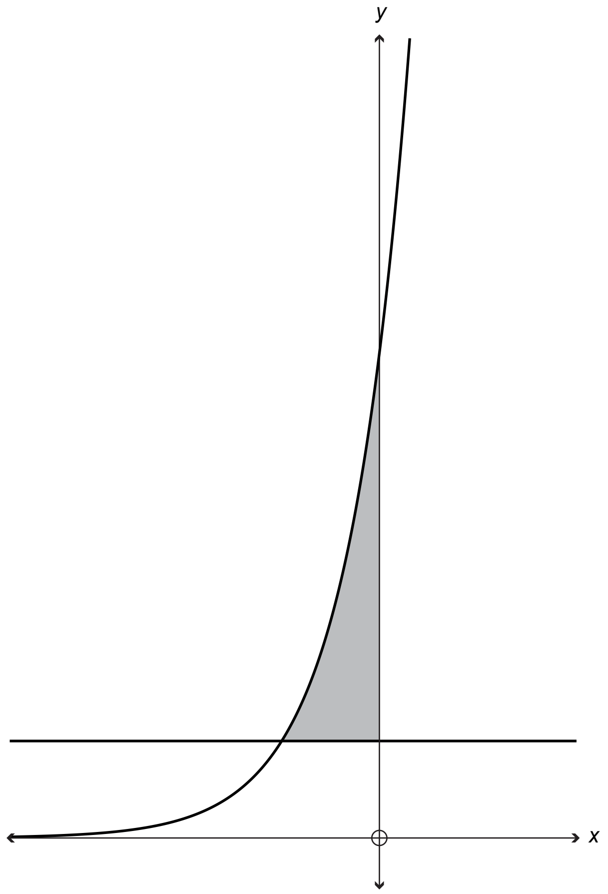


Me whakaatu ko te wāhi kauruku he $\frac{k}{2}\left(k - 1 + \ln\frac{1}{k}\right)$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

Whakaaturia āu mahinga katoa.

(e) The graph below shows the functions $y=\left(ke^{x}\right)^{2}$ and $y=k,$ where k is a constant greater than 1.



Show that the shaded area is $\dfrac{k}{2}\left(k-1+\ln\dfrac{1}{k}\right).$

You must use calculus and show the results of any integration needed to solve the problem.

Clearly show each step of your working.

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

English translation of the wording on the front cover

Level 3 Calculus 2021

91579M Apply integration methods in solving problems

Credits: Six

91579M

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2 – 21 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area ( ). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.